Range Identification for Perspective Dynamic Systems using Nonlinear Observers

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Contents

1. Background Information

Nonlinear Observers

For a general nonlinear system, observer design techniques include:

1) Assuming bounded or local/global Lipschitz conditions.

2) Lyapunov based design.

3) Linear approximation based technique.

4) Via transformations.

Observer Canonical Form

Consider a nonlinear system in the form of

$$\begin{aligned} \dot{x} &= Ax + a(y,t), \\ y &= Cx. \end{aligned} \tag{1}$$

The nonlinear observer:

$$\dot{\hat{x}} = A\hat{x} + a(y,t) + K(y - c\hat{x}), \tag{2}$$

such that the error dynamics is:

$$\dot{e} = (A - kC)e. \tag{3}$$

Perspective Dynamic Systems (PDS)

A typical PDS consists of:

1) Motion Dynamics (such as the following affine motion):

$$\begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \quad (4)$$

2) Homogeneous Output Function:

$$y_1(t) = \frac{X(t)}{Z(t)}, \quad y_2(t) = \frac{Y(t)}{Z(t)}. \quad (5)$$

Example: vision-based sensing and perception.

Define $y_3(t) = 1/Z(t)$. The derivative of $y(t) = [y_1, y_2, y_3]$ is:

$$\begin{cases} \dot{y}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \qquad\qquad - a_{32}y_1y_2 + (b_1 - b_3y_1)y_3, \\ \dot{y}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \qquad\qquad - a_{32}y_2^2 + (b_2 - b_3y_2)y_3, \\ \dot{y}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})y_3 - b_3y_3^2. \end{cases} \tag{6}$$

Range Identification of a PDS

Two important issues in the PDS include:

1) **Range Identification**: Assuming that the motion parameters $a_{i,j}$ and b_i for $i, j = 1, 2, 3$ are known, to estimate the position of an object with unknown initial condition from observations on the imaging surface.

2) **Motion Estimation**: Assuming that the motion parameters $a_{i,j}$ and b_i for $i, j = 1, 2, 3$ are unknown, to identify the motion parameters to the extent possible from observations on the imaging surface.

2. Existing Nonlinear Observers for PDS

The following three nonlinear observers have been proposed in the literature:

1) The **IBO** that is motivated from the adaptive control theory.

2) A state observer (**SMO**), which is a combination of the sliding mode control method, the adaptive method, and the discontinuous observer techniques.

3) A range identification observer (**RIO**) that facilitates a Lyapunov-based analysis and is motivated from the recent disturbance observer results.

The above three observers are all **Lyapunov function based** and assume **local Lipschitz condition**.

Systems that the IBO is applicable

IBO is suitable for the following class of nonlinear systems:

$$\begin{cases} \dot{x}_1 = w^T(x_1, u)x_2 + \phi(x_1, u), \\ \dot{x}_2 = g(x_1, x_2, u), \\ \ y = x_1, \end{cases} \tag{7}$$

where the matrix $w^T(x_1, u)$ and the vector $g(x_1, x_2, u)$ are in general nonlinear functions of their parameters.

IBO Observer

$$\begin{cases} \dot{\hat{x}}_1 = GA(\hat{x}_1 - x_1) + w^T(x_1, u)\hat{x}_2 + \phi(x_1, u), \\ \dot{\hat{x}}_2 = -G^2 w(x_1, u)P(\hat{x}_1 - x_1) + g(x_1, \hat{x}_2, u), \\ \hat{x}(t_i^+) = M\dfrac{\hat{x}(t_i^-)}{\|\hat{x}(t_i^-)\|}, \end{cases} \tag{8}$$

where the sequences of t_i is defined as

$$t_i = \min\{t : t > t_{i-1} \text{ and } \|\hat{x}(t)\| \geq \gamma M\}. \tag{9}$$

and the matrix P is a positive definite solution of the Lyapunov equation $A^T P + PA = -Q$. In (9), scalar M is an assumed upper bound for the state estimate $\|\hat{x}(t)\|$ and γ is a fixed constant scalar with $\gamma > 1$. The G in (8) is a constant scalar gain.

3. Linear Approximation of Nonlinear Systems

Replace a nonlinear system by a sequence of LTV approximations

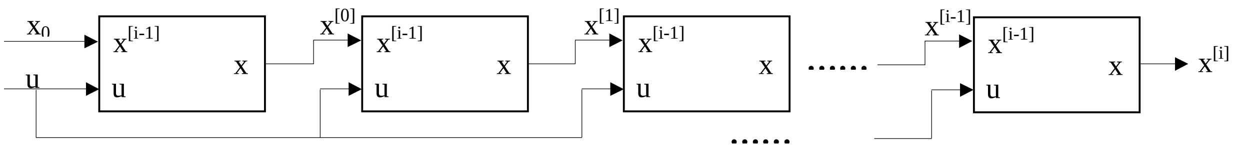
- Given a nonlinear system:

$$\begin{aligned} \dot{x}(t) &= A(x)x(t) + B(x)u(t), \\ y(t) &= C(x)x(t), \end{aligned} \tag{10}$$

- LTV approximations:

$$\begin{cases} \dot{x}^{[0]}(t) = A(x_0)\, x^{[0]}(t) + B(x_0)\, u^{[0]}(t), \\ \dot{x}^{[i]}(t) = A(x^{[i-1]}(t))\, x^{[i]}(t) + B(x^{[i-1]}(t))\, u^{[i]}(t), \\ x^{[0]}(t) = x_0, \text{ for } i = 0, \quad x^{[i]}(t) = x_0, \text{ for } i \geq 1. \end{cases} \tag{11}$$

- Block Diagram:



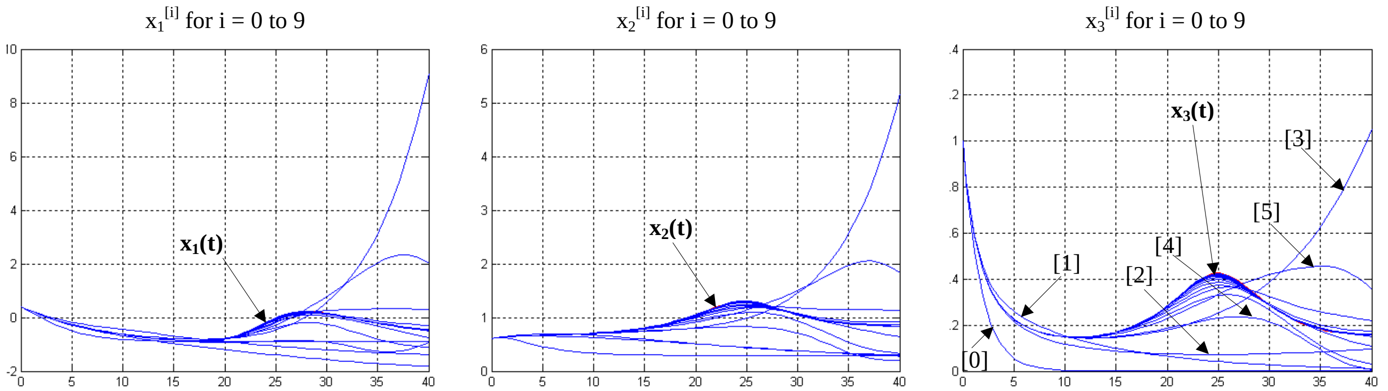
Example Illustration

Motion Dynamics:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} -0.2301 & 0.4043 & -0.5769 \\ 0.1276 & -0.3003 & 0.2839 \\ 0.2450 & -0.4247 & 0.4319 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \\ 0.3 \end{bmatrix}, \qquad (12)$$

$$[X_0, Y_0, Z_0]^T = [0.4, 0.6, 1.0]^T.$$

Using linear approximations:



(a) (b) (c)

4. LAO Observer

Linear Approximation Based Nonlinear Observer

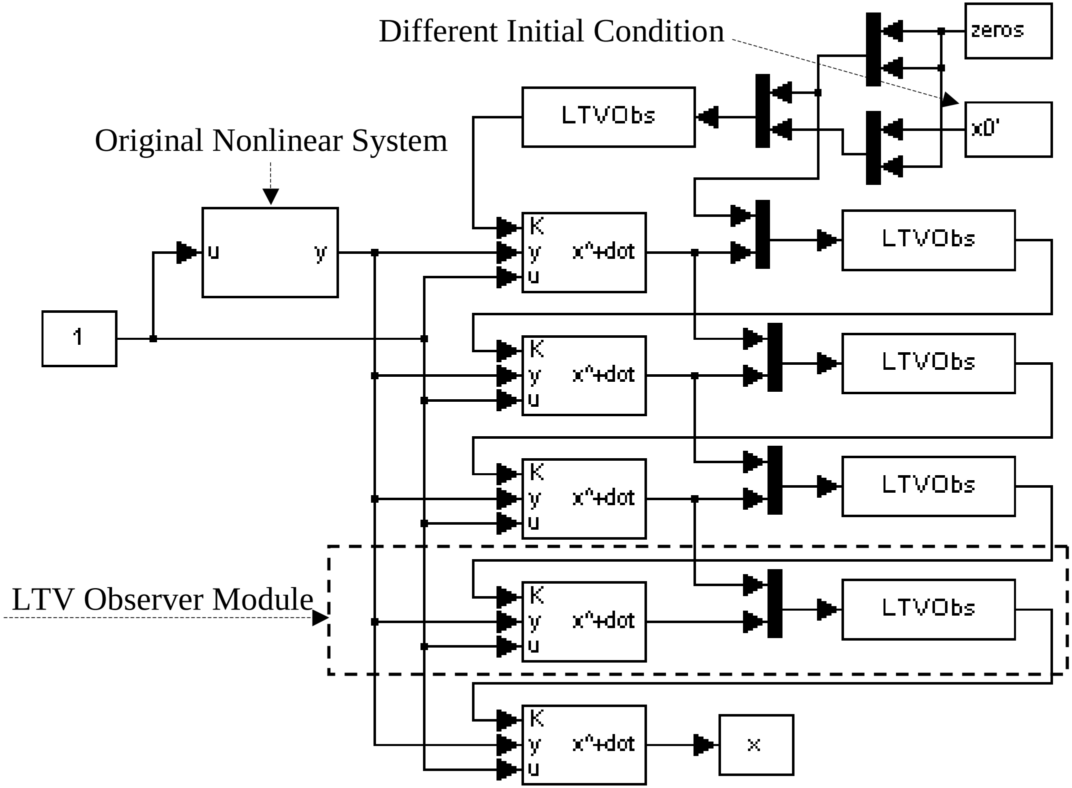
$$\text{LAO}\begin{cases} \dot{\hat{\bar{x}}}^{[0]}(t) = \bar{F}\,\hat{\bar{x}}^{[0]}(t) + \bar{G}(x_0')\,y(t) + \bar{B}\,u(t), \\ \dot{\hat{\bar{x}}}^{[i]}(t) = \bar{F}\,\hat{\bar{x}}^{[i]}(t) + \bar{G}(\hat{x}^{[i-1]})\,y(t) + \bar{B}\,u(t), \\ x^{[0]}(0) = x_0',\ \text{for } i = 0, \quad x^{[i]}(0) = x_0',\ \text{for } i \geq 1, \end{cases} \tag{13}$$

with

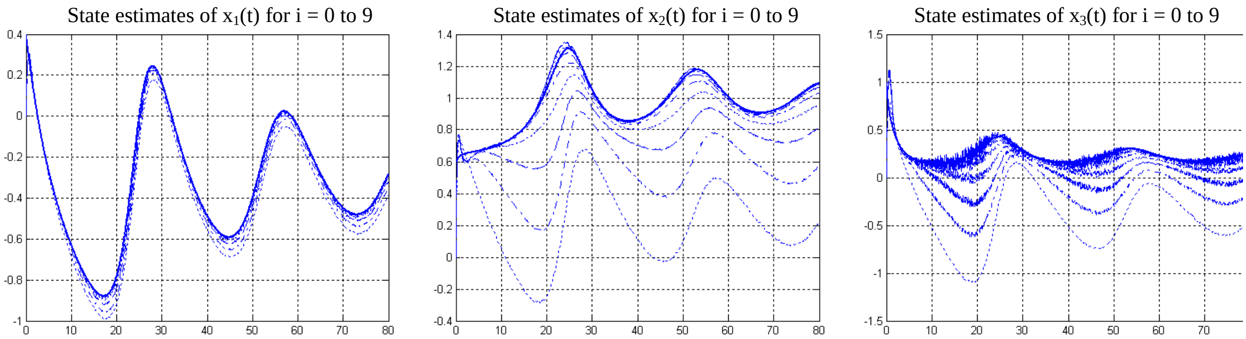
$$\hat{x}^{[i]}(t) = P(t)\,\hat{\bar{x}}^{[i]}(t), \quad \text{for } i \geq 0,$$

where $u(t)$ and $y(t)$ are the input and output of the original nonlinear system. $\hat{x}^{[i-1]}(t)$ is the state estimation from the (i-1)-th subsystem.

Simulation Diagram



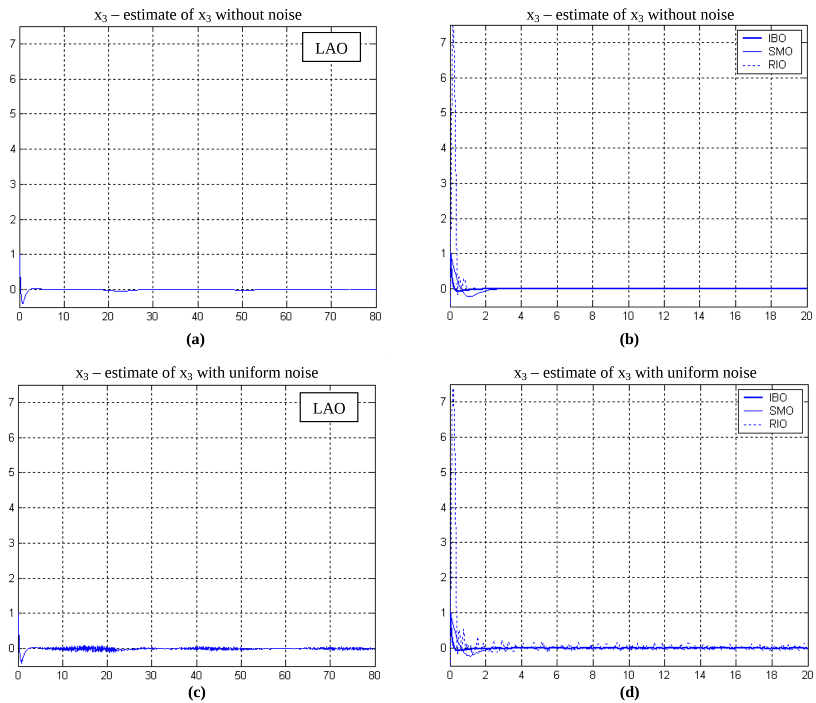
Simulation Results



(a) **(b)** **(c)**

$\hat{x}^{[i]}_{1,2,3}(t)$ with uniform noise bounded by $\pm 10^{-2}$

Comparisons of the Four Observers

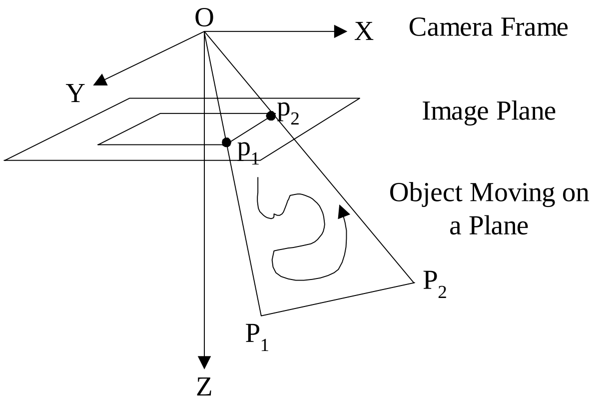


(a, b) without noise; (c, d) with uniform noise bounded by $\pm 10^{-2}$

5. Range Identification with Single Homogeneous Output

Motivation

Consider a special situation when an object is moving on a plane P_1OP_2, whose projection on the image plane is a line p_1p_2 that has either a constant $y_1(t)$ or a constant $y_2(t)$.



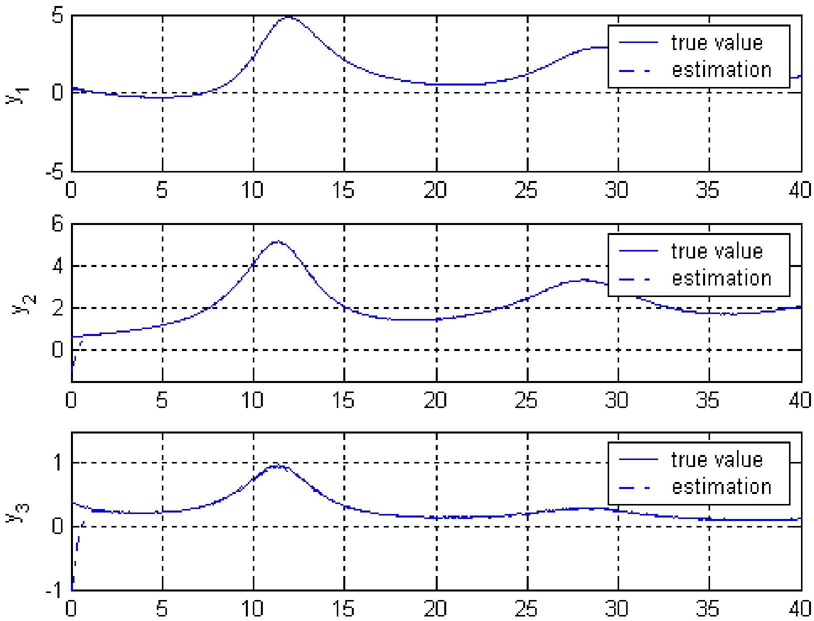
$\text{IBO}_{y_1+y_2}$

$$
\text{IBO}_{y_1+y_2}: \begin{cases}
\begin{bmatrix} \dot{\hat{y}}_1 \\ \dot{\hat{y}}_2 \end{bmatrix} = G\,A \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} b_1 - b_3 y_1 \\ b_2 - b_3 y_2 \end{bmatrix} \hat{y}_3 \\
\quad + \begin{bmatrix} a_{13} + (a_{11} - a_{33}) y_1 + a_{12} y_2 \\ a_{23} + a_{21} y_1 + (a_{22} - a_{33}) y_2 \end{bmatrix} \\
\quad - \begin{bmatrix} a_{31} y_1^2 + a_{32} y_1 y_2 \\ a_{31} y_1 y_2 + a_{32} y_2^2 \end{bmatrix}, \\
\dot{\hat{y}}_3 = -G^2 \begin{bmatrix} b_1 - b_3 y_1 & b_2 - b_3 y_2 \end{bmatrix} P \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\
\quad -(a_{31} y_1 + a_{32} y_2 + a_{33}) \hat{y}_3 - b_3 \hat{y}_3^2,
\end{cases} \tag{14}
$$

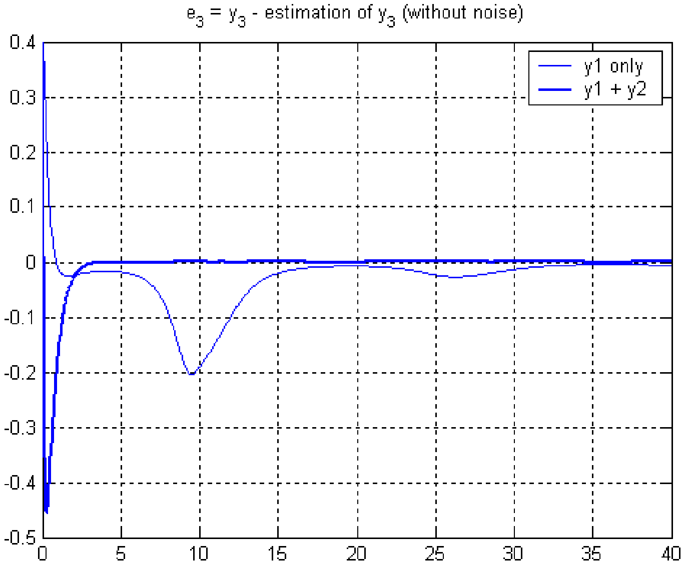
IBO y_1

$$
\underset{y_1}{\mathrm{IBO}}:\left\{\begin{aligned}
\dot{\hat{y}}_1 &= GA\,e_1 + [a_{12}-a_{32}y_1,\ b_1-b_3y_1]\begin{bmatrix}\hat{y}_2\\ \hat{y}_3\end{bmatrix} \\
&\quad + [a_{13}+(a_{11}-a_{33})y_1 - a_{31}y_1^2]\,, \\
\begin{bmatrix}\dot{\hat{y}}_2\\ \dot{\hat{y}}_3\end{bmatrix} &= -G^2\begin{bmatrix}a_{12}-a_{32}y_1\\ b_1-b_3y_1\end{bmatrix}P\,e_1 \\
&\quad + \begin{bmatrix}a_{23}+a_{21}y_1+(a_{22}-a_{33})\hat{y}_2\\ -(a_{31}y_1+a_{32}\hat{y}_2+a_{33})\hat{y}_3\end{bmatrix} \\
&\quad + \begin{bmatrix}-a_{31}y_1\hat{y}_2 - a_{32}\hat{y}_2^2 + (b_2-b_3\hat{y}_2)\hat{y}_3\\ -b_3\hat{y}_3^2\end{bmatrix},
\end{aligned}\right. \qquad (15)
$$

Simulation Results of IBO y_1



Compare $\text{IBO}_{y_1+y_2}$ vs. IBO_{y_1}



Summary

- LAO Observer: More general but has a slower converging speed compared to several other nonlinear observers designed specifically for a PDS.
- With Single Output: Converging speed of the observer for the single observation case is slower than those with full observations. However, both observers have similar robust performance.

Future Directions

- LTV observer design for the LAO observer.
- LAO observer using single homogeneous output.
- LAO observer applied to more general nonlinear systems, possibly a PDS with more general imaging surfaces.