Midterm Exam - DUE March 31, 2004.
ECE6330, "Nonlinear and Adaptive Control", Spring 2004[1]

Open book, open notes, no time limit. Answer each question and turn in all your work. Do not consult with classmates.

Conceptual: Answer each of the following using words, equations, sketches, and examples as appropriate. Approximate writing time 2 hours.

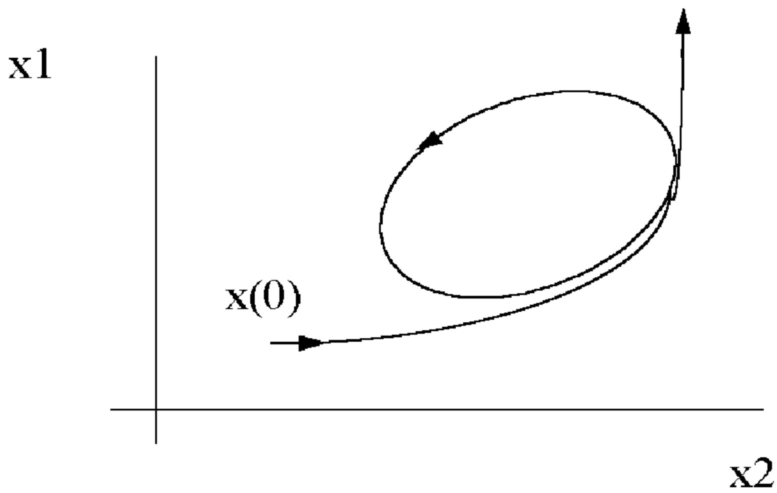
1. Summarize as many as possible available stability notions/definitions (including input-state stable; input-output stable, L_p stable etc) according to our class lectures or weekly reading materials. Explain in your words why there are so many different stability notions. (30 points)

2. Describe the possible use of *Bellman-Gronwall Lemma*, *LaSalle's Invariant Set Theorem*, *Center Manifold Theorem* and *Babalat's Lemma* and find for each an application example. (20 points)

3. You programmed a computer to solve and plot the solution of the autonomous system

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2),\\ \dot{x}_2 &= f_2(x_1, x_2).\end{aligned}$$

   You know the system satisfies existence and uniqueness conditions. Is the phase plane plot shown below possible? What if the system depends explicitly on t (i.e., is non-autonomous, so that $f_i = f_i(t, x_1, x_2)$? (10 points)



[1] Instructor: YangQuan Chen. Email: `yqchen@ieee.org`

Computational: Show all your work. Approximate writing time 4 hours.

1. (20 points) For the system

$$\ddot{x} + \dot{x}^5 = x^2 u$$

with state x and input u, show that:

   (a) The linearized system is "marginally stable" (Note: you must define the equilibrium point in terms of x, $\dot{x}$, and u).

   (b) The linearized system is not controllable.

   (c) Using a simple state feedback law $u = -x$ produces a closed-loop nonlinear system that has an asymptotically stable equilibrium point at the origin.

2. (30 points) Consider the system

$$\begin{aligned} \dot{x}_1 &= -\epsilon x_1, \\ \dot{x}_2 &= -x_1 - 1 + e^{-x_2}, \end{aligned}$$

where $\epsilon > 0$.

   (a) Find any equilibrium points and determine their stability (Hint: Don't forget the indirect method).

   (b) Use

$$V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$

   as a Lyapunov function and determine a region of attraction of x_e.

   (c) Use

$$V(x) = \frac{\alpha}{2\epsilon}x_1^2 + \frac{1}{2}x_2^2$$

   as a Lyapunov function and determine a region of attraction of x_e. Compare this result to that obtained in (b). (Hints: Write $\dot{V}(x) = -x^T Q(x) x$. This is negative definite if the determinant of $Q(x)$ is strictly positive. Also, if you get a transcendental equation don't try to solve it. Just describe the solution graphically as the intersection of a line and a function).

3. (30 points) Consider the system

$$\begin{aligned} \dot{x} &= 4x^2 y - f_1(x)(x^2 + 2y^2 - 4) \\ \dot{y} &= -2x^3 - f_2(y)(x^2 + 2y^2 - 4) \end{aligned}$$

where the continuous functions $f_1(x)$ and $f_2(y)$ have the same sign as their argument and satisfy $f_i(0) = 0$. Show that the system tends towards a limit cycle independent of the explicit expressions of f_1 and f_2. Hint: Use $V(x, y) = \frac{1}{2}(x^2 + 2y^2 - 4)^2$ as a candidate Lyapunov function.

4. (40 points) Write a short tutorial on the relay automatic tuning method for PID controller design based on the describing function analysis. Attach a worked out example, step by step, on how you can design a PI/PID controller for a typical chemical plant $1/(1+s)^n$ where $n = 20$. Attach your Matlab/Simulink code and all relevant plots. (Bonus: you can work out a number of examples for different values of n. Plot the PI/PID gains vs. n. Discuss the trend in these plots.)