

Lecture Notes: Week 7a

Topic: (Youla) Controller parameterization

ECE/MAE 7360

Optimal and Robust Control

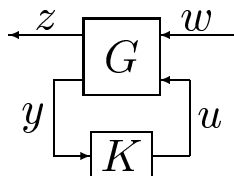
(Fall 2003 Offering)

Instructor: Dr YangQuan Chen, CSOIS, ECE Dept.,

Tel. : (435)797-0148.

E-mail: yqchen@ieee.org or, yqchen@ece.usu.edu

Controller Parameterization



$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}.$$

Suppose (A, B_2) is stabilizable and (C_2, A) is detectable.

Youla parameterization:

all controllers K that internally stabilize G .

- Suppose $G \in \mathcal{RH}_\infty$. Then

$$K = Q(I + G_{22}Q)^{-1}, \quad Q \in \mathcal{RH}_\infty$$

and $I + D_{22}Q(\infty)$ nonsingular.

K stabilizes a stable plant G_{22} iff $K(I - G_{22}K)^{-1}$ is stable. So let $Q = K(I - G_{22}K)^{-1}$.

- General Case: Let F and L be such that $A + LC_2$ and $A + B_2F$ are stable. Then $K = \mathcal{F}_\ell(J, Q)$:

$$J = \left[\begin{array}{c|cc} A + B_2F + LC_2 + LD_{22}F & -L & B_2 + LD_{22} \\ \hline F & 0 & I \\ -(C_2 + D_{22}F) & I & -D_{22} \end{array} \right]$$

with any $Q \in \mathcal{RH}_\infty$ and $I + D_{22}Q(\infty)$ nonsingular.

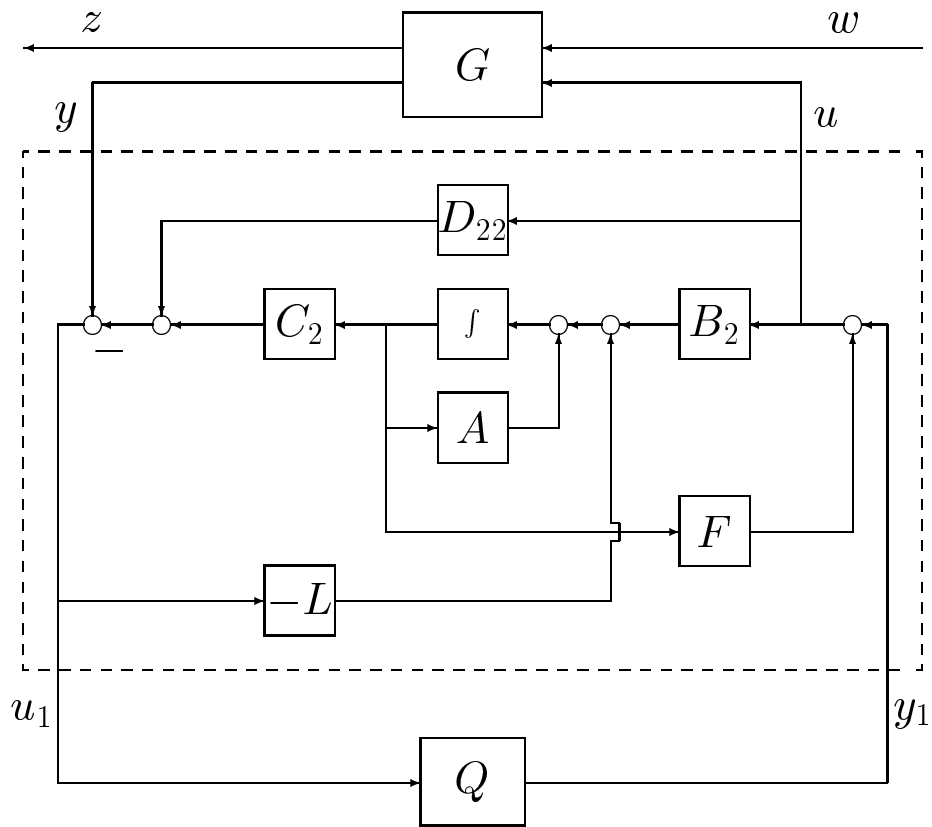


Figure 0.1: Structure of Stabilizing Controllers

- Closed-loop Matrix:

$$\mathcal{F}_\ell(G, K) = \mathcal{F}_\ell(G, \mathcal{F}_\ell(J, Q)) = \mathcal{F}_\ell(T, Q).$$

$$= \{T_{11} + T_{12}QT_{21} : Q \in \mathcal{RH}_\infty, I + D_{22}Q(\infty) \text{ invertible}\}$$

where T is given by

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} A + B_2F & -B_2F & B_1 & B_2 \\ 0 & A + LC_2 & B_1 + LD_{21} & 0 \\ \hline C_1 + D_{12}F & -D_{12}F & D_{11} & D_{12} \\ 0 & C_2 & D_{21} & 0 \end{array} \right].$$

- Coprime factorization approach: Let $G_{22} = NM^{-1} = \tilde{M}^{-1}\tilde{N}$ be rcf and lcf of G_{22} over \mathcal{RH}_∞ , respectively. And let $U_0, V_0, \tilde{U}_0, \tilde{V}_0 \in \mathcal{RH}_\infty$ satisfy the Bezout identity:

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Then

$$\begin{aligned} K &= (U_0 + MQ_y)(V_0 + NQ_y)^{-1} \\ &= (\tilde{V}_0 + Q_y\tilde{N})^{-1}(\tilde{U}_0 + Q_y\tilde{M}) \\ &= \mathcal{F}_\ell(J_y, Q_y), \quad Q_y \in \mathcal{RH}_\infty \end{aligned}$$

where

$$J_y := \begin{bmatrix} U_0V_0^{-1} & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N \end{bmatrix}$$

and $(I + V_0^{-1}NQ_y)(\infty)$ is invertible

Lecture 16 - Controller Structures

K. J. Åström

1. Introduction
2. Feedback and Feedforward
3. Linear Schemes
4. Nonlinear Schemes
5. Gain Scheduling and Adaptation
6. Summary

Theme: Building complex control systems.

Introduction

Many common issues in design of machines, electronics, computer software, mechatronics

- How to deal with complexity
- Modularization
- Standardization
- Structures
- Paradigms, Design principles
- Top Down and Bottom Up

Bottom Up Design

- A way to view systems
- A number of building blocks
- Ideas to combine them
- What are the building blocks of control?
- What principles can be used to select and combine them?
- The danger: Can it be done better?
- Commissioning: Close loops one by one.

Bottom Up Design of Control Systems

Components

- Controllers
- Observers
- Estimators
- Filters
- Limiters
- Dead zones
- Selectors

System principles

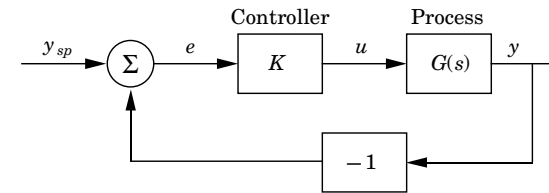
- Feedback
- Feedforward
- Model following
- Cascade
- Split range
- Gain scheduling
- Adaptation

Top Down Design of Control Systems

- Model complete system
- Design an integrated system
- System concepts
 - State feedback
 - Observers
 - Model predictive control
- Commissioning: Needs careful consideration.

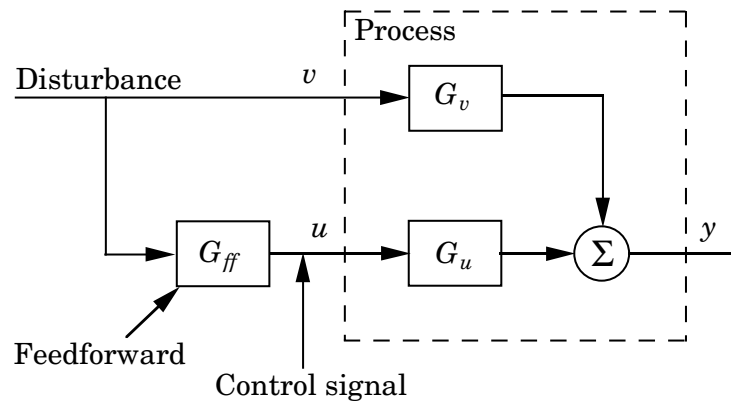
Feedback

A very powerful idea with dramatic impact



- + Reduce effect of disturbances
- + Reduce effect of process variations
- + Linearize nonlinear systems
- + Does not require accurate process model
 - Measurement noise is injected into the system
 - Risk for instability

Feedforward

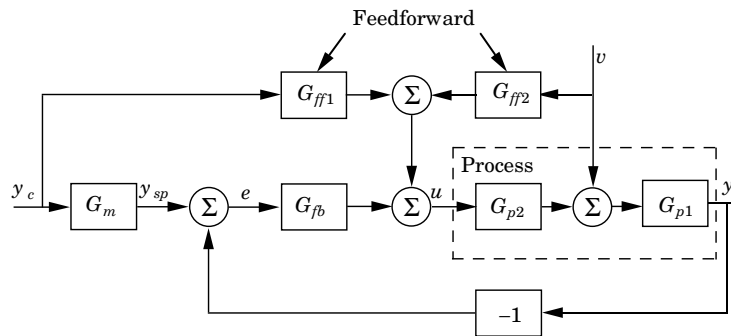


- + Reduce effects of disturbances that can be measured
- + Improve response to reference signals
- + No risk for instability

Feedback and Feedforward

- | | |
|---|---|
| • Feedback | • Feedforward |
| • Closed loop | • Open loop |
| • Acts only when there are deviations | • Acts before deviations show up |
| • Market Driven | • Planning |
| • Robust to model errors
$ S < 1$ for some some frequencies | • Not robust to model errors
$ S = 1$ for all frequencies |
| • Risk for instability | • No risk for instability |

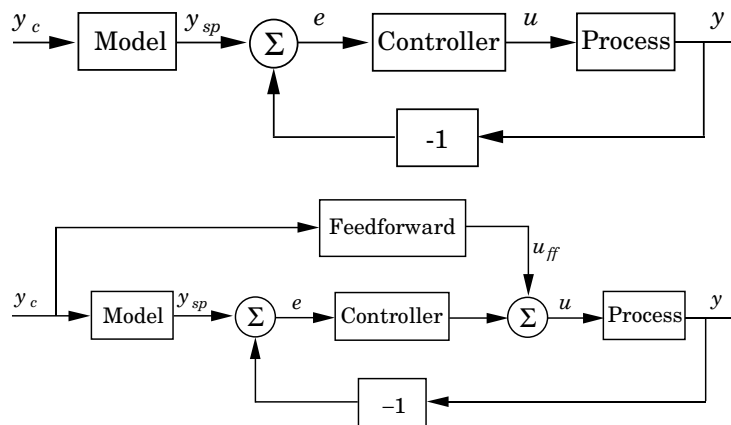
Combination of Feedback and Feedforward



Linear Schemes

- Model following - Systems with two degrees of freedom (2DOF)
- Filters
- Cascade control
- State feedback
- Observers
- Attenuation of disturbances with specific character
- The Smith Predictor
- Model Predictive Control

Model Following - 2DOF



Applications of Model Following

- Coordination in multi-axis motion control
- Robotics
- Path following
- Mixing in chemical processes
- Coordinated production changes

Filters

Typical filters

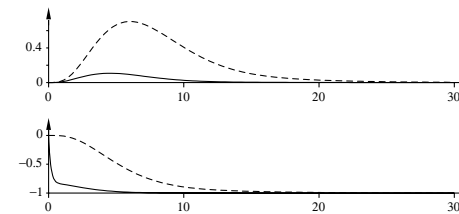
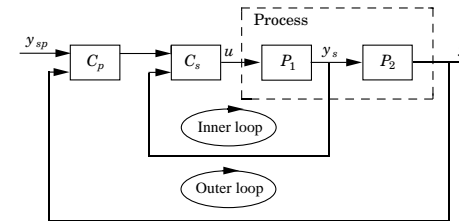
- Low pass
- High pass
- Band pass
- Notch
- Body bending filters

Typical applications

- Reduce disturbances
- Improve robustness (high frequency roll-off)
- Smooth reference signals

Cascade Control

How to use several sensors. State feedback is the ultimate case!



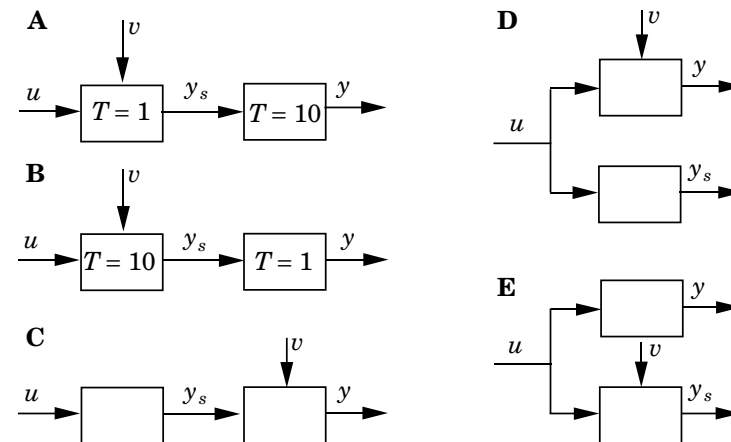
When is Cascade Control Useful?

Key idea: make tight feedback around essential places where there are essential perturbations (disturbances or uncertainties)

Guidelines:

- Well defined relation between primary and secondary variables
- Essential disturbances and process variations in inner loop
- Inner loop faster
- Tight feedback (high gain and high bandwidth) in inner loop

When is Cascade Control Useful?

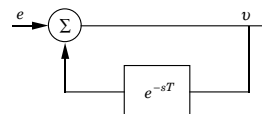
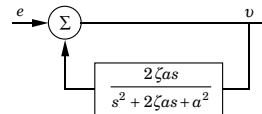
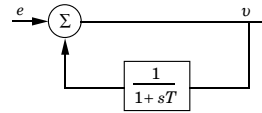


Attenuation of Disturbances with Specific Character

Idea: Exploit model of disturbances (internal model principle)

- Constant disturbances (Integral Action)
- Sinusoidal disturbances
- Periodic disturbances

A disturbance observer is an alternative.



Systems with Time Delays

- The derivative of the output gives poor prediction for systems with time delay
- Better predictions are possible by using past control signals $u(t - \tau)$, $0 < \tau < T)_d$

Replace the regular PID controller

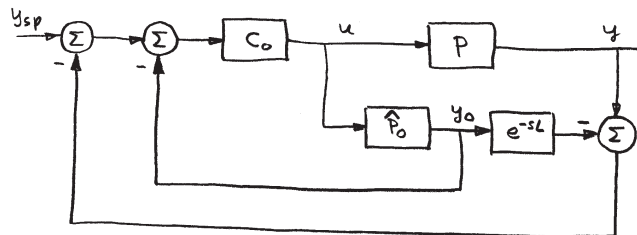
$$u = ke + \frac{1}{k_i} \int e(s) ds - k_d \frac{dy_f}{dt}$$

by the PPI (Predictive PI) controller

$$u = ke + \frac{1}{k_i} \int e(s) ds - k_p \int_{t-L}^t u(s) ds$$

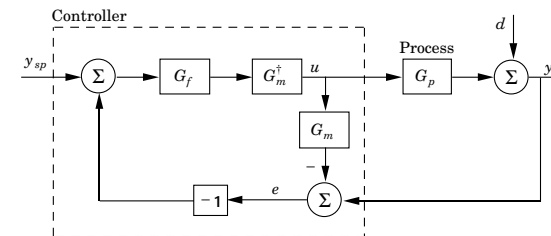
A simple form of the Smith predictor

The Smith Predictor



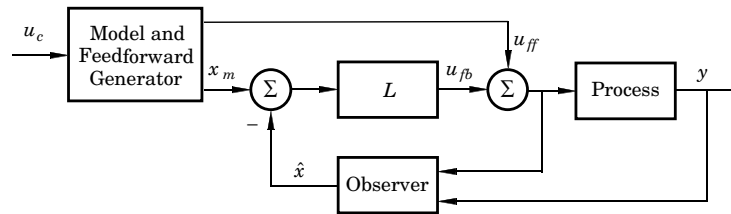
Design controller C as if there were no time delays in the process.

Model Predictive Control



- Beautifully simple
- Are there some snags?
 - Cancellations may degrade performance
 - Does not work if process is unstable
 - Less general than state feedback
- Widely used in process industries

State Feedback and Observers



- Use model to estimate variables that are not directly measurable
- States are the variables required to account for storage of mass, momentum and energy
- Estimate the state
- Feedback from full state deviation
- Feedforward to generate u_m and y_m

Nonlinear Schemes

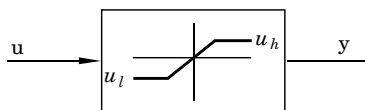
- Limiters
- Split range
- Ratio control
- Selectors
- Fuzzy control
- Gain scheduling
- Neural networks
- Adaptation

Limiters

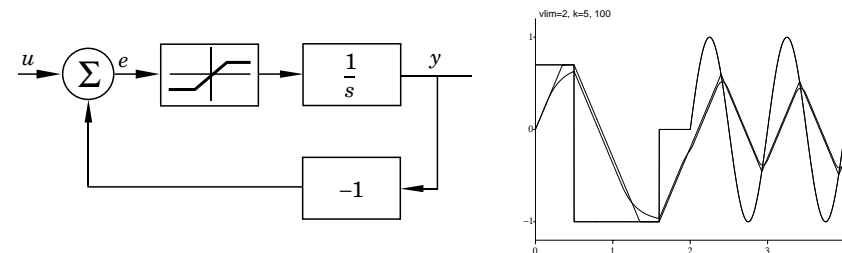
Limiters are often used

- To avoid saturation
- An element in circuits for windup protection
- To protect equipment to rapid changes

A simple amplitude limiter

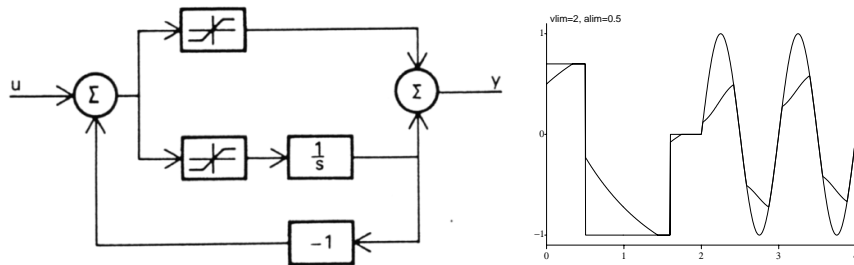


Rate Limiter



A rate limiter causes delays (JAS)

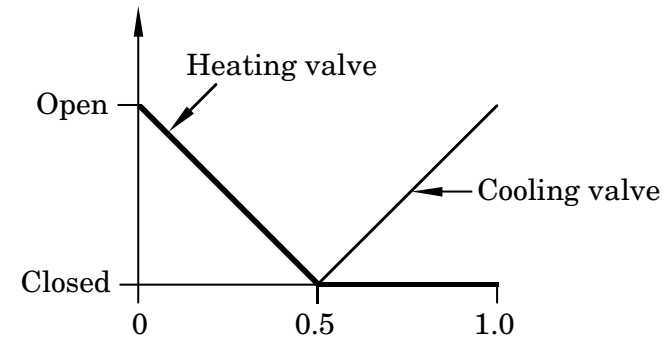
Jump and Rate Limiter



Commonly used in the power industry for load changes to save boilers.

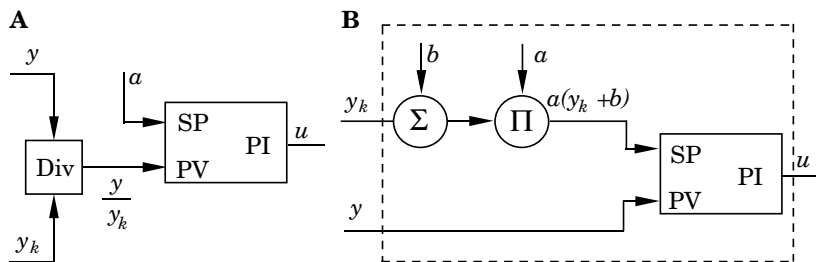
Split Range

A simple way to use one controller to control two actuators. Commonly used for heating and cooling.



Ratio Control

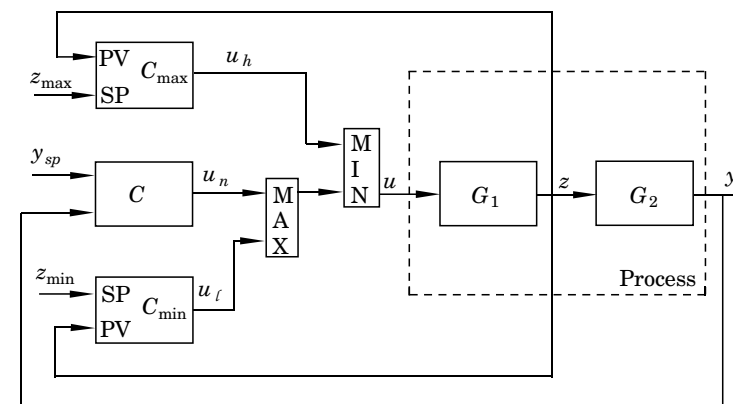
Arrangement to obtain two flows that are proportional to each other, e.g. oil and air in boilers



The scheme B is preferable! Why?

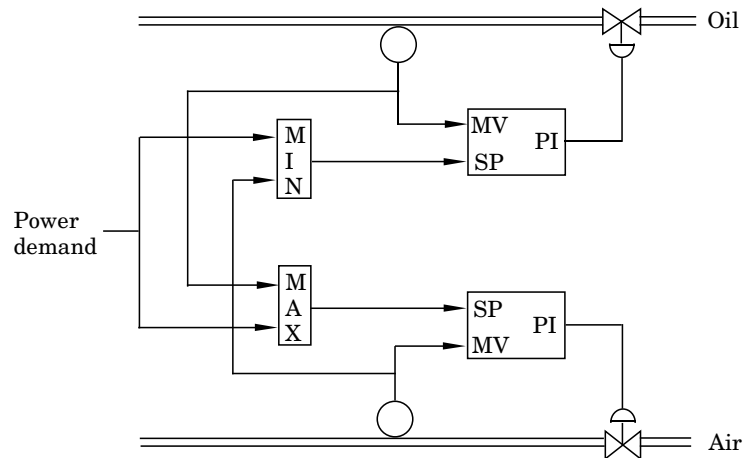
Selector Control

Scheme used to achieve several control objectives, e.g. control temperature unless pressure is too high. A way to constrain process variables during operation.



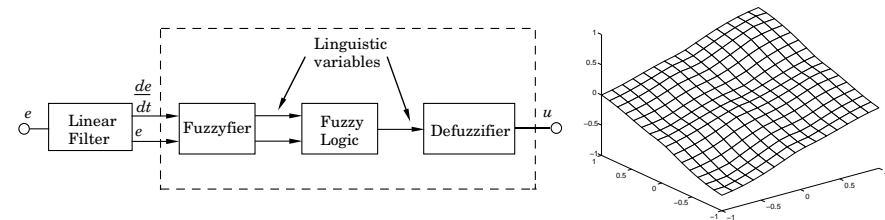
Control of Fuel and Air in a Boiler

An elegant solution



Fuzzy Control

- Rule based control
- Linguistic variables *high, low, medium*
- Membership functions
- If temperature *high* then increase flow *a little*



Fuzzy Control

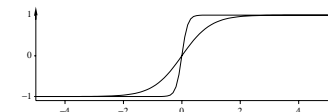
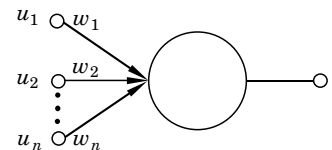
- A nonlinear state feedback
 - How do we get the states?
 - What does the nonlinearity look like?
 - Rules and interpolation
 - Why so few rules
 - When is it useful
- Excellent to automate successful manual operations
- Intuitive
- A lot of controversy: The *No Model Myth*
- Fuzzy control is more useful than its detractors claim but less useful than the propagandists claim
- Neuro-fuzzy

Neural Networks

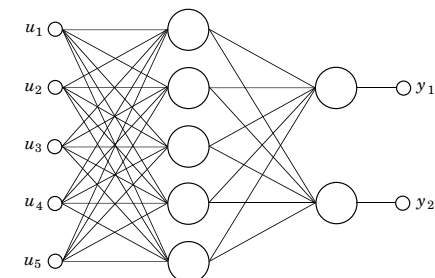
Representation of functions of many variables

$$y(t) = f\left(\sum a_i u_i(t)\right)$$

Real and artificial neurons

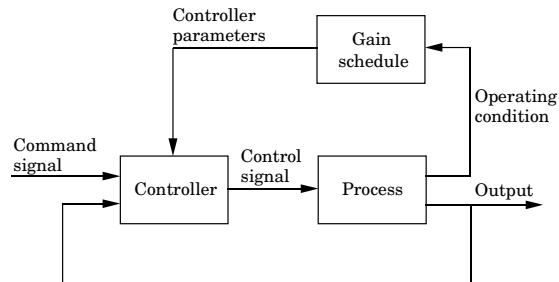


Feedforward neural network



A nonlinear function with a learning mechanism!

Gain Scheduling



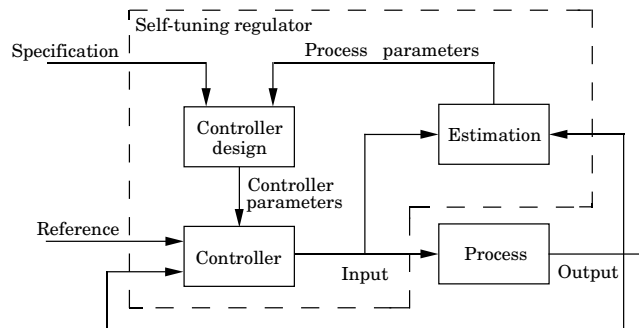
Example of scheduling variables

- Production rate
- Machine speed
- Mach number and dynamic pressure
- Room occupancy

Uses of Gain Scheduling

- Many uses
 - Linearization of actuators
 - Surge tank control
 - Control over wide operating regions
- Important issues
 - Choice of scheduling variables
 - Granularity of scheduling table
 - Interpolation schemes
 - Bump-less parameter changes
 - Man machine interfaces
- Importance of auto-tuning

Adaptation



- Certainty Equivalence
- Many control and estimation schemes
- Dual control
 - Control should be directing as well as investigating!

Uses of Adaptation

- Tuning Tools
- Automatic Tuning
- Gain Scheduling
- Adaptive feedback
- Adaptive feedforward
- Integrated systems

