ECE7750 Distributed Control Systems:FISP1
Disturbance Propagation and packet loss in Vehicle Control

Zhongmin Wang

Center for Self-Organizing and Intelligent Systems
Dept. of Electrical and Computer Engineering
Utah State University

Feb 7, 2005

References

- Pete Seiler, Aniruddha Pant, and Karl Hedrick, Disturbance Propagation in Vehicle Strings, IEEE Trans on Automatic Control, Vol 49, No 10. Oct 2004.
- Pete Seiler and Raja Sengupta, Analysis of Communication Losses in Vehicle Control Problems, Prceedings of the American Control Conference, Arlington, VA, June, 2001, 1491-1496.

String Stability in Automated Highway Systems(AHS)

- The throughout of the transportation network can be greatly increased if all the vehicles are controlled to follow its predecessor with a safe but tight distance at a desired velocity.
- The disturbance in preceding vehicles will propagate to following vehicles. It can cause actuator saturation in following vehicles and make the whole system unstable. This problem is called string stability problem in platoon.
- String Stability problem falls into the category of networked controls system problems if each vehicle can obtain the other vehicle's information by wireless communication.
- It is a example to show how to design a local controller to make the whole system stable.

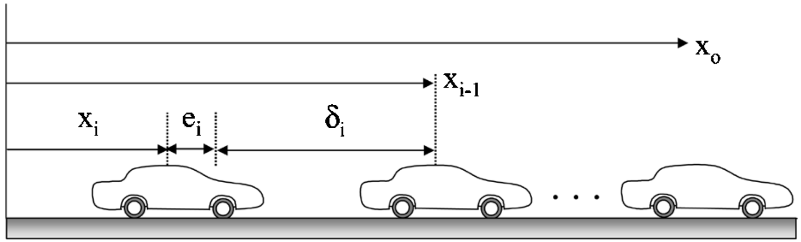

Fig. 1. AHS platoon.

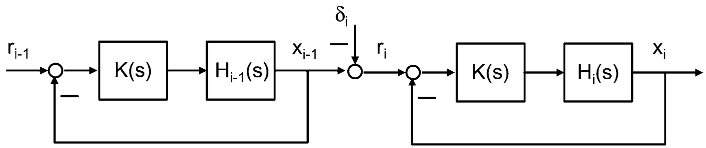

Fig. 2. Coupled feedback loops.

Some research result

If vehicles use only relative spacing information with its predecessor:

- It is widely known that string stability can not be obtained if each vehicle wants to maintain a constant distance behind their predecessors by any linear controller.
- If the spacing is proportional to the vehicle's velocity, string stability may be achieved by linear controller.
- If each vehicle use different controllers, the string stability may be achieved.
- It shows that if the predecessor's information and the leader's information can be used, the string stability can be achieved.

Error propagation using relative spacing

Assumption:

- All the vehicles have same model, $H(s)$.
- $H(s)$ is linear, SISO, with two integrators.
- All vehicles have the same control law.
- The desired spacing is constant.
- Initial position $x_i(0) = -i\delta$.
- No extra input disturbance.

Error propagation using relative spacing, con't

The spacing error dynamics are:

$$E_i(s) = \frac{1}{1+H(s)K(s)} x_0(s) := S(s)X_0(s); \tag{1}$$

$$E_i(s) = \frac{H(s)K(s)}{1+H(s)K(s)} E_{i-1} := T(s)E_{i-1}(s); \tag{2}$$

- $S(s)$ is the sensitivity function. $T(s)$ is the complementary sensitivity function. There is a classical tradeoff between making $|S(j\omega|$ and $|T(j\omega|$ small.

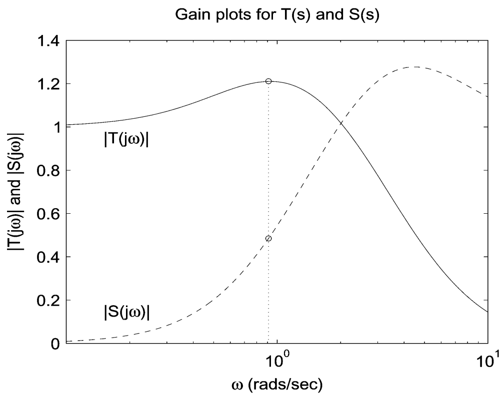

Fig. 3. Plots of $|T(j\omega)|$ and $|S(j\omega)|$.

Error propagation using relative spacing, con't

There is a theorem that implies that there is a frequency, ω, such that $|T(j\omega)| > 1$.

Theorem 0.1 *Assume that $H(s)$ is a rational transfer function with at least two poles at the origin. If the associated feedback system is stable, then the complementary sensitivity function must satisfy*

$$\int_0^\infty ln|T(j\omega)|\frac{d\omega}{\omega^2} \geq 0$$

- This integral relation is similar to the more common Bode sensitivity integral.
- Since $H(s)$ is strictly proper, $T(j\omega) \rightarrow 0$ as $\omega \rightarrow \infty$ and hence $ln|T(j\omega)| < 0$ at high frequencies. As a result, theorem 0.1 implies that $|T(j\omega)| > 1$ for some frequency ω.

Error propagation using relative spacing, con't

Example:
Assume

$$H(s) = \frac{1}{s^2(0.1s+1)}, k(s) = \frac{2s+1}{0.05s+1}$$

The $\|T\|_\infty = 1.21$ for $\omega = 0.93$rad/s.

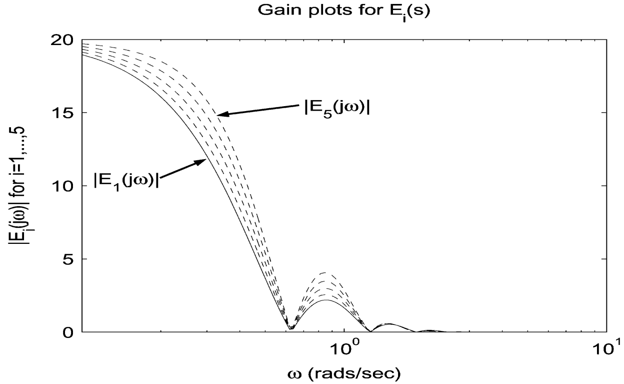

Fig. 4. Frequency domain plots of spacing errors with the predecessor following strategy.

Following with predecessor and leader information

To make the string stable with a linear controller, we need to add leader information in the controller.
The controller is designed as follows:

$$U_i(s) = K_p(s)E_i(s) + K_l(s)(X_0(s) - X_i(s) - \frac{i\delta}{s})$$

This controller tries to keep the errors with respect to the preceding vehicle and with respect to the lead vehicle small.
The the error dynamics are:

$$E_i(s) = \frac{1}{1 + H(s)(K_p(s) + K_l(s))} x_0(s) := S_{lp}(s)X_0(s); \tag{3}$$

$$E_i(s) = \frac{H(s)K_p(s)}{1 + H(s)(K_p(s) + K_l(s))} E_{i-1} := T_{lp}(s)E_{i-1}(s), 2 \leq i \leq N. \tag{4}$$

We can easily design $K_l(s)$ and $K_p(s)$ so that $\|T_{lp} < 1$.

Following with predecessor and leader information, con't

In the above example, if $K_l(s) = K_p(s) = (1/2)K(s)$, $\|T_{lp} = 0.605$. Thus, all frequency content of propagating errors is attenuated.

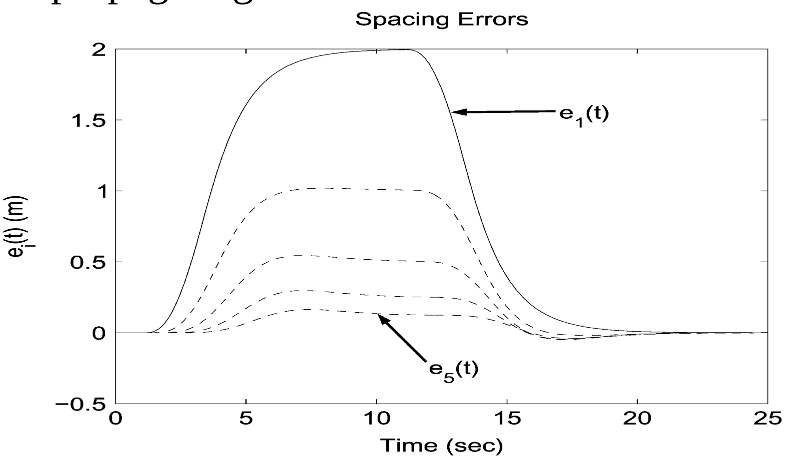

Fig. 6. Time domain plots of spacing errors with the predecessor and leade following strategy.

Analysis of Disturbance and Communication Losses in Vehicle Control Problems, an introduction

A wireless communication system is needed if the leader's information is necessary. We investigate the effect of random packet loss in the feedback loop due to a non-ideal communication network.
Characteristics of communication network:

- The communication network is between Controller and System out.
- The packet loss is a random, independent random process.
- Delay introduced.
- Assume no network jitter.

Stability for a discrete time jump linear system

Definition 1. *The system given by (8) with* $u \equiv 0$ *is:*

1. *mean-square stable (MSS) if for every initial state* (x_0, θ_0), $\lim_{k\to\infty} E[\|x(k)\|^2] = 0$.
2. *stochastically stable (SS) if for every initial state* (x_0, θ_0), $E\left[\sum_{k=0}^{\infty} \|x(k)\|^2\right] < \infty$.
3. *exponentially mean square stable (EMSS) if for every initial state* (x_0, θ_0), *there exists constants* $0 < \alpha < 1$ *and* $\beta > 0$ *such that* $\forall k \geq 0$, $E\left[\|x(k)\|^2\right] < \beta\alpha^k \|x_0\|^2$.
4. *almost surely stable if for every initial state* (x_0, θ_0), $P\left[\lim_{k\to\infty} \|x(k)\| = 0\right] = 1$.

System model

The system model for each vehicle is:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

$$y_c(k) = \theta(k)y(k) + (1-\theta(k))y_c(k-1)$$

$y_c(k)$ is the communicated data. $\theta(k)$ is a Bernoulli process with $P_r[\theta(k) = 0] = p$ and $P_r[\theta(k) = 1] = 1 - p$.

- When a packet is lost, the controller will use the latest received information.
- We assume the controller has knowledge of $\theta(t)$.
- We only investigates a string with two vehicles

This problem is investigated under the framework of discrete time Markovian Jump Linear Systems(MJLS). A necessary condition to make sure the system is stable is given by a LMI formulation.

Questions?