ECE7750 FISP: Estimation with missing measurement in wireless sensor network (Connection to Kalman Filter Structured ILC)

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References

This presentation is based on the following papers:

- Bruno Sinopoli et al., "Kalman filtering with intermittent observations," IEEE Trans. Automatic Control, Vol. 49, No. 9, Sep. 2004.
- Zidong Wang et al., "Variance-Constrained filtering for uncertain stochastic systems with missing measurements," IEEE Trans. Automatic Control, Vol. 48, No. 7, Jul. 2003.
- S. Craig Smith and Peter Seiler, "Estimation with lossy measurements: Jump esimators for jump systems," IEEE Trans. Automatic Control, Vol. 48, No. 12, Dec. 2003.
- P. M. Lynch, J. F. Figueroa and J. DePaso, "A prototype intelligent control structure using intermittent multiple independent measurements," Proceedings IEEE 1990 Southeastcon.
- P. M. Lynch and R. Vangal, "Tracking partially occluded two dimensional shapes," Proceedings SPIE 1989.
- Kwang Soon Lee and Jay H. Lee, "Constrained model-based predictive control combined with iterative learning for batch or repetitive process," Proceedings 2nd ASCC 1997.
- Kwang Soon Lee and Jay H. Lee, "Convergence of constrained model-based predictive control for batch processes," IEEE TAC, Vol. 45, No. 10, 2000.

Basics

• What is Kalman filtering ? $\Rightarrow$ (Sub)optimal Estimation: key idea: derivative of trace of matrix by Kalman gain matrix.
$\Rightarrow$ Basic question: Why derivative of the trace ?. Can we make derivative of a matrix by a matrix ?

• Linear KF: standard form.

• Extended KF: nonlinear system.

• Linearized KF: nonlinear but linearized along the nominal trajectory.

• etc., : Adaptive KF, etc.....

Basics: Standard linear discrete KF

- System:

$$x_{k+1} = A_k x_k + B_k u_k + w_k; \;\; \tilde{z}_k = C_k x_k + v_k$$

where $w_k \sim N(0, Q)$ and $v_k \sim N(0, R)$.

- Propagation:

$$\begin{aligned} \bar{x}_k &= A_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1} \\ \bar{P}_k &= A_{k-1}\hat{P}_{k-1}A_{k-1}^T + Q \end{aligned} \quad (1)$$

- Correction:

$$\begin{aligned} K_k &= \bar{P}_k C_k^T (C_k \bar{P}_k C_k^T + R)^{-1} \\ \hat{x}_k &= \bar{x}_k + K_k(\tilde{z}_k - C_k \bar{x}_k) \\ \hat{P}_k &= (I - K_k C_k)\bar{P}_k \end{aligned} \quad (2)$$

where K_k is Kalman gain matrix.

Missing measurements in wireless sensor network

Following two papers deal with the missing measurement problem in wireless sensor network.

- Bruno Sinopoli et al., "Kalman filtering with intermittent observations," IEEE Trans. Automatic Control, Vol. 49, No. 9, Sep. 2004.
- S. Craig Smith and Peter Seiler, "Estimation with lossy measurements: Jump esimators for jump systems," IEEE Trans. Automatic Control, Vol. 48, No. 12, Dec. 2003.

KF with Missing measurements

In the system below,

$$x_{k+1} = A_k x_k + w_k; \;\; \tilde{z}_k = C_k x_k + v_k \tag{3}$$

with missing measurement, $\tilde{z}_k = v_k$ because $C_k x_k$ is not measured. So, the "correction" process is governed by

$$\begin{aligned} K_k &= \bar{P}_k C_k^T (C_k \bar{P}_k C_k^T + R)^{-1} \\ \hat{x}_k &= \bar{x}_k + K_k (v_k - C_k \bar{x}_k) \\ \hat{P}_k &= (I - K_k C_k) \bar{P}_k. \end{aligned} \tag{4}$$

Thus, the estimated state $\hat{x}_k$ is biased by v_k. Now, how to fix this problem ? This problem has been studied for a long time in the name of "intermittent", "missing measurement" or "missing observation".

Lynch et al., : 1989 and 1990

Key idea: To produce an estimator which effectively ignores a measurement, an assumption is made: the measurement is present but that "noise" associated with the measurement is very large.

So, the measurement noise is expressed as: $v'_k = T_k v_k$ where T_k is an diagonal matrix which is dependent on the measurement like:

$t_{ij} = 0$ if $i \neq j$,

$t_{ij} = 1$ if $i = j$ and feature measurement is valid,

$t_{ij} = \epsilon \gg 1$ if $i = j$ and feature measurement is occluded.

Then, covariance matrix of the measurement noise is given as $R'_k = T_k R T_k^T$.

Lynch et al., : 1989 and 1990

Therefore,

$$\begin{aligned} K_k &= \bar{P}_k C_k^T (C_k \bar{P}_k C_k^T + T_k R T_k^T)^{-1} \\ &= \bar{P}_k C_k^T (T_k^T)^{-1} (T_k^{-1} C_k \bar{P}_k C_k^T (T_k^T)^{-1} + R)^{-1} T_k^{-1} \end{aligned}$$

Defining $S_k = T_k^{-1}$, which is given as:
$s_{ij} = 0$ if $i \neq j$,
$s_{ij} = 1$ if $i = j$ and feature measurement is valid,
$s_{ij} = 0$ if $i = j$ and feature measurement is occluded,
we obtain

$$K'_k = \bar{P}_k C_k^T S_k (S_k C_k \bar{P}_k C_k^T S_k + R)^{-1} S_k$$

Lynch et al., : 1989 and 1990

Finally, by several simplifications, the correction process is governed by

$$\begin{aligned} \hat{x}_k &= \bar{x}_k + K_{ks}(S_k\tilde{z}_k - C_{ks}\bar{x}_k) \\ \hat{\bar{P}}_k &= (I - K'_k C_k)\bar{P}_k \end{aligned} \tag{5}$$

where $C_{ks} = S_k C_k$ and

$$K_{ks} = \bar{P}_k C_{ks}^T S_k (C_{ks}\bar{P}_k C_{ks}^T S_k + R)^{-1}.$$

Wang et al., : 2003 TAC

In this paper, the following model uncertain system with missing measurement is considered.

$$x_{k+1} = (A_k + \Delta_A)x_k + w_k; \ \tilde{z}_k = C_k x_k + v_k \tag{6}$$

The following correction is used

$$\hat{x}_{k+1} = G\hat{x}_k + K(\tilde{z}_k - \gamma_k C_k \hat{x}_k)$$

where γ is 1 or 0 depending on the availability of the measurement. Now, it is a problem to design G and K. Main result is formulated in Theorem 1. See attached reference paper.

What can we learn from this paper ?

- Under nominal A is stable, G and K can be calculated by modified Riccati Eq. But, question, why G ?
- Error covariance does not diverge (i.e., the state is estimated within boundary). But, how to reduce the size of error covariance matrix ?
- The same approach of this paper can be applied to model uncertain ILC. But I have found it is not easy for several reasons.

Smith et al., : 2003 TAC

In this paper, the following model uncertain system with missing measurement is considered.

$$x_{k+1} = Ax_k + Bw_k;\ \tilde{z}_k = C_\theta x_k + v_k \tag{7}$$

where C_θ could be 1 or 0.

• Exactly the same KF structure as Lynch is used in this paper. But, the Kalman gain is scheduled based on the past information. See Fig. 2 of the reference paper.

• This paper focuses on wireless sensor network. So, probably mostly related with CSOIS MASnet project.

What can we learn ? Theoretically not so attractive. But, for the wireless network, maybe useful in future work.

Sinopoli et al., : 2004 TAC

This paper is UC Berkeley Sensor network group work. But, in fact, this paper handles theoretical development in intermittent KF.

- This paper also uses the exactly same KF structure as Lynch.
- In section 3, modified algebraic Riccati equation is given. By this point, nothing new.
- Main contribution of this paper is to find the critical probability over which the error covariance matrix does not diverge. All results of this paper come from the attack to Eq. (15). What are properties in Eq. (15) ?

What can we learn ? The result probably can be used in ILC. Then, we can numerically provide the critical probability of the measurement in ILC. But, currently I cannot see the result from this paper. Actually, in ILC, the output is measured in each iteration. But, in each iteration, some of the time dependent measurement could be missed. But, in this paper, from the observation $\tilde{z}_k = \gamma C_\theta x_k + v_k$, if $\gamma = 1$, then x_k is measured otherwise, x_k is not measured. So, the vector x_k is total measured or total not measured. But, in ILC given as $Y_k = HU_k + v_k$, it is more practical meaningful to assume that some of Y_k is measured and some of them are not measured. So, Sinopoli's result cannot be directly applied to ILC. This is an interesting research topic, but I think it is also not easy.

Main results: Intermittent ILC

- In ILC, mostly, Lee and Lee well described the KF problem in the super-vector framework. Here, in our stochastic super-vector ILC, we are interested in intermittent measurement.
- Missing measurements in ILC ?
- What is the estimated state in KF ?
- How to design the KF structure in ILC ?

KF frame in ILC

From $Y_{k+1} = HU_{k+1}$ and $Y_k = HU_k$, by defining $E_k \triangleq Y_k - Y_d$ and $\Delta_k^u \triangleq U_{k+1} - U_k$, we obtain the following relationship easily:

$$E_{k+1} = E_k + H\Delta_k^u \tag{8}$$

The measurement is E_k because Y_k is measured. So, we have a state-space form like

$$\begin{aligned} E_{k+1} &= AE_k + B\Delta_k^u \\ z_k &= CE_k \end{aligned} \tag{9}$$

where $A = C = I$ and $B = H$. Now, the control input $\Delta_k^u = U_{k+1} - U_k$, so we have $U_{k+1} = \Delta_k^u + U_k$. This is associated with the ILC update rule: $U_{k+1} = U_k + \Gamma E_k$. Thus, notice that $\Delta_k^u = \Gamma E_k$. Next, what is the noises ? In measurement, there is a measurement noise due to $\tilde{Y}_k = Y_k + v_k$. So, $\tilde{z}_k = CE_k + w_k$. In process, during the ILC update $U_{k+1} = U_k + \Gamma E_k$, we can include the process noise v_k. So, finally we have

$$\begin{aligned} E_{k+1} &= AE_k + B\Delta_k^u + v_k \\ z_k &= CE_k + w_k \end{aligned} \tag{10}$$

Kalman Filter in super-vector ILC

- Propagation:

$$\begin{aligned} \bar{E}_k &= A\hat{E}_{k-1} + B\Delta^u_{k-1} \\ \bar{P}_k &= A\hat{P}_{k-1}A^T + Q \end{aligned} \tag{11}$$

- Correction:

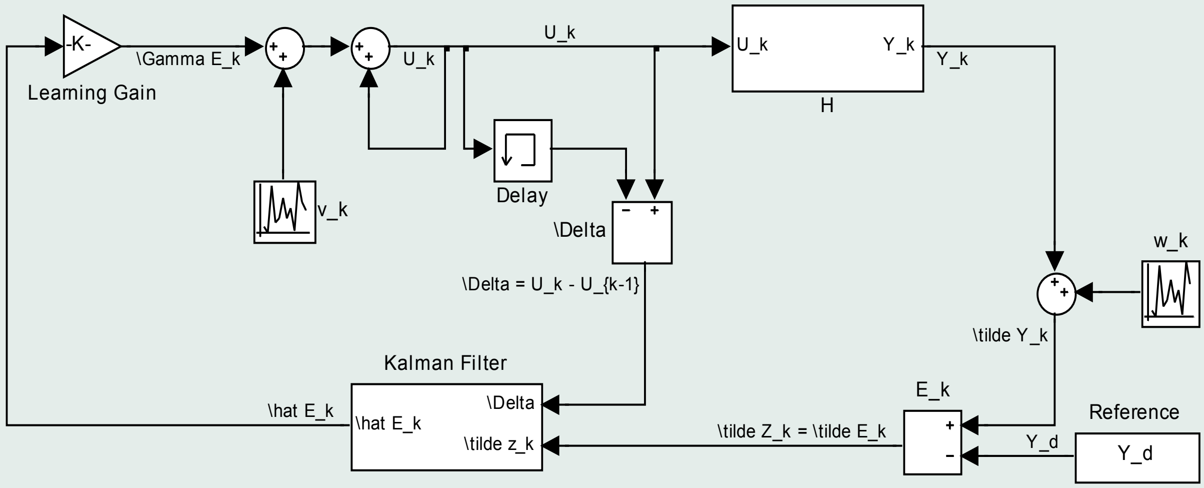
$$\begin{aligned} K_k &= \bar{P}_k C^T (C\bar{P}_k C^T + R)^{-1} \\ \hat{E}_k &= \bar{E}_k + K_k(\tilde{z}_k - C\bar{E}_k) \\ \hat{P}_k &= (I - K_k C)\bar{P}_k \end{aligned} \tag{12}$$

Thus, with $A = C = I$ and $B = H$, we have:

$$\begin{aligned} \bar{E}_k &= \hat{E}_{k-1} + H\Delta^u_{k-1} \\ \bar{P}_k &= \hat{P}_{k-1} + Q \\ K_k &= \bar{P}_k C^T (\bar{P}_k + R)^{-1} \\ \hat{E}_k &= \bar{E}_k + K_k(\tilde{z}_k - \bar{E}_k) \\ \hat{P}_k &= (I - K_k)\bar{P}_k \end{aligned} \tag{13}$$

ILC with KF

In (13), the inputs are $\Delta^u_{k-1} = U_k - U_{k-1}$ and $\tilde{z}_k$, and output is the estimated error $\hat{E}_k$. The following figure shows the iterative learning control system with Kalman filter for the estimation of E_k.



Intermittent ILC

In previous KF ILC block diagram, the output Y_k could be occluded. In this work, as done in the reference papers, we simply model the output as: $Y_k' = \Lambda Y_k$, where λ_{ij} is
$\lambda_{ij} = 0$ if $i \neq j$,
$\lambda_{ij} = 1$ if $i = j$ and feature measurement is valid,
$\lambda_{ij} = \epsilon \gg 1$ if $i = j$ and feature measurement is invalid.

- Probably, Lynch's work is simply used, and/or Wang and Sinopoli's works can be adopted for intermittent ILC. This remains as future works.

Conclusions

• For the intermittent ILC, the Kalman filter structure was developed in this work.
• Existing previous works could be modified for ILC.
• But, several technical difficulties should be solved first for example, in ILC $Y_k' = \Lambda Y_k$, Λ is matrix not scalar.
• Model uncertainty problem is hard to be integrated into Kalman filtered ILC even though Wang et al., made some progresses in here. So, we will not consider the model uncertainty in this work.
• Is there any connection between wireless sensor network and iterative learning control ?