Low Earth-orbit satellite attitude stabilization with fractional regulators

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Fractional Order Control methods were applied to a three-axis reaction wheels satellite attitude control system. To show the advantages of this method, a comparative study between a Linear Quadratic Regulator and a Fractional Order Control was established through two principal fractional control laws. The aim is to establish an efficient control law which satisfies a given specification and maintains sufficient stability and accuracy even under the strong effects of intrinsic parameters uncertainties, and also external perturbations.

1. Introduction

Earth-pointing satellites are expected to maintain their local-vertical/local-horizontal (LVLH) attitude in the presence of different environmental disturbance effects. Most of the time, to ensure precise pointing, the satellite requires a reaction wheel system to counteract the attitude drifts caused by those perturbations, especially seculars ones, such as torques due to passive gravity gradient, and aerodynamic and solar forces.

In attitude control design, different control approaches have been used, e.g. Proportional Integral Derivative (PID) (Witford and Forrest 1996, Sidi 1997), Linear Quadratic Regulation (LQR) (Musser and Elbert 1986), pole placement techniques (Wie 1998), robust control (Valentin-Charbonnel et al. 1999, Skullestad and Gilbert 2000), etc. All those methods, expressed most of the time in different attitude terminologies, are using Euler angles for small attitude commands, while for large attitude manoeuvres, quaternion (Kim et al. 1995) and direction cosine errors (Sidi 1997) are used.

A comparative study by Won (1999) shows the differences in performances and stability criteria between controllers: PD, $H_2$, $H_\infty$ and a mixed $H_2/H_\infty$. The differences between the previous control methods lie in the gain matrix. The present work has introduced a fractional controller that contains additional parameters to act on the systems' characteristics without changing the control gains.

The paper deals with fractional attitude control method to stabilize the attitude movement of an Earth-pointing satellite under the effect of external disturbances, using three-axis reaction wheels as actuators. The dynamics of the satellite are described by a quasi-bilinear multivariable-coupled system. This study relates to the optimal stabilization of the system by state feedback, i.e. the choice of the optimal non-integer derivative order intervening in the synthesis of the fractional control law, and ensuring a good compromise between stability and the different criteria of performance: time response, precision, etc. Another factor is considered in the framework of this study: the degree of stability or robustness. The fractional order controller introduced here guarantees good robustness (Devy Vareta 2000) with respect to uncertainties on the intrinsic parameters of the system, such as matrix inertia and external perturbation.

The paper is organized as follows. In Section 2, the general non-linear equations model of an Earth-pointing satellite attitude dynamics are developed. In Section 3, the attitude equations are linearized about the nadir attitude position as the origin. This leads to a quasi-bilinear multivariable-coupled system. Then, for small manoeuvres, the quasi-bilinear term is neglected to obtain a linear system. In Section 4, the different control
methods used are presented, i.e. the LQR and the Fractional Order Controllers (FOC), for which the theoretical stability, in case of a linear system, is investigated. Subsequently, in Section 5, simulation results are shown and a comparative study established. Finally, in Section 6, a conclusion and current studies are given.

2. Non-linear model of satellite attitude dynamics

The attitude motion of the satellite is represented by the Euler equations for rigid body motion under the influence of external moments, such as the control moment generated by the actuators. Attitude control requires coordinate transformation from LVLH to the Satellite Coordinate System (SCS) system defined as follows: the LVLH coordinate system \((X_o, Y_o, Z_o)\) is a right orthogonal system centred in the satellite’s centre of mass (SCM). The roll axis, \(X_o\), points along the velocity vector, the pitch axis, \(Y_o\), points in the direction of the negative orbit normal, and the yaw axis, \(Z_o\), points in the nadir direction. The SCS system \((X_s, Y_s, Z_s)\) is a right orthogonal system centred in the SCM, parallel to principal moment of inertia axis of the satellite. \(Z_s\) is parallel to the smallest moment of inertia axis; \(Y_s\) is parallel to the largest moment of inertia axis; \(X_s\) completes the right orthogonal system. Consider a satellite with three reaction wheels. The general non-linear attitude dynamics model can be described as follows (Wertz 1978, Sidi 1997, Tsiotras et al. 1999):

\[
\begin{align*}
\vec{I}_s^s(t) &= -\dot{h}_s(t) - \omega_s^s(t) \times \vec{I}_s^s(t) \\
&= -\omega_s^s(t) \times h_s(t) + M_s^s(t) + P(t),
\end{align*}
\]

where \(\vec{I}_s^s\) is the total moment of inertia matrix for the satellite without reaction wheels inertia \((3 \times 3)\), \(\omega_s^s(t)\) is the inertial angular velocity vector in SCS \((3 \times 1)\), \(h_s(t)\) is the angular momentum vector of the wheel cluster, \(M_s^s(t)\) is the torque due to the Earth’s gravity gradient, and \(P(t)\) is the disturbance torque due to aerodynamics, solar pressure and other environmental factors. It is assumed to be (Sidi 1997, Tsiotras et al. 1999):

\[
P(t) = \begin{bmatrix}
4 \times 10^{-6} + 2 \times 10^{-6} \sin(\omega_0 t) \\
6 \times 10^{-6} + 3 \times 10^{-6} \sin(\omega_0 t) \\
3 \times 10^{-6} + 3 \times 10^{-6} \sin(\omega_0 t)
\end{bmatrix},
\]

where \(\omega_0\) is the orbital angular rate.

To keep the satellite attitude earth pointing, the SCS axes must remain aligned with LVLH axes. The transformation matrix, expressed with Euler angles \((\phi, \theta, \psi)\), respectively, roll, pitch and yaw, is given by (Wertz 1978, Sidi 1997):

\[
T_{VH/S} = \begin{bmatrix}
C_\phi C_\theta & C_\phi S_\theta & -S_\phi \\
-C_\theta S_\phi + S_\theta C_\psi C_\phi & C_\theta C_\phi + S_\theta S_\psi C_\phi & S_\theta S_\phi \\
S_\theta S_\psi + C_\theta S_\phi C_\psi & -S_\theta C_\phi + C_\theta S_\psi C_\phi & C_\theta C_\phi
\end{bmatrix},
\]

where \(S\) and \(C\) are, respectively, the sine and cosine.

The gravity gradient torque \(M_g^s(t)\) is given by (Hughes 1986, Sidi 1997):

\[
\begin{align*}
M_{gx} &= \frac{3}{2} \omega_0^2 (I_z - I_x) \sin(2\theta) \cos^2(\phi) \\
M_{gy} &= \frac{3}{2} \omega_0^2 (I_z - I_x) \sin(2\theta) \cos(\phi). \\
M_{gz} &= \frac{3}{2} \omega_0^2 (I_z - I_x) \sin(2\theta) \sin(\phi)
\end{align*}
\]

To describe the satellite kinematics, two important factors are to be taken into account: angular velocity of the body axis frame (SCS) with respect to the reference LVLH frame \(\omega_{vh}^{\phi\theta\psi}(p, q, r)\), and the angular velocity of the body frame with respect to inertial axis frame \(\omega_{i}^{\phi\theta\psi}(\omega_x, \omega_y, \omega_z)\). These quantities are related to the derivative of the Euler angles as follows (Sidi 1997):

\[
\begin{align*}
p &= \dot{\phi} - \psi S_\theta \\
q &= \dot{\theta} C_\phi + \dot{\psi} C_\phi S_\theta \\
r &= \psi C_\theta + \theta S_\phi
\end{align*}
\]

and

\[
\omega_i^s = \omega_{vh}^s + T_{VH/S} \begin{bmatrix} 0 & -\omega_0 & 0 \end{bmatrix}^T.
\]

3. Linearized equations of motion

Assuming small variations of the Euler angles \((\phi, \theta, \psi)\), then the transformation matrix becomes:

\[
T_{O/S} = \begin{bmatrix}
1 & \psi & -\theta \\
-\psi & 1 & \phi \\
\theta & -\phi & 1
\end{bmatrix}.
\]

On the other hand, one obtains that:

\[
\dot{\phi} \approx p, \quad \dot{\theta} \approx q, \quad \dot{\psi} = r
\]

and

\[
\omega_x = \dot{\phi} - \omega_0 \psi, \quad \omega_y = \dot{\theta} - \omega_0, \quad \omega_z = \dot{\psi} + \omega_0 \dot{\phi}.
\]
Then the equations of motion (1) and (5) can be linearized about the origin, giving a quasi-bilinear multivariable system:

\[ \dot{x}(t) = Ax(t) + Bu(t) + G\left(\int_0^t u(\xi)d\xi\right) + BP(t) \]

where

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\omega_0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -4\omega_0^2\sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 3\omega_0^2\sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \omega_0^2\sigma_3 & -\omega_0(1+\sigma_3) & 0 \\ 0 & 0 & 0 & -\omega_0/IZ & 0 \\ 0 & 0 & 0 & \omega_0/Ix & 0 \\ -\omega_0/IZ & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1/Ix & 0 & 0 & 0 & 0 \\ 0 & 1/Iy & 0 & 0 & 0 \\ 0 & 0 & 1/Iz & 0 & 0 \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

where \( \sigma_i = (I_i - I_k)/I_i \) for the \((i,j,k)\) index sets \((1,2,3)\), \((2,3,1)\), and \((3,1,2)\).

Moreover, assuming that the angular velocity components \(p, q\) and \(r\) are also small, and for slight manoeuvres, one can neglect the following products (Wertz 1978, Won 1999, Kim et al. 2001):

\[ x_i h_{ni}, i = 1, \ldots, 6. \]

The equation of motion (9) can then be written in the standard form of a linear equations system:

\[ \dot{x}(t) = Ax(t) + Bu(t) + BP(t); \ x(0) = x_0. \] (10)

Based on the previous assumption, the methodology is as follows: first, the linearized system (10) will be considered, for which an LQR controller will be developed, minimizing a given performance index. This same control law worked out will be applied to the quasi-bilinear system (9). Then, based on the already established control, a fractional controller of type \((-Kx^{(\alpha)}\)) will be designed. Here \(\alpha\) is a real number (not necessarily an integer). Comments and simulation results will be provided.

4. Control methods

The theory of automatic control includes several methods: PID, pole placement, \(H_2, H_\infty\), etc. The two main methods used here, and which are the subject of a comparative study, are LQR and FOC.

4.1. Linear quadratic method

Consider the controlled process described by the state equation (10) to find an optimal control (Musser and Elbert 1986, Zhou et al. 1996) law \(u^*(t)\) that minimizes the cost function:

\[ J = \frac{1}{2} \int_0^T \left( x^T(t)Qx(t) + u^T(t)Ru(t) \right) dt, \]

where the matrices \(R\) and \(Q\) are positive definite.

For this state regulator problem, a solution exists only if (10) represents a completely controllable system, i.e. the rank of the composite matrix \(Q_c\) is \(n\), where

\[ Q_c = [B, AB, \ldots, A^{n-1}B]. \]

The solution of such a problem is given by:

\[ u^*(t) = -R^{-1}B^TPx(t), \]
where $P$ is the solution of the well-known Ricatti equation:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0.$$ 

4.2. Fractional control method

As mentioned above, the satellite attitude dynamics is described, when neglecting the quasi-bilinear term, by system (10):

$$\dot{x}(t) = Ax(t) + Bu(t)^{(a)} + w(t); \quad x(0) = x_0,$$  \hspace{1cm} (11)

where $w(t) = BP(t)$ is the perturbation term and $u(t)$ is the Fractional Control Law applied to stabilize the system (11), given by:

$$u = -K(x - x_r)^{(a)},$$  \hspace{1cm} (12)

where $x_r$ is the attitude reference and is zero for nadir pointing.

The linear fractional system is obtained in the form:

$$\dot{x}(t) = Ax(t) - BKx(t)^{(a)} + w(t); \quad x(0) = x_0.$$  \hspace{1cm} (13)

The fractional derivative of a function $f(t)$ is defined by the Riemann–Liouville formula (Samko et al. 1987) as:

$$\mathcal{D}^a\alpha f(t) = \frac{1}{\Gamma(m - a)} \int_c^t (t - \tau)^{m - \alpha - 1} f(\tau) d\tau,$$  \hspace{1cm} (14)

where $\mathcal{D}^a\alpha$ are the fractional $\alpha$-time derivative action, $c$ and $t$ are the limits of the operation, $\Gamma$ is the well-known Euler's gamma function, and $m < \alpha < m + 1$.

Numerically, we will adopt the definition given by Grünwald–Letnikov (Lubich 1986) as:

$$D^a\alpha f = f^{(\alpha)}(t) = \lim_{h \to 0} \frac{(\Delta_h^a f)(t)}{h^a},$$

where

$$(\Delta_h^a f)(t) = \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f(t - kh)$$

and

$$\binom{\alpha}{k} = \frac{\Gamma(\alpha + 1)}{\Gamma(k + 1) \Gamma(\alpha - k + 1)}.$$ 

Thus,

$$x_i^{(a)} = h^\alpha \sum_{k=0}^n c_k x_{i,n-k}; \quad c_k = \left(1 - \frac{1 + \alpha}{k}\right) c_{k-1}, c_0 = 1.$$ 

Then

$$u_{i,n} = h^{-\alpha} \sum_{k=1}^n \sum_{i=0}^n k_{ij} c_i x_{i,n-i},$$

where $k_{ij}$ are gains matrix elements.

In the following, only the fractional orders such as $\alpha = 1/p, p \in N^*$ will be considered. Then

$$\dot{x}(t) = Ax(t) - BKx(t)^{(1/p)} + Dw(t).$$  \hspace{1cm} (15)

Note:

$$\left(x^{(1/p)}\right)^m(t) = x(t),$$

$$\left(x^{(1/p)}\right)^{m+1}(t) = x^{(1/p)}(t),$$

$$\vdots$$

$$\left(x^{(1/p)}\right)^{m-1}(t) = x^{(1/p)}(t).$$  \hspace{1cm} (16)

From (15) and (16), one obtains the original state representation (17). It is original because in Matignon (2002) and Dorcak et al. (2001), for example, one starts from a monovariable scalar system described by a fractional differential equation, which one puts in the fractional state form. Here, a system of fractional differential equations is considered describing a fractional multivariable coupled system, which we investigate using a generalized state representation whose left side expresses a fractional derivative of order $\alpha = 1/p$.

$$\begin{pmatrix}
(x^{(i/p)})^0(t) \\
(x^{(i/p)})^1(t) \\
(x^{(i/p)})^2(t) \\
\vdots \\
(x^{(i/p)})^{m-1}(t)
\end{pmatrix}^{(1/p)} =
\begin{pmatrix}
0 & 1d & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1d & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1d & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 1d \\
A & -BK & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
(x^{(i/p)})^0(t) \\
(x^{(i/p)})^1(t) \\
(x^{(i/p)})^2(t) \\
\vdots \\
(x^{(i/p)})^{m-1}(t)
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
0 \\
\vdots \\
0 \\
0
\end{pmatrix}^T w(t)$$  \hspace{1cm} (17)
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Note:

\[ X(t) = \left[ (x^{(1/p)})^0(t), (x^{(1/p)})^1(t), \ldots, (x^{(1/p)})^{p-1}(t) \right]^T \]

and

\[
\begin{pmatrix}
0 & Id & 0 & 0 & 0 & 0 \\
0 & 0 & Id & 0 & 0 & 0 \\
0 & 0 & 0 & Id & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & Id \\
A & -BK & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\theta_0 \\
\theta_0 \\
\theta_0 \\
\theta_0 \\
\theta_0 \\
\theta_0
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
\vdots \\
0 \\
D
\end{pmatrix}
\]

wherein \( \Lambda, \) \( \text{Id} \) and \( 0 \) are, respectively, the two vectors of identity and zero of the same dimension than vector \( x \), i.e. in our case \((6, 6)\), and \( \Lambda \) is a square matrix of dimension \((p \times \text{dim}(x))\).

However, in \( D, \) \( 0 \) represents a zero rectangular matrix of the same dimension as \( D \). We can then write:

\[ X^{(1/p)} = \Lambda X + Dw(t). \] (19)

Before giving the solution of such a system, a study of stability is essential. In contrast to linear systems of integer order given by \( \dot{x}(t) = Ax(t) + Bu(t) \), for which the stability condition requires that the eigenvalue of the matrix \( A \) be of negative real parts, the fractional system \((19)\) requires different conditions stated in the following theorem due to Matignon (2002).

**Theorem** (Matignon 2002): the system \( X^{(1/p)} = \Lambda X + Dw(t) \) is stable, if and only if:

\[ |\arg(\text{spectre}(\Lambda))| > \frac{\pi}{2p}. \]

The problem of stabilization by state feedback is equivalent to finding a matrix \( K \) that stabilizes \((19)\), i.e. which checks the stability condition given by theorem above.

Once the study of stability is being completed, one gives the analytical solution of system \((19)\) that uses a class of special functions:

\[ x(t) = E_{1/p}(\Lambda, t)x(0) + (\varepsilon_{1/p}(\Lambda, .)Dw)(t), \] (20)

where \( E_p(\Lambda, t) \) is the one-parameter Mittag–Leffler function (Dzhebashyan 1966) defined by:

\[ E_{1/p}(\Lambda, t) = \sum_{k=0}^{\infty} \frac{\Lambda^k}{\Gamma(1 + k/p)}. \] (21)

And where \( \varepsilon_{1/p}(\Lambda, .) \) is the fundamental distribution of the fractional derivative operator \( D^{1/p} \) given by:

\[ \varepsilon_{1/p}(\Lambda, t) = \sum_{k=0}^{\infty} \frac{\Lambda^k t^{(1+k)/p-1}}{\Gamma(1 + k/p)}. \] (22)

Or more generally for \( \alpha \in \mathbb{R} \):

\[
\begin{cases}
\varepsilon_\alpha(\Lambda, t) = \sum_{k=0}^{\infty} \frac{\Lambda^k t^{(1+k)/\alpha}}{\Gamma(1 + \alpha k)} \\
E_\alpha(\Lambda, t) = \sum_{k=0}^{\infty} \frac{\Lambda^k t^{(1+k)/\alpha}}{\Gamma(1 + \alpha k)}
\end{cases}
\]

where \( x(0) \) is the initial vector conditions in which only the components \((\phi(0), \theta(0), \psi(0), \phi(0), \theta(0), \psi(0))\) are non-null. The other fractional initial components are selected null because up to now those quantities do not have any physical interpretation.

The control laws that are considered here are those for which \( 0 < |\alpha| < 1, \alpha \in \mathbb{Q} \).

5. Simulation and comparison of an LQR and a fractional controller

This section proposes, in addition to the LQR control, various fractional control strategies that were the subject of several tests. Note that the calculation of the solution directly according to \((20)\) is numerically difficult. To employ it, it is necessary to consider asymptotic methods of calculation. We thus prefer to use an algorithm (not introduced here) based on the fourth-order Runge–Kutta method, as adapted to the fractional systems. The simulation parameters are the orbital rate \( \omega_0 = 0.00104 \text{ rad/sec} \) and the total moment of inertia matrix for the satellite (Kailil 2000):

\[
I_s = \begin{bmatrix}
4.730759 & 0 & 0 \\
0 & 4.518349 & 0 \\
0 & 0 & 3.494047
\end{bmatrix} \text{ Kg.m}^2.
\]

The initial conditions are chosen as:

\[ x(0) = (1^\circ \ 1^\circ \ 1^\circ \ 0.01^\circ/\text{s} \ 0.01^\circ/\text{s} \ 0.01^\circ/\text{s}). \]
5.1. Fractional control law in state space: $u(t) = -Kx^{(a)}$

Figures 1 and 2 show the Euler angles versus time graph for $\alpha$ between $-0.5$ and $0.5$. Note that the angles are settling down to zero for different $\alpha$. However, the settling time for negative values of $\alpha$ is shorter than those for positive ones. Table 1 gives the steady state error taken at 150 s for some intermediate values of $\alpha$. Note that error passes from $8.257 \times 10^{-4}$ for $\alpha = -0.5$, to $0.0866$ for $\alpha = 0.5$.

The good accuracies obtained for the negative values are because for those values we introduce a fractional integrator that ensures good performances but which requires more energy. Therefore, even if there is a loss in terms of steady accuracy, this is compensated for by the energy saving (figure 3). This is a good compromise and the choice must be made depending on the mission purpose and parameters.

To highlight the compromise between accuracy and energy, the control action due to the reaction wheels required for 150 s for each $\alpha$ is plotted. Figure 4 shows the evolution of the control action, respectively, in roll, pitch and yaw axes with respect to $\alpha$. It can be seen that the curve has a parabolic behaviour with a minimum at $\alpha = 0.3$. On the other hand, the steady-state errors for different values of $\alpha$ are plotted (figure 4). The best
accucess are obtained for $\alpha = -0.5$. This gain in accuracies is compensated for by a loss of energies (figure 3).

Another important result that justifies and consolidates the choice of the fractional controller laws concerns its robustness with respect to uncertainties in satellite intrinsic parameters. Figures 1 and 2 have no undershoots. To show the degree of sensibility in terms of undershoots, another gain matrix with new initial conditions is chosen.

Several tests are made. The following investigates the effect of uncertainty in the pitch moment of inertia estimation. The results are given for $I_\theta = 6.518349$ kgm$^2$, which is equivalent to an uncertainty of $\Delta I_\theta = 2$. As shown in figure 5, uncertainty on the nominal state lets the overshoot increase in pitch angle by almost 200% for $\alpha = 0$, whereas it increases by only 47.95% for $\alpha = 0.5$, which explains the robustness of the fractional control law $u(t) = -KX^{(\alpha)}$ for positive values of $\alpha$.

5.2. Proportional Fractional Derivative $PD^{(1+\alpha)}$

As shown above, the control law $u(t) = -KX^{(\alpha)}$ is more accurate than the LQR controller for negative values of $\alpha$; however, it requires more energy. For positive values, there was a gain in energy in contrast to a loss in accuracy. By the following fractional control law we try to palliate this problem, i.e. keeping the accuracy with a minimum increase in energy. Moreover, after several tests and the running of programmes, it was concluded that the interesting values of $\alpha$ are between 0 and 0.1.

The control law is given by:

$$u(t) = -K_1x_1 - K_2x_2^{(\alpha)},$$

where $x_1 = (\phi \ \theta \ \psi)^T$ and $x_2 = (\dot{\phi} \ \dot{\theta} \ \dot{\psi})^T$.

This control law is the equivalent to a $(PD^{(1+\alpha)})$ controller, which is a particular case of a $(PI^{(1)}D^{(0)})$ controller, if the state is selected as being $(\phi \ \theta \ \psi)^T$. It can be expected that the $(PI^{(1)}D^{(0)})$ controller enhances system performance because it has more tuning knobs introduced. This last control law will be investigated in further studies. However, note the non-conventional way of robust control based on fractional order calculus.

Figures 6 and 7, respectively, show the Euler angles and angular velocities versus time. The first subplot is for roll motion; the second subplot is for pitch motion; the third subplot is for yaw motion, all for different values of $\alpha$. Better precision and a shorter settling time are obtained when $\alpha$ increases. The evolution of angular and velocity settling times are also plotted.
for different $\alpha$ (to reach $0.01^\circ$ in angles and $0.001^\circ$/s in velocities) (Figures 8 and 9).

Tables 2 and 3 have some examples of settling times for different values of $\alpha$. The angle curves show a minimum for $\alpha = 0.04$. The percentage in pitch angle decreases from approximately 57%, and 16% in velocity, between the minimum ($\alpha = 0.04$) and the initial value for $\alpha = 0.0$ (LQR).

Figure 10 shows the reaction wheels control action versus time graphs. Note that the curves of different $\alpha$ are almost superimposed, which mean that the control action slightly varies with $\alpha$ (Figure 11).

Table 4 shows examples of control actions required for the specified time in the roll, pitch and yaw axes.

The control action grows almost linearly with slight slopes: 0.00342 in roll, 0.00277 in pitch and 0.00233 in yaw. Note the slight difference between different fractional controllers and the LQR controller ($\alpha = 0$) that requires the smallest control effort.

Note that this difference is negligible with respect to the settling time and the accuracy obtained with the fractional controllers.

5.3. Robustness of the fractional-order controller

This section investigates the degree of sensibility of the system to the perturbation variations. The following perturbations are applied to the system:

\[
P(t) = \begin{bmatrix}
4 \times 10^{-6} + 2 \times 10^{-6} \sin(\omega t) \\
6 \times 10^{-6} + 3 \times 10^{-6} \sin(\omega t) \\
3 \times 10^{-6} + 3 \times 10^{-6} \sin(\omega t)
\end{bmatrix} + \begin{bmatrix}
2.0 \times 10^{-6} \\
2.0 \times 10^{-6} \\
2.0 \times 10^{-6}
\end{bmatrix}
\]
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Reaction Wheels Control Action for different alpha

Figure 11. Reaction wheels control the action required for different fractional controllers.

Table 4 Reaction wheels control action required for different $\alpha$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Time (s)</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0075544</td>
<td>0.0068162</td>
<td>0.0062403</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>0.0076162</td>
<td>0.0068664</td>
<td>0.0062821</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>0.0076487</td>
<td>0.0068917</td>
<td>0.0063040</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>0.0076839</td>
<td>0.0069204</td>
<td>0.0063281</td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>0.0077596</td>
<td>0.0069825</td>
<td>0.0063804</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 Percentage of settling time variation (examples).

<table>
<thead>
<tr>
<th></th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Settling time (s)</td>
<td>% Settlement time</td>
<td>Settling time (s)</td>
</tr>
<tr>
<td>0.0</td>
<td>95.58</td>
<td>13.24</td>
<td>161.96</td>
</tr>
<tr>
<td>0.02</td>
<td>70.52</td>
<td>4.97</td>
<td>115.6</td>
</tr>
<tr>
<td>0.04</td>
<td>58.67</td>
<td>2.66</td>
<td>79.22</td>
</tr>
<tr>
<td>0.07</td>
<td>51.54</td>
<td>1.7</td>
<td>65.55</td>
</tr>
<tr>
<td>0.08</td>
<td>46.68</td>
<td>1.21</td>
<td>57.74</td>
</tr>
</tbody>
</table>

Figure 12. Percentage of settling time variations.

Table 5 shows the settling time and their variation ratio for some values of $\alpha$. Figure 12 gives the variation of these ratios with respect to $\alpha$ (from 0 to 0.04). It is clear that the ratio tends towards zero when $\alpha$ increases, which means that the system becomes less sensitive to perturbation variations. Note that for pitch angle, the percentage graph presents a peak around $\alpha = 0.008$ (29.31%), after which it settles down exponentially.

6. Conclusion

Earth-pointing satellite attitude stabilization with fractional control laws methods has been presented. First, the theory relating to such a law of control has been developed, and a generalized state representation to a system feedback, using fractional derivative, has been given. A comparative study between fractional laws and an optimal control law (LQR) were then established. Two fractional control strategies have been proposed and examined to stabilize satellite attitude movement. The first represents control by state feedback completely fractional. The second is equivalent to a $PD^{(1+\alpha)}$ controller. The study was made for values of $\alpha$ in the interval $[-1, 1]$ in the first case and for $[0, 0.1]$ in the second case.

In both cases, the fractional-order controllers are more flexible and gave a better opportunity to adjust the dynamical properties of the fractional order control system by means of additional parameters that are acted upon: the order of fractional derivative $\alpha$, which constitutes an interpolation of the integer order derivative, and then allows the enhancement of the quality of the control.

Results show that fractional-order controllers are more robust with respect to both systems' parameters, uncertainties and external disturbances. They are also more accurate and faster in terms of time response than LQR control for some specific values of $\alpha$.

Current studies are in progress dealing with some new algorithms and other fractional methods.

References


KAILIL, A., 2000, Architecture et Analyse Dynamique par la Methode des Elements Finis de la plate forme d'un microsatellite. DESA, CRES, Mohammadia Engineers School, Morocco.


