1. Introduction

**Purpose of this Work:**
- To Propose an Auto-Tuning Method for a Fractional PI\(^\lambda\)D\(^\mu\) Controller.

- The introduction of the orders \(\lambda\) and \(\mu\) allows the fulfillment of a robustness constraint without increasing the complexity of the design method (simple equations).
- The method uses the relay test to obtain the information of the plant to control, due to its reliability and simplicity.

\[ C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \]

\[ C_{opt}(s) = k_p \left(1 + \frac{1}{T_1 s} \right)^{-\lambda} \left(1 + \frac{T_2 s}{1 + \frac{1}{N}} \right)^\mu \]
2. The Relay Test

- Relay Auto-Tuning Scheme:

  Process: \( G(s) \)

  Condition for Oscillation:
  \[
  \arg(G(j\omega)) = -\pi + \phi_0 \theta_0,
  \quad |G(j\omega)| = \frac{\omega_c}{N(\omega)}
  \]

Where:
- \( \omega_c \) is the frequency of interest.
- How to select the right value of \( \theta_0 \) which corresponds to \( \omega_c \)?

  Iterative Process
  \[
  \theta_n^\ast = \frac{\omega_n - \omega_{n-1}}{\omega_{n-1} - \omega_{n-2}}(\theta_{n-1} - \theta_{n-2}) + \theta_{n-1}
  \]

  \( n \): Current iteration number

  \((\theta_1, \theta_2); (\omega_1, \omega_2)\): Two Initial Values of the Delay and Their Corresponding Frequencies

3. The Auto-Tuning Method Proposed (I)

- SPECIFICATIONS OF DESIGN:
  - Crossover Frequency \( \omega_c \)
  - Phase Margin \( \phi_m \)
  - Robustness Property (flat phase)

- Relay Auto-Tuning Scheme:

  \[
  C_{ad}(s) = k_p \left( 1 + \frac{1}{T_1 s} \right)^{\lambda} \left( 1 + \frac{T_2 s}{1 + \frac{T_3 s}{N(\omega)}} \right)^{\mu}
  \]

- \( \lambda \): Lag Compensator with pole at the origin=PI
- \( \mu \): Lead Compensator
- \( N(\omega) \): Noise Filter

3. The Auto-Tuning Method Proposed (II)

1) Robustness Constraint: flat phase of the open-loop system

- Constant overshoot for gain variations

- \( \phi^\ast = \text{arg} \left( PT^\ast(s) \right) = \lambda \left( \text{atan}(\lambda_1 \omega) - \pi/2 \right) \)

- \( \psi^\prime = \frac{d\psi}{d\omega} \big|_{\omega=\omega_c} = \lambda_1 \frac{1}{1 + (\lambda_1 \omega_c)^2} \)

- Slope of the phase of the plant, estimated by the relay test

3. The Auto-Tuning Method Proposed (III)

2) * Crossover frequency specification, \( \omega_c \)

* Phase margin specification, \( \phi_m \)

- \( \lambda \): Lead Compensator

- \( \mu \): ROBUSTNESS CONSTRAINT (AGAIN)
3. The Auto-Tuning Method
Proposed (IV)

LEAD COMPENSATOR ($\alpha > 0$)

$$C(s) = k\left(\frac{s + 1/A}{s + 1/\lambda}\right)^\alpha = k_x x^{\alpha}\left(\frac{\lambda s + 1}{\lambda x s + 1}\right)^\alpha,$$

$0 < x < 1, \alpha \in \mathbb{R}$

$$\frac{C(s)}{k_x x^{\alpha}} \bigg|_{x=x_m} = C(s) = \left(\frac{(\lambda x s + 1)^{\alpha}}{(x\lambda s + 1)^\alpha}\right)^\alpha = \left(\frac{1}{\sqrt{x}}\right)^\alpha$$

$$\text{Arg}(C(s))_{x=x_m} = \phi_m = \alpha \sin \left(\frac{1-x}{\sqrt{1+x}}\right)$$

$\omega_{zero} = 1/\lambda, \omega_m = \frac{1}{\lambda \sqrt{x}}$

$\omega_{pole} = 1/xx\lambda$

$\alpha$: Flexibility in the Design

Modulation of the Phase Curve

$\phi_m$: Does Not Impose the Value of $x$

3. The Auto-Tuning Method
Proposed (V)

LEAD COMPENSATOR ($\alpha > 0$)

$\alpha \geq \alpha_{min}$

Lead Region

$$(\alpha, a, b_1), (\alpha, s, \lambda); k_c = k_s/k_x^{\alpha}$$

$\alpha_{min}$: Maximum Distance Between Zero-Pole (Phase Flatness)

Doing Some Calculations:

$$x = \frac{a(a-1)+b^2}{b(\alpha-1)}$$

$$\lambda = \frac{a(a-1)+b^2}{b\omega_c}$$

3. The Auto-Tuning Method
Proposed (VI)

$$C(s) = k_{std}(\frac{\lambda_3 s + 1}{s})^{\lambda} \left(\frac{\lambda_2 s + 1}{x\lambda_2 s + 1}\right)^\mu$$

$$C_{std}(s) = k_p \left(1 + \frac{1}{T_i s}\right)^\lambda \left(1 + \frac{T_d s}{1 + \frac{s}{T_d}}\right)^\mu$$

$$C_{std}(s) = \frac{k_p}{(T_i)^\lambda} \left(\frac{T_i s + 1}{s}\right)^\lambda \left(\frac{T_d (1 + \frac{1}{x}) s + 1}{\frac{1}{T_d s} + 1}\right)^\mu$$

$$T_i = \lambda_1, \quad k_p = \frac{k_p}{\lambda_1^\lambda}, \quad \frac{T_d (1 + \frac{1}{x})}{T_i} = \lambda_2, \quad T_d = \frac{\lambda_1^\lambda}{x\lambda_2},$$

$N = \frac{1}{\lambda^2}$

3. The Auto-Tuning Method
Proposed (VII)

To sum the auto-tuning method up:

- The relay test: *magnitude and phase of the plant at $\omega_1, \omega_2, \omega_3$
  - estimation of the slope of the phase of the plant
- PI$(s)$: cancell the slope of the phase of the plant, giving the minimum lag phase possible=robustness constraint
- PD$(s)$: fulfills the frequency specifications of $\omega_c$ and $\phi_m$, following a robustness constraint
- The parameters of the fractional PI$D^\mu(s)$ controller are obtained by simple equations that can be solved by a PLC (industrial application)
- The implementation of the fractional controller: we are currently working on it
4. Illustrative Example of Application (I)

UNKNOWN PLANT:

\[ G(s) = \frac{0.55}{s(0.6s + 1)} e^{-0.4s} \]

SPECIFICATIONS OF DESIGN:
- Robustness Constraint
- Crossover Frequency \( \omega_c = 2.3 \text{ rad/sec} \)
- Phase Margin \( \phi_m = 72^\circ \)

RELAY TEST:
- Slope of the phase of the plant = 0.2566

\[ PF^*(s) = \left( \frac{0.4348s + 1}{a} \right)^{1.1803} \]

4. Illustrative Example of Application (II)

UNKNOWN PLANT:

\[ G(s) = \frac{0.55}{s(0.6s + 1)} e^{-0.4s} \]

SPECIFICATIONS OF DESIGN:
- Robustness Constraint
- Crossover Frequency \( \omega_c = 2.3 \text{ rad/sec} \)
- Phase Margin \( \phi_m = 72^\circ \)

RELAY TEST:
- Slope of the phase of the plant = 0.2566

\[ PF^*(s) = \left( \frac{0.4348s + 1}{a} \right)^{1.1803} \]

4. Illustrative Example of Application (III)

UNKNOWN PLANT:

\[ G(s) = \frac{0.55}{s(0.6s + 1)} e^{-0.4s} \]

SPECIFICATIONS OF DESIGN:
- Robustness Constraint
- Crossover Frequency \( \omega_c = 2.3 \text{ rad/sec} \)
- Phase Margin \( \phi_m = 72^\circ \)

RELAY TEST:
- Slope of the phase of the plant = 0.2566

\[ PF^*(s) = \left( \frac{0.4348s + 1}{a} \right)^{1.1803} \]

4. Illustrative Example of Application (IV)

UNKNOWN PLANT:

\[ G(s) = \frac{0.55}{s(0.6s + 1)} e^{-0.4s} \]

SPECIFICATIONS OF DESIGN:
- Robustness Constraint
- Crossover Frequency \( \omega_c = 2.3 \text{ rad/sec} \)
- Phase Margin \( \phi_m = 72^\circ \)

RELAY TEST:
- Slope of the phase of the plant = 0.2566

\[ PF^*(s) = \left( \frac{0.4348s + 1}{a} \right)^{1.1803} \]
4. Illustrative Example of Application (V)

**UNKOWN PLANT:**
\[ G(s) = \frac{0.55}{s(0.6s + 1)} e^{-0.1s} \]

**SPECIFICATIONS OF DESIGN:**
- Robustness Constraint
- Crossover Frequency \( \omega_c = 2.3 \text{ rad/sec} \)
- Phase Margin \( \phi_m = 72^\circ \)

5. Conclusions and Current Works

**In This Work:**
- An auto-tuning method for the fractional PI\(^\lambda\)D\(^\mu\) controller has been proposed.
- This method is based on a robustness criterium.
- The method is reliable and easy to apply. The controller parameters are directly obtained by very simple equations.
- Very suitable for industrial application.

**Current Works:**
- Implementation and experimental results in a PLC.

4. Illustrative Example of Application (VI)

ON AUTOTUNING OF FRACTIONAL CONTROLLERS

**AUTO-TUNING OF FRACTIONAL PI\(^\lambda\)D\(^\mu\) CONTROLLERS**

C.A. Monje*, B.M. Vinagre*, V. Feliú**, Y.Q. Chen***

*Escuela de Ingenierías Industriales. Universidad de Extremadura (Badajoz), Spain. e-mail: cmonje@unex.es; bvinagre@unex.es
**Escuela Técnica Superior de Ingenieros Industriales. Universidad de Castilla-La Mancha, Spain. e-mail: Vicente.Feliu@uclm.es
***CSOIS, Utah State University, Logan, Utah, USA. e-mail: yqchen@ece.usu.edu