Ubiquitous Fractional Order Controls?

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14:30-15:30, July 21, 2006, Auditório E
2nd IFAC Workshop on Fractional Differentiation and its Applications (IFAC FDA’06), July 19-21, 2006, Porto, Portugal.
CSOIS research impacts (1998-2004)

- **Educational**
  - 2 PhD graduated with 2 others expected this year
  - 38 MS and ME students graduated
  - Numerous ECE and MAE Senior Design Projects

- **Scholarly**
  - Five faculty collaborating between three different departments
  - Four books
  - Over 100 refereed journal and conference publications
  - 18 visiting research scholars from 7 countries (3 month to 1 year visits)

- **Economic**
  - 14 full-time staff employed (average of 7 FTE per year)
  - 8 PhD students employed
  - 64 MS and ME students employed
  - 31 Undergraduate students employed
  - 12 Other staff employed
  - A payroll of over $5 million in salaries paid to students, faculty, and staff
  - Purchases of over $1.5M in the U.S. economy

CSRA Research:

Center for Self-Organizing and Intelligent Systems

- CSOIS is a research center in USU’s Department of Electrical and Computer Engineering that coordinates most CSRA (Control Systems, Robotics and Automation) research
- Officially Organized 1992 - Funded for 7 (seven) years by the State of Utah’s Center of Excellence Program (COEP)
- Horizontally-Integrated (multi-disciplinary)
  - Electrical and Computer Engineering (Home dept.)
  - Mechanical Engineering
  - Computer Science
- Vertically-integrated staff (20-40) of faculty, postdocs, engineers, grad students and undergrads
- Average over $2.0M in funding per year since 1998
- Three spin-off companies since 1994.

CSOIS Core Capabilities and Expertise

- Control System Engineering
  - Algorithms (Intelligent Control)
  - Actuators and Sensors
  - Hardware and Software Implementation
- Intelligent Planning and Optimization
- Real-Time Programming
- Electronics Design and Implementation
- Mechanical Engineering Design and Implementation
- System Integration

*We make real systems that WORK!*
CSRA/CSOIS Courses

- Undergraduate Courses
  - MAE3340 (Instrumentation, Measurements); ECE3620/40 (Laplace, Fourier)
  - MAE5310/ECE4310 Control I (classical, state space, continuous time)
  - MAE5620 Manufacturing Automation
  - ECE/MAE5320 Mechatronics (4cr, lab intensive)
  - ECE/MAE5330 Mobile Robots (4cr, lab intensive)
- Basic Graduate Courses
  - MAE/ECE6340 Spacecraft attitude control
  - ECE/MAE6320 Linear multivariable control
  - ECE/MAE6350 Robotics
- Advanced Graduate Courses
  - ECE/MAE7330 Nonlinear and Adaptive control
  - ECE/MAE7350 Intelligent Control Systems
  - ECE/MAE7360 Robust and Optimal Control
  - ECE/MAE7750 Distributed Control Systems

CSOIS Research Strengths

- ODV (omni-directional vehicle) Robotics
- MAS-net (mobile actuator and sensor networks)
- Fractional Dynamic Systems and Control
- Iterative Learning Control Techniques
- Smart Mechatronics, Computer Vision
- Unmanned Autonomous Vehicles (UAVs)
  - Formation Control and Information Consensus Building

Some Robots Built At USU

USU ODV Technology

- USU has worked on a mobility capability called the “smart wheel”
- Each “smart wheel” has two or three independent degrees of freedom:
  - Drive
  - Steering (infinite rotation)
  - Height
- Multiple smart wheels on a chassis creates a “nearly-holonomic” or omni-directional (ODV) vehicle
T1 Omni Directional Vehicle (ODV)

Maneuverability of the ODV design concept

Mobility enhancements achieved by ODV design

“Putting Robots in Harm’s Way, So People Aren’t”

Ten ODIS Manufactured
**CSOIS Research Strengths**

- ODV (omni-directional vehicle) Robotics
- MAS-net (mobile actuator and sensor networks)
- **Fractional Dynamic Systems and Control**
- Iterative Learning Control Techniques
- Smart Mechatronics, Computer Vision.
- Unmanned Autonomous Vehicles (UAVs)
  - Formation Control and Information Consensus Building

**Outline**

- Introduction - CSOIS and Research Strength
- **Fractional Order Control – Better?**
  - Fractional order calculus, systems, controls
  - Example-1: FO PID
  - Example-2: FO Boundary Control
- Fractional Order System Controls – Ubiquitous?
- Concluding Remarks

**What Is Non-Integer Order Calculus?**

- “In-between” thinking, e.g.,
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the fuzzy logic;
  - Between integer order splines, there are “fractional order splines”
  - Between integer high order moments, there are noninteger order moments
  - Between “integer dimensions”, there are fractal dimensions
  - Fractional Fourier transform (FrFT)
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)
Why and How and When

• Why – Many reasons. Dynamic systems modeling and controls

• How – Analog versus digital realization methods. Many.

• When – Now. Ubiquitous. Take a try since we have the new tool. The beginning of a new stage

<table>
<thead>
<tr>
<th>1695</th>
<th>1960s</th>
</tr>
</thead>
<tbody>
<tr>
<td>static models</td>
<td>dynamical models</td>
</tr>
<tr>
<td>geometry, algebra</td>
<td>differential and integral calculus</td>
</tr>
</tbody>
</table>

Modeling: heat transfer

\[
\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{k^2 \partial y(x, t)}{\partial t}, \quad (t > 0, \ 0 < x < \infty)
\]

\[
y(0, t) = m(t)
\]

\[
y(x, 0) = 0
\]

\[
\lim_{x \to \infty} y(x, t) < \infty
\]

Transfer function:

\[
\frac{d^2 Y(x, s)}{dx^2} = k^2 s Y(x, s)
\]

\[
Q(0, s) = M(s)
\]

\[
\lim_{x \to \infty} Y(x, s) < \infty
\]
\[ Y(x, s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}} \]

- \[ A(s) = Y(0, s) = M(s) \]
- \[ B(s) = 0 \]

\[ Y(x, s) = M(s)e^{-kx\sqrt{s}} \]

\[ G(s) = \frac{Y(x, s)}{M(s)} = e^{-kx\sqrt{s}} \]

**think about transfer function** \( e^{-\sqrt{s}} ! **

---

**FO Controller + IO Plant**

**System transfer function**: \( G(s) = \frac{k}{s^{(1+\alpha)}} J \) being the payload inertia. Phase margin of controlled system:

\[ \Phi_m = \arg \left[ C(j\omega)G(j\omega) \right] + \pi \]

Controller: \( C(s) = k_1 \frac{s^{(1+\alpha)}}{s^{(1+\alpha)}} \)

\[ k_2 = \rho \quad \text{giving a constant phase margin:} \]

\[ \Phi_m = \arg \left[ C(j\omega)G(j\omega) \right] + \pi = \arg \left[ (j\omega)^{-(1+\alpha)} \right] + \pi = \pi - (1 + \alpha) \frac{\pi}{2} \]

**Step response**:

\[ y(t) = \mathcal{L}^{-1} \left\{ \frac{k k_1 / J}{s^{1+\alpha} + k k_1 / J} \right\} = \left( \frac{k k_1}{J} \right) t^{1+\alpha} e^{1+\alpha} \left( \frac{k k_1}{J} \right)^{1+\alpha} \]

---

**Oustaloup’s CRONE Control**

**CRONE**: French abbreviation for “Contrôle Robuste d’Ordre Non Entier” (non-integer order robust control) (Since 1981)

Based on concept of “Fractal Robustness”: the isodamping and the vertical sliding form of frequency template in the Nichols chart.

Given plant \( G(s) \), how to design \( C(s) \)? The ideal situation is to make \( G(s)C(s) = (\tau s)^n \) so that the characteristic equation is: \( 1 + (\tau s)^n = 0 \), which is “Fractal Robust”.

Real life applications: car suspension control, flexible transmission, hydraulic actuator etc.
Fractional order PID control

- 90% are PI/PID type in **Ubiquitous** industry.

\[ u(t) = K_p e(t) + T_i \frac{1}{I_D} e(t) + \frac{1}{J_D} e(t) \].


YangQuan Chen, Dingyu Xue, and Huifang Dou. “Fractional Calculus and Biomimetic Control”. IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shengyang, China.

YangQuan Chen, Dingyu Xue, and Huifang Dou. “Fractional Calculus and Biomimetic Control”. IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shengyang, China.

**Realizations: Analog**

- IO Controller + IO Plant
- FO Controller + IO Plant (Example 1)
- FO Controller + FO Plant (Example 2)
- IO Controller + FO Plant


A modified Oustaloup’s approximation method

A new approximate realization method for fractional derivative in the frequency range of interest \((\omega_h, \omega_e)\)

\[
G(s) = K \left( \frac{ds^\alpha + bs\omega_h}{d(1-\alpha)s^\alpha + bs\omega_h + d\alpha} \right) \prod_{k=-N}^{N} \frac{1+s/\omega_k}{1+s/\omega_k}
\]

where the zeros, poles and the gain can be

\[
\omega_k = \left( \frac{d}{b\omega_h} \right)^{\alpha+2k \choose 2N+1} \quad \omega_k = \left( \frac{b}{d\omega_h} \right)^{\alpha+2k \choose 2N+1} \quad K = \left( \frac{d\omega_h}{b} \right)^{\alpha \prod_{k=-N}^{N} \omega_k}
\]


Realizations: CT (Oustaloup) Approx.

Realizations: DT Approx. (fo_c2d.m)


Missing: Joint time-frequency domain approximation
**Example-1: FO Controller + IO Plant**


**The Benchmark Position Servo System**

The LTI state-space form of the system is given by:

\[
\begin{bmatrix}
\dot{x}_p \\
\dot{x}_v 
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & 0 
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_v 
\end{bmatrix} +
\begin{bmatrix}
\frac{k_T}{J_M} \\
\frac{k_T}{J_M} 
\end{bmatrix}
\begin{bmatrix}
\theta \\
T 
\end{bmatrix}
\]

**Servomechanism System's Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_\theta)</td>
<td>1202</td>
<td>(20)</td>
</tr>
<tr>
<td>(k_T)</td>
<td>10</td>
<td>(50)</td>
</tr>
<tr>
<td>(J_M)</td>
<td>0.5</td>
<td>(25)</td>
</tr>
<tr>
<td>(T)</td>
<td>50</td>
<td>(R)</td>
</tr>
<tr>
<td>(R)</td>
<td>20</td>
<td>(J)</td>
</tr>
</tbody>
</table>

**Best IO PID**

- For the optimally searched IO PID using ITAE:
  \[ G_{c1}(s) = 41.94 + \frac{21.13}{s} - 8.26s \]

- For the optimally searched IO PID using ISE:
  \[ G_{c2}(s) = 110.09 + \frac{10.65}{s} + 30.97s \]

**Best FO PID**

- Optimal FO PID using ITAE:
  \[ G_{c3}(s) = 135.12 + \frac{0.01}{s^{0.7}} - 31.6s^{0.6} \]

- Optimal FO PID using ISE:
  \[ G_{c4}(s) = 61.57 + \frac{91.95}{s^{0.5}} + 2.33s^{0.6} \]
How to Decide $\lambda$ and $\mu$?

we build two tables of optimal ITAE and ISE, respectively, with respect to $\lambda$ and $\mu$ which are enumerated from 0.5 to 1.5 with step of 0.1

Which $N$ Is Good Enough?

Robustness Against Load Variations

Fig. 4. Searching the best fractional orders ($N = 4$)

Fig. 5. Searching the best fractional orders ($N = 6$)

Fig. 6. Step responses comparisons with different $N$'s

(a) ITAE($\lambda, \mu$)  (b) ISE($\lambda, \mu$)

(a) ITAE($\lambda, \mu$)  (b) ISE($\lambda, \mu$)

(a) $N = 2$  (b) $N = 6$

(a) +50% load variation  (b) -50% load variation

Fig. 7. Step responses comparison with $N = 4$ (Dotted: Best PID; Solid line: Best FO PID)
**Best IO PI**

- The optimal IO PI using ITAE
  \[ G_{c5}(s) = 107.35 + \frac{0.14}{s} \]

- The optimal IO PI using ISE
  \[ G_{c6}(s) = 106.82 + \frac{3.36}{s} \]

**Fig. 8.** Best IO PI Controllers. Solid line: ITAE; Broken line: ISE

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**Best FO PI**

- Optimal FO PI using ITAE
  \[ G_{c7}(s) = 39.82 + \frac{72.3}{s^{0.05}} \]

- Optimal FO PI using ISE
  \[ G_{c8}(s) = -48.38 + \frac{198.26}{s^{0.2}} \]

**Fig. 10.** Best FO PI Controllers. Solid line: ITAE; Broken line: ISE

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**Decide \( \lambda \)**

**Fig. 9.** Searching the best fractional order (\( N = 4 \))

---

**Robustness Against Load Variations**

- +50% load variation
- -50% load variation

**Fig. 11.** Step responses comparison with \( N = 4 \) (Dashed line: Best IO PI; Solid line: Best FO PI)
Robustness to Mechanical Nonlinearities

A: Using square wave as the reference input signal (period $T=40$ sec.) and adding Coulomb friction 0.1

B: with the deadband width of 0.5

C: the case of deadzone with its parameter set as +/- 0.5

Fig. 12. Responses comparison with Coulomb friction (Dashed line: Best IO Controllers; Solid line: Best FO Controllers)

Fig. 13. Responses comparison with backlash (Dashed line: Best IO Controllers; Solid line: Best FO Controllers)

Fig. 14. Responses comparison with deadzone (Dashed line: Best IO Controllers; Solid line: Best FO Controllers)

Fig. 15. Responses comparison when $k_\theta$ increases 50% (Dashed line: Best IO Controllers; Solid line: Best FO Controllers)
Robustness to Elasticity Parameter Change

B: elasticity parameter \( k_\theta \) decreases 50%

Observations from Example-1

- The best FO PID outperforms the best IO PID
- The best FO PI outperforms the best IO PI
- The best FO PI outperforms the best IO PID

Example-2: FO control + FO plant

Consider a cable made from special materials, with one end fixed and the other end free, governed by

\[
\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} = b_2 \frac{\partial^2 u(x, t)}{\partial x^2}, \quad 1 < \alpha < 2, \quad x \in [0, 1], \quad t \geq 0,
\]

\( u(x, t) \): displacement of the cable at \( x \in [0, 1] \) and \( t \geq 0 \). Boundary conditions:

\[
\begin{align*}
    u(0, t) &= 0, \\
    u_x(1, t) &= f(t)
\end{align*}
\]

\( f(t) \): boundary control force at the free end

Initial conditions:

\[
\begin{align*}
    u(x, 0) &= u_0(x), \\
    u_t(x, 0) &= v_0(x)
\end{align*}
\]
Research questions asked

- When $\mu = 1$, the controller is called integer order controller and has been widely used in the boundary control of wave equations and beam equations.
- When $0 < \mu < 1$, can this controller stabilize the system?
- What advantages does a fractional order controller have over integer order controllers?

How to simulate?

Observations from Example-2

- Fractional order boundary controllers are applicable.
- The best fractional order boundary controller are better than the best integer order boundary controller when the wave equation is time-fractional.

Note: Boundary controllers considered in this example are still in very simple forms. How to design best fractional order controllers structures best suited for the fractional wave equation will be a new future research topic.
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  - Fractional order calculus, systems, controls
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Ubiquitous (1) - hysteresis
Biomimetic Materials and Biomimetic Actuators
- EAP (electroactive polymers), a.k.a. artificial muscle
- ferroelectric and relaxor materials
- piezoceramic and piezopolymetric materials
- liquid crystal elastomers
- electro and magnetostrictive materials
- shape memory alloys/polymers
- intelligent gels etc. However, little has been reported on the controls of actuators made with these biomimetic materials.

Compensation of nonlinearity with memory
- e.g., hysteresis, backlash.
- My Assertion: Fractional calculus may better help us.

Phase Control Approach to Hysteresis Reduction
Juan Manuel Cruz-Hernández, Member, IEEE, and Vincent Hayward, Member, IEEE.

Abstract—This paper describes a method for the design of compensators able to reduce hysteresis in transducers, as well as input measures to quantify and compare controller performance. Rate independent hysteresis, as represented by the Preisach model of hysteresis, is seen as an input-output phase lag. The compensation is based on controllers derived from the “phasor,” a unitary gain operator that shifts a periodic signal by a single phase angle. A “variable phasor” is shown to be able to handle minor hysteresis loops. Practical implementations of these controllers are given and discussed. Experimental results exemplify the use of these techniques.

Index Terms—Compensation, hysteresis, intelligent materials, phase control, piezoelectric transducers, smart materials, transducers.
"smart material" based Fractor™

Analog Fractional Order Controller in a Temperature Control Application

Proc. of the 2nd IFAC FDA’06, July 19-21, 2006, Porto, Portugal.

Big Picture, or, the message for you to take home

- The big picture for the future is the intelligent control of biomimetic system using biomimetic materials with fractional order calculus embedded. In other words, it is definitely worth to have a look of the notion of "intelligent control of intelligent materials using intelligent materials." Advocating this picture is the major purpose or contribution, if any, of this talk.

Ubiquitous (2) – Independent Loopshaping? A Conjecture

- It is well known due to Bode that, for finite dimensional linear time invariant single input and single output rational systems, the gain and phase are inter-related. For robust loop shaping, it is not possible to do loop shaping for gain and loop phase plots independently. However, using fractional order control, this is possible. In other words, fractional order controllers have the potential to do loop shaping of phase and gain independently.
Waterbed Effect

The Law:
\[ \int_0^\infty \ln|S(j\omega)|d\omega = \pi \sum_{i=1}^{N_p} \text{Re}(p_i) \]

- All systems must obey Bode’s Sensitivity Integrals
- Increased Performance over some frequencies = Increased sensitivity in others

\[ S = (1 + L)^{-1} \] Sensitivity Function
\[ T = L(1 + L)^{-1} \] Complimentary Sensitivity Function

Physical Interpretation
- Removal of sensitivity “dirt” from lower frequencies creates higher sensitivity at higher frequencies
- This will lead to instability and/or non-optimal performance at higher frequencies

Question on “waterbed effect”
- In infinite dimensional setting, will this “waterbed effect” be invalid?
- If yes, it is a good news for FOC. But I do not know. Maybe. We need mathematicians to handle this research question.
- It is “yes” for nonlinear controls that have been shown to break the famous (curse?) “waterbed effect”.
Joint loop shaping and phase shaping?

Flat phase condition: \[ \frac{d \angle G(s)}{ds} \bigg|_{s=j \omega_c} = 0 \]


Ubiquitous (3) – FO PI/D tuning

- PI/D controllers are **ubiquitous** in industry.
- Endless possibilities depending on what prior plant knowledge we have.
- Will be a sustainable research topic like integer order PI/D controllers.

\[ G_c(s) = K_p + \frac{K_i}{s^\lambda} + K_ds^\mu \]


Ubiquitous (4) – “mentally low cost” Robust Control

- Robustness is a “ubiquitous” requirement.
- Many existing robust control methods (H_infinity, LMI, H_2, SSV mu synthesis etc.) are “mentally expensive” to understand and follow by busy engineers.
- CRONE provided a “mentally low cost” robust control easily accepted by practitioners. It has been very successful and impressive.
- People may worry about the approximation implementation of FOC. The rationale here is: The robust controller \( C \) is designed to attack the uncertainty \( \Delta P \) of the plant \( P \). However, when implementing \( C \), we introduced another \( \Delta C \). See this paper for the issue — I. H. Keel and S. Bhattacharyya. “Robust, fragile, or optimal?” IEEE Trans. on Automatic Control, 42:1098-1105, 1997.
- So, a more authentic analog FOC element might relieve this worry? Or, let us think in this way – “Low cost (analog) robust control with fractional element”.
- So, shall we convert the robust controller obtained from “expensive” design techniques to “low cost analog robust controllers using FOC element”? Maybe logical.


Ubiquitous (4.5) – A FAQ

- A FAQ is “Ya--! Your FOC (or fractional order filters) is good and interesting but in the end you need to implement it in finite dimensional transfer function (filter) of a high order. Why not simply start with the high order finite dimensional TF?” (implied suspicion: FOC is kind of redundant)

- **Answers:**
  - Less tuning knobs (engineers like this)
  - More insights (note: finite dimensional, lumped parameter modeling, analysis, control is just “for our own convenience”)
  - Etc. (e.g., “mentally economic?!”)

Ubiquitous (5) – A paradigm shift from LPS to DPS

- “Lumped parameter, LTI, finite dimensional, causal, deterministic, integer order, dynamic systems” – Time to shift the paradigm
  - Distributed parameter systems
    - Infinite dimensional
    - Fractional order?
      - Fractional order dynamic systems and control (FODSC) has a good future.
      - FODSC will be as popular as “fuzzy logic” soon. (Think in-between)
        - 2003 Chicago. I shared this thinking in the first meeting of ASME “Fractional Dynamics” subcommittee

A new viewpoint?

- FOC helps to understand the “distributed parameter systems” with “lumped parameter system” viewpoint using irrational transfer function?

  Modeling: heat transfer
  \[
  \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t} \\
  (t > 0, \quad 0 < x < \infty)
  \]
  \[
  y(0,t) = m(t) \\
  y(x,0) = 0
  \]
  \[
  \lim_{x \to \infty} y(x,t) < \infty
  \]

  Transfer function:
  \[
  \frac{\partial^2 Y(x,s)}{\partial x^2} = k^2 Y(x,s) \\
  G(0,s) = M(s) \\
  \lim_{x \to \infty} Y(x,s) < \infty
  \]

  \[
  Y(x,s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}}
  \]
  \[
  A(s) = Y(0,s) = M(s) \\
  B(s) = 0
  \]

  \[
  Y(x,s) = M(s)e^{-kx\sqrt{s}}
  \]
  \[
  G(s) = \frac{Y(x,s)}{M(s)} = e^{-kx\sqrt{s}}
  \]

  **think about transfer function** $e^{-\sqrt{s}}$!
What transfer function?

Order parameter for fluidization: “shear melting” of granular material

\[ \rho = \frac{Z_{sf}}{Z} \]

- \( Z_{sf} \) - total number of contacts per particle (coarse-grained)
- \( Z_{sf} \) - number of persistent contacts

OP characterizes the phase state of granular matter:
- solid: \( \rho = 1 \)
- liquid: \( \rho = 0 \)

2D simulations of granular shear flow between 2 plates. OP is shown by color.
Volfson, Tsujiura & Aronson, PRL 2003

Dust is everywhere
Spatial-fractional and time-fractional dynamics?

- collapse of a silo by stress concentration!

USU Material Research Laboratory

- Materials Processing,
  Heat Treating, Materials Joining, and Powder Metallurgy Studies using the Gleeble 1500D System

Source: [http://www.mae.usu.edu/faculty/leijun/gleeble.html](http://www.mae.usu.edu/faculty/leijun/gleeble.html)

NSF NER: Solid-state synthesis of nano-scale hydrogen storage materials by bulk mechanical alloying

Fractional order calculus?

- Dynamic force measurements vs. strokes

Data credit: Leijun Li

Fractional order vs. strokes

Big picture of nanoparticle manufacturing

- **Now**: given cycles, given stroke profile, see how particulate process evolves.
- **Future**: Production process development – given final particle grain size distribution, how to achieve this by using minimum number of cycles with possible cycle-to-cycle, or run-to-run (per several cycles) adaptive learning control with variable stroke profiles.

Fractional order ILC (iterative learning control)?

- D-alpha type ILC with a (really good) reason?!
Something big to come?

Students of mechanics of materials often raise the question, “Is there a constitutive model which is applicable to all materials?” And I respond: “Although our understanding of the material’s response is growing, there is no model available that can characterize all materials in all respects. To understand and characterize matter (materials) completely, one may need to become the matter itself! When that happens, there is no difference left, and a full understanding may follow.”

This realization is important because the pursuit toward increased comprehension and improved characterization of materials must continue!


Ubiquitous (6) – Long Range Dependence and FOSP

• Consider a second order stationary time series $Y = \{Y(k)\}$ with mean zero. The time series $Y$ is said to be long-range dependent if

$$r_γ(k) = E(Y(k)Y(0)) \leftrightarrow c_γ |k|^{-γ}, k \to \infty, 0 < \gamma < 1$$

$$f_γ(\xi) \leftrightarrow c_γ |\xi|^{-α}, 0 < \alpha < 1,$$

FOSP: fractional order signal processing

Hurst parameter

• The Hurst parameter $H$ characterizes the degree of long-range dependence in stationary time series.
• A process is said to have long range dependence when $0.5 < H < 1$
• Relationships:
  - $\alpha = 1 - \gamma$
  - $H = (1 + \alpha) / 2$
• 2), $d$ is the differencing parameter of ARFIMA $H = d + \frac{1}{2}$

Long-range dependence

• History: The first model for long range dependence was introduced by Mandelbrot and Van Ness (1968)
• Domains:
  - financial data
  - communications networks data
  - video traffic
  - biocorrosion data, QCM signal
  - etc.
Models with Hurst phenomenon

- Fractional Gaussian noise (FGN) models (Mandelbrot, 1965; Mandelbrot and Wallis, 1969a, b, c)
- Fast fractional Gaussian noise models (Mandelbrot, 1971)
- Broken line models (Ditlevsen, 1971; Mejia et al., 1972)
- ARFIMA/FIARMA models (Hosking, 1981, 1984)
- Symmetric moving average models based on a generalised autocovariance structure (Koutsoyiannis, 2000)

1/f noise

- Models of 1/f noise were developed by Bernamont in 1937:
  \[ S(f) = \frac{C}{|f|^\gamma} \]
  where \( C \) is a constant, \( S(f) \) is the power spectral density.
- 1/f noise is a typical process that has long memory, also known as pink noise and flicker noise.
- It appears in widely different systems such as radioactive decay, chemical systems, biology, fluid dynamics, astronomy, electronic devices, optical systems, network traffic and economics

We may define 1/f noise as the output of a fractional system. The input could be white noise.

Also, we can consider 1/f noise as the output of a fractional integrator. The system can be defined by the transfer function

\[ H(s) = \frac{1}{s^\alpha} \]

with impulse response \( h(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)} \quad 0 < \alpha < 1 \)

Therefore, the autocorrelation function of the output is

\[ R(\tau) = \sigma^2 \frac{|\tau|^{2\alpha-1}}{2\Gamma(2\alpha)\cos\alpha\pi} \]
Fractional Gaussian Noise (fGN)

- fGN is a kind of $1/f$ noise.
- fGN can be seen as the unique Gaussian process that is the stationary increment of a self-similar process, called fractional Brownian motion (fBM).
- The fBM plays a fundamental role in modeling long-range dependence processes.
- The increments time series
  \[ G_H(k) = B_H(k) - B_H(k-1), k \in \mathbb{Z} \]
  of the fBM process $B_H$ are called fGN.

Hurst parameter estimation methods

- R/S analysis
- Local Whittle
- Periodogram methods
- Dispensational analysis method
- Wavelet-based
- *Fractional Fourier Transform (FrFT) based

Comparison of some important Hurst parameter estimation methods tested with 100 fGN of known Hurst parameters from 0.01 to 1.00

FOSP Topics:

- Fractional derivative and integral
- Fractional linear system
- Autoregressive fractional integral moving average (ARFIMA)
- $1/f$ noise and $(1/f)^a$ noises
- Hurst parameter estimation
- Fractional Fourier Transform (FrFT), Multiscale, Multiresolution, Time-frequency approaches
- Fractional Cosine, Sine and Hartley transform
- Fractal dimension (FD).
- Fractional Splines
Ubiquitous (7) – Power law and power law Lyapunov

• “Power law is ubiquitous” – John Doyle 2001 IEEE CDC Plenary Talk http://www.cds.caltech.edu/~doyle/CDC2001/index.htm

• “When you talk about power law, you are talking actually about fractional order calculus!” – YangQuan Chen 2006 IFAC FDA06 Plenary Talk

• “Lyapunov is ubiquitous in control literature” – ibid.

Slide credit shared by Igor Podlubny.

A snap shot of discussion board of Igor Podlubny and YangQuan Chen in Sept. 2005

Lyapunov’s Direct Method

- Enables one to determine whether or not the equilibrium state of system

\[ \dot{x} = f(x, t) \] (2)

is stable without actually determining the solution.

- Involves finding a suitable scalar function \( V(x, t) \) and examining its time derivative along the trajectory of the system.

Slide credit shared by Igor Podlubny.
Then,

• (a) (i) and (iii) imply that the origin of differential equation (1) is stable;
• (b) (i), (ii) and (iii) imply the origin of d.e. (1) is uniformly stable;
• (c) (i) - (iv) imply that the origin of d.e. (1) is uniformly stable in the large.

**Power Law Lyapunov Method**

- Enables one to determine whether or not the equilibrium state of system
  \[ x^{(\alpha)} = f(x,t) \]
  is stable without actually determining the solution.
- Involves finding a suitable scalar function \( V(x,t) \) and examining its time derivative along the trajectory of the system.
Then,

- (a) (i) and (iii) imply that the origin of differential equation (#) is stable;
- (b) (i), (ii) and (iii) imply the origin of d.e. (#) is uniformly stable;
- (c) (i) - (iv) imply that the origin of d.e. (#) is uniformly stable in the large.

Intuitions

- You don’t have to be rich to be smart.
- You don’t have to be smart to use fractional order calculus.
- A dynamic system doesn’t have to make the “generalized energy” decay exponentially to be stable!

Call for
Power Law Lyapunov Method

Outline

- Introduction - CSOIS and Research Strength
- Fractional Order Control – Better?
  - Fractional order calculus, systems, controls
  - Example-1: FO PID
  - Example-2: FO Boundary Control
- Fractional Order System Controls – Ubiquitous?
- Concluding Remarks

Ubiquitous (∞) – Final Remarks
Endless possibilities since it is ubiquitous.
The beginning of a new stage

Do more and do better!!
Rule of thumb

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^\alpha$ noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters …

To probe further

http://www.tuke.sk/podlubny/
http://mechatronics.ece.usu.edu/foc/

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- Concepción A. Monje, José Ignacio Suárez, Chunna Zhao, Jinsong Liang, Hyosung Ahn, Tripti Bhaskaran, Theodore Ndzana, Christophe Tricaud, Rongtao Sun, Nikita Zaveri

Q/A Session

- Apologize for not citing carefully math/phyx FOC papers and for not referring to more complete FOC literatures
- Check http://mechatronics.ece.usu.edu/foc for more information.
- Mr. Rongtao Sun. “Fractional Order Signal Processing Methods And Their Applications In Electrochemical Biosensors”, ibid, 2006. (to complete)
IFAC FDA papers

- Anhong Zhou and YangQuan Chen*, “FRACTIONAL ORDER PROCESSING OF QUARTZ CRYSTAL MICROBALANCE BASED DNA BIOSENSOR SIGNALS.” ibid.
- José Ignacio Suárez, Blas M. Vinagre* and YangQuan Chen. “A FRACTIONAL ADAPTIVE SCHEME FOR LATERAL CONTROL OF AN AGV”. ibid.
- Concepción A. Monje, Blas M. Vinagre*, Vicente Feliu and YangQuan Chen. “ON AUTO-TUNING OF FRACTIONAL ORDER PI^lambda D^mu CONTROLLERS”. ibid.

Plenary Lecture #5 Abstract

- There is an increasing interest in dynamic systems and controls of noninteger orders or fractional orders. Clearly, for closed-loop control systems, there are four situations. They are 1) IO (integer order) plant with IO controller; 2) IO plant with FO (fractional order) controller; 3) FO plant with IO controller and 4) FO plant with FO controller. However, from engineering point of view, doing something better is the major concern. This talk will first show an example that the best fractional order controller outperforms the best integer order controller. Then, we try to argue why consider fractional order control even when integer (high) order control works comparatively well. We will also address issues in fractional order PID controller tuning. Using several real world examples, we further argue that, fractional order control is ubiquitous when the dynamic system is of distributed parameter nature.

IFAC FDA’2006 Plenary Lecture #5
Ubiquitous Fractional Order Controls?

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14:30-15:30, July 21, 2006, Auditório E
2nd IFAC Workshop on Fractional Differentiation and its Applications (IFAC FDA’06), July 19-21, 2006, Porto, Portugal.

• His current research interests include autonomous navigation and intelligent control of a team of unmanned ground or aerial vehicles, distributed real-time irrigation control, distributed parameter system controls with MAS-net (mobile actuator-sensor networks), fractional order control and signal processing, interval computation, and iterative/repetitive/adaptive learning control. Dr. Chen has been an Associate Editor in the Conference Editorial Board of IEEE Control Systems Society since 2002. He is a founding member of the ASME subcommittee of "Fractional Dynamics" in 2003. He is a senior member of IEEE, a member of ASME and a member of ISIF (International Society for Information Fusion).

YangQuan Chen’s background
Acting Director of CSOIS since June 2004.

• BS 1985, University of Sci. and Tech. of Beijing (UTSB), China
• MS 1989, Beijing Institute of Technology (BIT), China
• 1996-1999: National Univ. of Singapore
• 1999-2000: Seagate, Singapore Science Park
• 2000-present: Utah State University

• Keywords: Dynamic systems and control (linear, nonlinear, adaptive, robust, optimal, intelligent, distributed, biological, mechatronic, biomimetic ... integer or noninteger order)

• Publication: 12 granted and 2 pending US patents on HDD servomechanics, over xyz papers, over 50 industrial tech. reports., 2 research monographs (95,99) and 4 textbooks (02, 04, 07).
• Now “enjoying” tenure process at Utah State University (s2002). Teaching/research/service.