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September 19, 2004

Technical Report No. : USU-CSOIS-TR-04-01
(Accepted by IEEE Transactions on Magnetics, 10/22/2004)

Reports are available from http://mechatronics.ece.usu.edu/reports/

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Abstract

This paper focuses on the state-periodic adaptive compensation of cogging and Coulomb friction for permanent magnet linear motors (PMLM) executing a task repeatedly. The cogging force is considered as a position dependent disturbance and the considered Coulomb friction is non-Lipschitz at zero velocity. The key idea of our disturbance compensation method is to use one trajectory-period past information along the state axis to update the current adaptation law. The new method consists of three different steps: Firstly, in the first repetitive trajectory, an adaptive compensator is designed to guarantee the ℓ_2-stability of the overall system; secondly, from the second repetitive trajectory and onwards, a trajectory-periodic adaptive compensator is designed to stabilize the system; and finally, to make use of the stored past state-dependent cogging information, a search process is utilized for adapting the current cogging coefficient. The validity of our adaptive cogging and friction compensator is illustrated through the actual PMLM model based simulation.

Index Terms

Cogging force, Coulomb friction force, state-dependent disturbance, adaptive control, trajectory-periodic adaptation.

I. Introduction

Permanent magnet (PM) motors are the most popularly used electromechanical devices for accurate speed and position control of the linear system or rotary system. In parallel with the popularity of PM motors, the nonlinear

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torques inherent to PM motors have been addressed in numerous literatures [1], [2], [3]. In particular, in [4], load torques, friction effects, and cogging torques are addressed as inherent torques of the permanent magnet stepper motors; and in [5], [6], friction, cogging and reluctance forces are modelled for iron-core permanent magnet linear motors. As explained in [7], the cogging forces are due to the interaction between the permanent magnets and the steel teeth of the primary section; and the friction force is a velocity-dependent nonlinear disturbance, which is inherent to most of the electromechanical systems.

In permanent-magnet linear motors (PMLMs), nonlinear mechanical disturbances such as back-lash are greatly reduced; while the cogging forces are considered as the main disturbance [3], [5]. However, static friction force such as Coulomb friction is still a dominant disturbance and should be compensated for accurate speed and position control of PMLMs. Thus, in this paper, we focus on the compensation for the disturbance of cogging force and the Coulomb friction. These disturbances are compensated by the trajectory-periodic adaptation based on Lyapunov stability analysis on the time-axis.

Cogging forces are position dependent periodic disturbances due to the slotted nature of the primary core [3], [6], and generally it is modelled as Fourier expansion [1], [2]. However, in control strategies, it has been modelled as a simple sinusoidal signal such as:

$$F_{cogging} = A\sin(\omega x + \varphi) \tag{1}$$

and the unknown parameters such as A, ω and φ have been compensated by certain parameter adaptation scheme [4], [5], [8]. However, this approach does not represent high order terms in the Fourier series, hence it cannot compensate the cogging force completely. In this paper, we do not assume any model such as (1); instead, it is considered that the cogging force could be any kind of Fourier expansion such as:

$$F_{cogging} = \sum_{i=1}^{\infty} A_i \sin(\omega_i x + \varphi_i), \tag{2}$$

where A_i is the amplitude, ω_i is the state-dependent cogging force frequency, and φ_i is the initial phase. In order to compensate cogging force of (2), it is suggested to make use of the periodicity of cogging disturbance on the repetitive trajectory. Note that cogging force waveform is periodic over a pole-pitch in PMLMs [2].

In control community, Coulomb friction force has been studied widely in [9], [10]; and many efforts have been devoted in late $80's$ and early $90's$ to compensating friction force [11], [12], [13], [14], [15]. After these early works, several adaptive friction compensation controllers have been suggested [16], [17], [18], [19]. We can see that the friction compensation is still considered as a hot topic.

In this paper, we focus on the state-periodic adaptive compensation of cogging and Coulomb friction for PMLM's executing a given task repeatedly. The cogging force is considered as a position dependent disturbance and the considered Coulomb friction is non-Lipschitz at zero velocity. The key idea of our disturbance compensation method is to use one trajectory-period past information along the state axis to update the current adaptation law. The new method consists of three different steps: Firstly, in the first repetitive trajectory, an adaptive compensator is designed to guarantee the ℓ_2-stability of the overall system; secondly, from the second repetitive trajectory and onwards, a trajectory-periodic adaptive compensator is designed to stabilize the system; and finally, to make use of the stored past state-dependent cogging information, a search process is utilized for adapting the current cogging coefficient. The validity of our adaptive cogging and friction compensator is illustrated through numerical simulation based on an actual PMLM model.

The paper is organized as follows: In Section II, a new adaptive state-dependent cogging and friction compensator is designed based on Lyapunov stability analysis. Simulation tests are performed in Section III. Conclusions are given in Section IV.

II. State-Dependent Adaptive Cogging Compensation

In this section, the state-dependent periodic adaptive cogging and friction compensator is designed. The cogging force of (2) can be written as: $-a(x)$, where $a(x)$ is the function of x. Coulomb friction is modelled as:

$$F_{fric} = -b\text{sgn}(v), \tag{3}$$

which is discontinuous at zero velocity. In this paper, the dynamics of a PMLM, which was introduced in [5], is slightly modified for ease of our presentation of our main ideas. In [5], the following equations are given:

$$\begin{aligned} u_v(t) &= k_e\dot{x}(t) + Ri(t) + L\frac{di(t)}{dt} && (4)\\ i(t) &= \frac{1}{k_f}f(t) && (5)\\ f(t) &= m\ddot{x}(t) + F_{load}(t) + F_{fric}(t) + F_{ripple}(x) + F_n(t), && (6) \end{aligned}$$

where $u_v(t)$ and $i(t)$ are time-varying motor terminal voltage and the armature current; $x(t)$ is the motor position; R is the resistance; L is the armature inductance; m is the moving mass; k_f is the force constant; k_e is the back EMF; F_{load} is the applied load torque; F_{ripple} is the position dependent cogging force; and F_n includes all other uncertainties and disturbances except F_{load}, F_{ripple}, and F_{fric}. In our analysis, we ignore load and small disturbances such as F_{load} and F_n, and the armature inductance L due to its small value compared to the resistance R. Then, (4), (5), and (6) are expressed in a simple form such as:

$$\ddot{x}(t) = -\frac{k_f k_e}{Rm}\dot{x} - \frac{1}{m}F_{fric} - \frac{1}{m}F_{ripple} + \frac{k_f}{Rm}u_v(t) \tag{7}$$

From now on, in this paper, let us use $a(x)$ to denote the cogging force instead of using F_{ripple}, and it is supposed that coefficients k_f, k_e, m, and R are known from the motor technical specification sheet (these values can be found in Table 1 of [5]). Finally, the following servo control problem is considered:

$$\dot{x}(t) = v(t) \tag{8}$$

$$\dot{v}(t) = -\frac{p}{m}v - \frac{1}{m}a(x) - \frac{1}{m}b\mathrm{sgn}(v) + u, \tag{9}$$

where x is the position; $a(x)$ is the unknown position-dependent cogging disturbance (i.e., F_{ripple} in (7)); b is the unknown friction coefficient; v is the velocity; u is the control input ($u := \frac{k_f}{Rm}u_v(t)$); and $p := \frac{k_f k_e}{R}$.

First, before proceeding to present our main results, the following definitions and assumptions are necessary.

Definition 2.1: The total passed trajectory is given as:

$$s = \int_0^t \frac{|\mathrm{d}x|}{\mathrm{d}\tau}\mathrm{d}\tau = \int_0^t |v(\tau)|d\tau,$$

where x is the position, and v is the velocity. In [20], it was defined as the curvilinear abscissa associated with the trajectory of the relative motion. In our definition, since s is the summation of absolute position increase along the time axis, s is a monotonously growing signal. Physically it is the total passed trajectory, hence, it has the following property:

$$s(t_1) \geq s(t_2), \text{ iff } \ t_1 \geq t_2.$$

With notation s, the position corresponding to $s(t)$ is denoted as $x(s)$ and the cogging force corresponding to $s(t)$ is denoted as $a(s)$.

Definition 2.2: Since the cogging force arises as a result of the mutual attraction between the magnets and cores of the translator, the cogging force is periodic with respect to position [5]. Based on Definition 2.1, the following relationship is derived:

$$a(s) = a(s - s_p), \text{ and } x(s) = x(s - s_p), \tag{10}$$

where s_p is periodicity of the trajectory.

Definition 2.3: In Definition 2.2, s_p was defined as periodic trajectory. So, $s(t) - s_p$ is one trajectory past point from $x(t)$ on the s axis. Let us denote the time corresponding to $x(t) - s_p$ with T_t. Then, $t - T_t$ is the time-elapse to complete one periodic trajectory from the time T_t to time t. This time-elapse is called a “cycle.” Particularly, it is called a “trajectory cycle” at time t and denoted as P_t. So, $P_t = t - T_t$. It is called “the search process” to find P_t at time instant t (note: the search process can be performed by interpolation).

Furthermore, time t is always monotonically increasing, and in our adaptive controller, the discrete time points are used. The monotonically increasing time sequence is denoted as $t_i, i = 0, \cdots, \infty$, where t_0 is the initial time when the PMLM starts to move. Then, the following relationship is immediate:

$$s(t_{i+1}) \geq s(t_i).$$

From now on, for accurate notation, the position corresponding to the time t_i is denoted as: $x(t_i)$ and its total passed trajectory by the time t_i is denoted as: $s(t_i)$. Henceforward, one trajectory past time from the time instant t_i is denoted as T_{t_i}, and its corresponding cycle is denoted as P_{t_i} (i.e, $P_{t_i} = t_i - T_{t_i}$).

Assumption 2.1: Throughout the paper, it is assumed that the current position and current time of the PMLM are measured. Let us denote the current position at time t_i as $x(t_i)$, where x is the position corresponding to t_i. Then, T_{t_i} is always calculated, hence P_{t_i} is calculated at time instant t_i.

With the above definitions and assumption, the following property is observed.

Property 2.1:

$$x(t_i) = s(t_i) - m' s_p, \tag{11}$$

where m' is the integer part of $s(t_i)/s_p$.

Remark 2.1: As will be shown in the following theorem, for ease of implementation, the actual state-dependent cogging force $a\left(s(t_i)\right)$ is not estimated on the state axis although in theory we can do so. Instead, $a(t_i)$ is estimated on the time axis. So, to find $a\left(s(t_i) - s_p\right)$, the following formula is used:

$$a\left(s(t_i) - s_p\right) = a\left(t_i - P_{t_i}\right) \tag{12}$$

Here, P_{t_i} is calculated in Assumption 2.1 (recall that P_{t_i} can be used to indicate exactly one-trajectory past position).

From (11) and (12), we also have the following property:

Property 2.2: The current cogging force is equal to one-trajectory past cogging force. From the relationship:

$$\begin{aligned} a\left(s(t_i) - s_p\right) &= a\left(x(t_i) + m s_p - s_p\right) \\ &= a\left(x(t_i)\right) \\ &= a\left(t_i - P_{t_i}\right) \end{aligned} \tag{13}$$

the following equality is derived: $a\left(x(t_i)\right) = a\left(t_i - P_{t_i}\right)$.

Now, based on the above discussions, the following stability analysis is performed in this paper. Our compensation approach is summarized as follows:

- When $s(t_i) < s_p$, the system is controlled to be bounded input bounded output (in ℓ_2-norm).
- When $s(t_i) \geq s_p$, the system is stabilized to follow the desired speed at the desired position. By a trajectory periodic adaptation, the unknown external disturbances (the summation of the cogging and friction forces) are

estimated.

The following notations are used:

$$e_x(t_i) = x(t_i) - x_d(t_i);\ e_v = v(t_i) - v_d(t_i);$$

$$e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i)); e_b(t_i) = b - \hat{b}(t_i),$$

where $\hat{a}(s(t_i)) = \hat{a}(t_i)$ (note: t_i is the current time corresponding to the current total passed trajectory $s(t_i)$). Here, let us change $e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i))$ into time domain such as:

$$\begin{aligned} e_a(s(t_i)) &= a(s(t_i)) - \hat{a}(s(t_i)) \\ &= a(t_i) - \hat{a}(t_i) \\ &= e_a(t_i). \end{aligned} \tag{14}$$

In the same way, the following relationships are true:

$$e_b(s(t_i)) = e_b(t_i);\ x(s(t_i)) = x(t_i);\ x_d(s(t_i)) = x_d(t_i);$$

$$v_d(s(t_i)) = v_d(t_i);\ v(s(t_i)) = v(t_i)$$

The control objective is to track or servo the given desired position $x_d(t_i)$ and the corresponding desired velocity $v_d(t_i)$ with tracking errors as small as possible. In practice, it is reasonable to assume that $x_d(t_i)$, $v_d(t_i)$ and $\dot{v}_d(t_i)$ are all bounded. From now on, based on relationship: $a(x(t_i)) = a(t_i - P_{t_i}) = a(t_i)$, $a(x(t_i))$ is equalized to $a(t_i)$ as done in (14); and let us omit subscript i from t_i and P_{t_i}. So, $a(x)$ is replaced by $a(t)$ in the following theorems.

The feedback controllers are designed as: When $s \geq s_p$,

$$u = \frac{\hat{a}(t) + \hat{b}\mathrm{sgn}(v(t))}{m} + \frac{p}{m}v + \dot{v}_d(t) - \alpha S(t) - \lambda e_v(t), \tag{15}$$

and when $s < s_p$,

$$u = \frac{\hat{a}(t)}{m} + \frac{p}{m}v + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t), \tag{16}$$

with

$$S(t) := e_v(t) + \lambda e_x(t), \tag{17}$$

where α and λ are positive gains; $\hat{a}(t)$ is an estimated cogging force from an adaptation mechanism to be specified later; $\hat{b}$ is the estimated friction coefficient; $\dot{v}_d(t)$ is the desired acceleration; and $e_x(t) = x(t) - x_d(t)$ is the position tracking error. Also be reminded that $e_x(s(t)) = e_x(t)$; and $S(s(t)) = S(t)$.

Our adaptation law is designed as follows:

$$\hat{a}(t) = \begin{cases} \hat{a}(t-P_t) - \frac{K}{m}S(t) & \text{if} \quad s \geq s_p \\ z - g(v) & \text{if} \quad s < s_p \end{cases} \tag{18}$$

$$\dot{\hat{b}}(t) = \begin{cases} -\frac{S(t)}{m}\mathrm{sgn}(v) & \text{if} \quad s \geq s_p \\ 0 & \text{if} \quad s < s_p \end{cases} \tag{19}$$

where $\hat{a}(t-P_t) = \hat{a}(s-s_p)$ (note: P_t is the trajectory cycle defined in Definition 2.3); P_1 is the first trajectory cycle specified in Definition 2.4; K is a positive design parameter (it is called the periodic adaptation gain); z will be defined in the following paragraph; and $g(v)$ is a tuning function to be selected later based on certain guidelines.

Definition 2.4: The first trajectory cycle P_1 is the elapsed time to complete the first repetitive trajectory cycle from the initial starting time t_0. In other words, P_1 is the time corresponding to the total passed trajectory when $s(t_i) = s_p$.

In our analysis part, the following tuning function is required for $g(v)$:

$$0 < g'(v) < \infty, \tag{20}$$

where $g'(\cdot) = \frac{\partial g(\cdot)}{\partial \cdot}$; and the following tuning mechanism is required for z:

$$\dot{z} = g'(v)[\dot{v}_d - \eta e_x - \lambda e_v] - \frac{e_v}{m} \tag{21}$$

The above tuning function design will be given later.

Consider two cases: 1) when $0 \leq t < P_1$ $(0 \leq s \leq s_p)$ and 2) when $t \geq P_1$ $(s \geq s_p)$. The key idea is that, for case 1), it is required to show the finite time boundedness of equilibrium points. For case 2), it is necessary to show the stability or asymptotic stability of equilibrium points in the sense of Lyapunov. Let us investigate the case 2) first. Our major results are summarized in the following theorems with Remark 2.2.

Remark 2.2: From the relationship (14), it can be said that if $e_a(t) \to 0$ as $t \to \infty$, then $e_a(s) \to 0$ as $s \to \infty$. Thus, in what follows, the stability analysis of $a(x)$ is performed on the time axis.

Theorem 2.1: When $t \geq P_1$ $(s \geq s_p)$, the control law (15) and the periodic adaptation law (18) and (19) guarantee the stability of the equilibrium points $e_x(t)$, $e_v(t)$, $e_a(t)$, and $e_b(t)$ as $t \to \infty$ $(s \to \infty)$.

Proof: Consider the following Lyapunov-like function at $s(t)$, whose corresponding time is t:

$$V(t) = \frac{1}{2}\left(e_b(t)\mathrm{sgn}(v)\right)^2 + \frac{1}{2}S^2(t) + \frac{1}{2K}\int_{t-P_t}^{t} e_a^2(\tau)\mathrm{d}\tau, \tag{22}$$

where P_t is calculated by the search process as commented in Definition 2.3. Then, from (22), the difference of the positive Lyapunov-like functions at two discrete time points (note: time difference is P_t) can be calculated as:

$$\begin{aligned}
\triangle V(t) &= V(t) - V(t-P_t) \\
&= \frac{1}{2}\left(e_b(t)\mathrm{sgn}(v(t))\right)^2 - \frac{1}{2}\left(e_b(t-P_t)\mathrm{sgn}(v(t-P_t))\right)^2 \\
&\quad + \frac{1}{2}S^2(t) - \frac{1}{2}S^2(t-P_t) + \frac{1}{2K}\int_{t-P_t}^{t}\left[e_a^2(\tau) - e_a^2(\tau-P_t)\right]\mathrm{d}\tau \\
&= \int_{t-P_t}^{t}\left[e_b(\tau)\mathrm{sgn}(v(\tau))\dot{e}_b(\tau)\mathrm{sgn}(v(\tau)) + S(t)\dot{S}(t)\right]\mathrm{d}\tau \\
&\quad + \frac{1}{2K}\int_{t-P_t}^{t}\left[e_a^2(\tau) - e_a^2(\tau-P_t)\right]\mathrm{d}\tau \\
&= \int_{t-P_t}^{t}\left[e_b(\tau)\dot{e}_b(\tau) + S(t)\dot{S}(\tau)\right]\mathrm{d}\tau + \frac{1}{2K}\int_{t-P_t}^{t}[e_a^2(\tau) - e_a^2(\tau-P_t)]\mathrm{d}\tau
\end{aligned} \tag{23}$$

To simplify our presentation, let the first integral term on the right-hand side be denoted by A and the second integral term by B. That is

$$A := \int_{t-P_t}^{t}\left[e_b(\tau)\dot{e}_b(\tau) + S(\tau)\dot{S(\tau)}\right]\mathrm{d}\tau;\;\; B := \frac{1}{2K}\int_{t-P_t}^{t}\left[e_a^2(\tau) - e_a^2(\tau-P_t)\right]\mathrm{d}\tau.$$

Here, from $a(s-s_p) = a(t-P_t)$ in Remark 2.1, the following equalities are satisfied:

$$a(s-s_p) = a(t-P_t) = a(t) = a(s)$$

Then, by several algebraic calculations and using $a(t-P_t) = a(t)$, B can be changed as

$$\begin{aligned}
B &= \frac{1}{2K}\int_{t-P_t}^{t}\{[a(\tau)-\hat{a}(\tau)]^2 - [a(\tau-P_t)-\hat{a}(\tau-P_t)]^2\}\mathrm{d}\tau \\
&= \frac{1}{2K}\int_{t-P_t}^{t}[\hat{a}(\tau-P_t)-\hat{a}(\tau)]\left\{2\left[a(\tau)-\hat{a}(\tau)\right] + \left[\hat{a}(\tau)-\hat{a}(\tau-P_t)\right]\right\}d\tau \\
&= \frac{1}{2K}\int_{t-P_t}^{t}\beta(\tau)\left\{2\left[a(\tau)-\hat{a}(\tau)\right] - \beta(\tau)\right\}\mathrm{d}\tau,
\end{aligned} \tag{24}$$

where

$$\beta(\tau) := \hat{a}(\tau-P_t) - \hat{a}(\tau).$$

By applying (18), we have $\beta(t) = \frac{K}{m}S(t)$. Furthermore, using (9), (15), and

$$\begin{aligned}
\dot{e}_x &= \dot{x} - \dot{x}_d = e_v, \\
\dot{e}_v &= \dot{v} - \dot{v}_d \\
&= -\frac{p}{m}v - \frac{1}{m}a(t) - \frac{1}{m}b\mathrm{sgn}(v) + u - \dot{v}_d \\
&= \frac{1}{m}\left(-a(t) - b\mathrm{sgn}(v) + \hat{a}(t) + \hat{b}\mathrm{sgn}(v)\right) - \alpha S(t) - \lambda e_v(t) \\
&= \frac{-e_a(t) - e_b\mathrm{sgn}(v)}{m} - \alpha S(t) - \lambda e_v(t),
\end{aligned} \tag{25}$$

from (17)

$$\begin{aligned} \dot{S} &= \dot{e}_v + \lambda \dot{e}_x(t) \\ &= \frac{-e_a(t) - e_b \mathrm{sgn}(v)}{m} - \alpha S(t) - \lambda e_v(t) + \lambda e_v(t) \\ &= \frac{-e_a - e_b \mathrm{sgn}(v)}{m} - \alpha S(t). \end{aligned} \tag{26}$$

Then, using

$$\dot{e}_b = \dot{b} - \dot{\hat{b}} = -\dot{\hat{b}} \tag{27}$$

A can be expressed as

$$A = \int_{t-P_t}^{t} -e_b(\tau)\dot{\hat{b}} + S(\tau)\left[\frac{-e_a(\tau) - e_b(\tau)\mathrm{sgn}(v)}{m} - \alpha S(\tau)\right] \mathrm{d}\tau. \tag{28}$$

Thus, $\triangle V$ becomes

$$\begin{aligned} \triangle V &= A + B \\ &= \int_{t-P_t}^{t} \left\{ -e_b(\tau)\dot{\hat{b}} + S(\tau)\left[\frac{-e_a(\tau) - e_b(\tau)\mathrm{sgn}(v)}{m} - \alpha S(\tau)\right]\right\} \mathrm{d}\tau \\ &\quad + \frac{1}{2K}\int_{t-P_t}^{t} \beta \left\{2\left[a(\tau) - \hat{a}(\tau)\right] - \beta(\tau)\right\} \mathrm{d}\tau. \end{aligned} \tag{29}$$

Also, using $e_a(t) = a(t) - \hat{a}(t)$, $A + B$ is changed as:

$$A + B = \int_{t-P_t}^{t} \left[-e_b(\tau)\dot{\hat{b}} - \frac{S(\tau)e_b(\tau)\mathrm{sgn}(v)}{m} - \alpha S^2 - \frac{1}{2K}\beta^2\right] \mathrm{d}\tau. \tag{30}$$

Then, using $\dot{\hat{b}} = -\frac{S(t)\mathrm{sgn}(v)}{m}$ from (19)

$$\begin{aligned} A + B &= \int_{t-P_t}^{t} -\alpha S^2 - \frac{1}{2K}\beta^2 \mathrm{d}\tau \\ &= \int_{t-P_t}^{t} -\alpha S^2 - \frac{K}{2m} S^2 \mathrm{d}\tau. \end{aligned} \tag{31}$$

Therefore, since $\Delta V(t) \leq 0$, the proof is completed. ■

The above theorem only guarantees the stability property in the sense of Lyapunov. To explore the asymptotical stability, the following notation and lemma are provided. The total external disturbances including cogging force and friction force are denoted as:

$$c(t) = \frac{a(t) + b\mathrm{sgn}(v)}{m}$$

and its corresponding error is denoted as:

$$\begin{aligned} e_c(t) &= \frac{1}{m}\left[a(t) + b\mathrm{sgn}(v) - \hat{a}(t) - \hat{b}\mathrm{sgn}(v)\right] \\ &= \frac{e_a(t) + e_b\mathrm{sgn}(v)}{m} \end{aligned} \tag{32}$$

Lemma 2.1: In the following equation with initial state $x(0) = x_0 = 0$

$$y = \dot{x} + \tau x, \quad \tau > 0,$$

$y \to 0$ as $t \to \infty$ if and only if $x \to 0$ as $t \to \infty$.

Proof: The sufficient condition is immediate because $x = 0$ makes $y = 0$. The necessary condition is proved easily by calculating the solution. When $y = 0$, $x(t)$ is calculated as:

$$x(t) = x_0 + e^{-\tau t}.$$

So, if $x_0 = 0$, as $t \to \infty$, $x(t) \to 0$. ■

Now, let us consider the asymptotically stability condition of the equilibrium points e_x, e_v, and e_c in the following theorem.

Theorem 2.2: If the initial position (x_0) is at the desired initial position ($x_d(0)$), i.e., $e_x(0) = 0$, the control law (16) and the periodic adaptation law (18) guarantee the asymptotically stability of the equilibrium points: e_x, e_v, and e_c as $t \to \infty$ $(t \geq P_1,\ \text{or}\ s \geq s_p)$.

Proof: Here, LaSalle's invariant set theorem is used to prove the asymptotical stability. From (31), only $S = 0$ makes $\triangle V = 0$. Using the definition $S = e_v + \lambda e_x$ and relationship $e_v = \dot{e}_x$, we have

$$S = e_v + \lambda e_x = \dot{e}_x + \lambda e_x. \tag{33}$$

So, from Lemma 2.1, if $e_x(0) = 0$, only $e_x = 0$ makes $S = 0$. Also, since $e_x = 0$, we have $e_v = 0$ from $e_v + \lambda e_x = 0$. Therefore, e_x and e_v are asymptotically stable at equilibrium points. Now let us consider e_c in what follows. From (26), $\dot{S} = \frac{-e_a - e_b\mathrm{sgn}(v)}{m} - \alpha S = -e_c - \alpha S$, $\dot{S} = -e_c$ because $S = 0$. Then, by showing that $\dot{S} \to 0$ as $S \to 0$, $e_c = 0$ can be an asymptotical stable point. Our approaches are as follows. From the following definition

$$\dot{S} = \lim_{\Delta t \to 0} \frac{S(t + \Delta t) - S(t)}{\Delta t}, \tag{34}$$

we know that as $t \to \infty$, $S(t + \Delta t) \to 0$ and $S(t) \to 0$. However, from our original assumption of the periodicity such as $\Delta t = P_t$, if P_t is not zero, then $\Delta t \neq 0$, while $S(t + \Delta t) - S(t) \to 0$ as $t \to \infty$. Thus, in (34), $\dot{S} \to 0$

as $t \to \infty$, hence $-e_c \to 0$ as $t \to \infty$. However, if $-e_c \neq 0$, $\dot{S} \neq 0$. Then $S(t+\Delta t) - S(t) \neq 0$, which is a contradiction to $S(t+\Delta t) - S(t) = 0$. Therefore, it can be concluded that only $-e_c = 0$ makes $\dot{S} = 0$ and in the sequel, no trajectory can stay except $e_c = 0$ when $S = 0$. Since only $e_x = 0$, $e_v = 0$ and $e_c = 0$ make $S = 0$, from the invariant set theorem, the equilibrium points e_x, e_v, and e_c are asymptotically stable. This completes the proof of this theorem. ■

Remark 2.3: The asymptotical stability of e_c does not guarantee the asymptotical stability of e_a and e_b. In other words, even if the suggested theorem guarantees the asymptotical stability of e_x and e_v, it does not provide the asymptotical stability of e_a and e_b. However, the cogging disturbance and friction disturbance will still be compensated altogether successfully by Theorem 2.2.

Now, let us consider the case 1) when $t < P_1$ $(s \leq s_p)$ and the overall stability when $t \geq 0$ $(s \geq 0)$.

Theorem 2.3: If $\dot{a}$ and b are bounded, the equilibrium points of e_x, e_v, e_a, and e_b are stable (or e_c is asymptotically stable) as $t \to \infty$ $(s \to \infty)$.

Proof: In this case, let us use the following Lyapunov function:

$$V(t) = \frac{\eta}{2}e_x^2(t) + \frac{1}{2}e_v^2(t) + \frac{1}{2}e_a^2(t) + \frac{1}{2}e_b^2 \tag{35}$$

Then, the derivative of V is expressed by using (9) as:

$$\dot{V}(t) = \eta e_x e_v + e_v\left(-\frac{p}{m}v - \frac{1}{m}a(t) - \frac{1}{m}b\mathrm{sgn}(v) + u - \dot{v}_d\right) + e_a\dot{e}_a + e_b\dot{e}_b \tag{36}$$

From (16), (18), (19), and (21), using

$$\dot{e}_b = \dot{b} - \dot{\hat{b}} = 0$$

$$u = \frac{1}{m}\hat{a}(t) + \frac{p}{m}v + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t)$$

$$\dot{e}_a = \dot{a} - \dot{\hat{a}} = \dot{a} - \dot{z} + g'(v)\dot{v}$$

we have

$$\begin{aligned}\dot{V}(t) &= \eta e_x e_v + e_v\left[\frac{-a(t) - b\mathrm{sgn}(v) + \hat{a}(t)}{m} + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t) - \dot{v}_d\right] \\ &\quad + e_a\left[\dot{a} - \dot{z} + g'(v)\dot{v}\right]. \end{aligned} \tag{37}$$

Now, rewriting $\dot{v}$ of (9) such as:

$$\dot{v} = \frac{-a(x) - b\mathrm{sgn}(v)}{m} - \frac{p}{m}v + u$$

$$\begin{aligned} &= \frac{-a(x) - b\mathrm{sgn}(v) + \hat{a}(t)}{m} + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t) \\ &= \frac{-e_a - b\mathrm{sgn}(v)}{m} + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t) \end{aligned} \tag{38}$$

and using (21), and inserting (38) into (37), (37) is changed as:

$$\begin{aligned} \dot{V}(t) &= -\lambda e_v^2 - \frac{b\mathrm{sgn}(v)}{m} e_v - \frac{1}{m} e_a e_v + e_a \left[\dot{a} + \frac{1}{m} e_v + g'(v) \left(\frac{-e_a - b\mathrm{sgn}(v)}{m} \right) \right] \\ &= -\lambda e_v^2 - \frac{1}{m} b\mathrm{sgn}(v) e_v - \frac{1}{m} e_a^2 g'(v) - \frac{1}{m} e_a b g'(v) \mathrm{sgn}(v) + e_a \dot{a}. \end{aligned} \tag{39}$$

Let us investigate $-\lambda e_v^2 - \frac{1}{m} b\mathrm{sgn}(v) e_v$ and $-\frac{1}{m} e_a^2 g'(v) - \frac{1}{m} e_a b g'(v) \mathrm{sgn}(v) + e_a \dot{a}$ of (39) separately. The following relationship is derived:

$$\begin{aligned} -\lambda e_v^2 - \frac{1}{m} b\mathrm{sgn}(v) e_v &= -\lambda \left(e_v^2 + \frac{b\mathrm{sgn}(v)}{\lambda m} e_v \right) \\ &= -\lambda \left(e_v + \frac{b\mathrm{sgn}(v)}{2\lambda m} \right)^2 + \frac{b^2}{4\lambda m^2} \end{aligned} \tag{40}$$

and $-\frac{1}{m} e_a^2 g'(v) + (\dot{a} - \frac{1}{m} b g'(v) \mathrm{sgn}(v)) e_a$ is changed as

$$-\frac{g'(v)}{m} \left(e_a - \frac{m}{2g'(v)} \left(\dot{a} - \frac{b}{m} g'(v) \mathrm{sgn}(v) \right) \right)^2 + \frac{m}{4g'(v)} \left(\dot{a} - \frac{b}{m} g'(v) \mathrm{sgn}(v) \right)^2 \tag{41}$$

Hence, since

$$-\lambda e_v^2 - \frac{1}{m} b\mathrm{sgn}(v) e_v \leq \frac{b^2}{4\lambda m^2}$$

and

$$-\frac{1}{m} e_a^2 g'(v) + \left(\dot{a} - \frac{1}{m} b g'(v) \mathrm{sgn}(v) \right) e_a \leq \frac{m}{4g'(v)} \left(\dot{a} - \frac{b}{m} g'(v) \mathrm{sgn}(v) \right)^2,$$

the derivative of Lyapunov function is upper bounded such as:

$$\dot{V}(t) \leq \frac{b^2}{4\lambda m^2} + \frac{m}{4g'(v)} \left(\dot{a} - \frac{b}{m} g'(v) \mathrm{sgn}(v) \right)^2$$

Thus, it concludes that $\dot{V}$ is bounded when $t < P_1$ $(s < s_p)$ if $\dot{a}$ and b are bounded. Consequently, V is bounded since $\dot{V}$ is bounded. Therefore, e_x, e_v, e_a, and e_b are also bounded in l_2 vector norm topology at $t < P_1$ $(s < s_p)$. Furthermore, when $t \geq P_1$ $(s \geq s_p)$, the equilibrium points of e_x, e_v, e_a, and e_b are all (e_c is asymptotically stable with $e_x(0) = 0$) stable from equation (31); so the system (8)-(9) can be (asymptotically with $e_x(0) = 0$) stabilized by the control law (15)-(16) and the adaptation law (18)-(19) as $t \to \infty$. This completes the proof. ■

III. Simulation Illustrations

For simulation test, let us use the following reference position and velocity signals, which have the same period and amplitude as in Fig. 2 of [5]:

$$\begin{aligned} x_d(t) &= 0.25\sin(2\pi\frac{t}{T_d} - \frac{\pi}{2}) + 0.25 \\ v_d(t) &= 0.5\pi\frac{1}{T_d}\cos(2\pi\frac{t}{T_d} - \frac{\pi}{2}) \\ \dot{v}_d(t) &= -0.25(2\frac{\pi}{T_d})^2\sin(2\pi\frac{t}{T_d} - \frac{\pi}{2}) \end{aligned} \tag{42}$$

where $T_d = 4$ seconds. To check our method, the actual PMLM model is used in the simulation. We use actual parameter values of LD-3810 PMLM motor, which are given in Table 1 of [5]. Furthermore, to represent the realistic friction model, the following friction force is used in simulation:

$$F_{fric} = [f_c + (f_s - f_c)e^{(v/v_s)^2} + f_v v]\mathrm{sgn}(v), \tag{43}$$

where f_c is Coulomb friction coefficient (b in analysis), f_s is static friction coefficient, f_v is the viscous friction coefficient, v_s is the lubricant parameter. In (7), parameter values are given as: $m = 5.4\ kg$; $R = 16.8$ ohms; $k_f = 130$ N/Amp; and $k_e = 123$ volt/m./sec. (same values as used in [5]). As for friction force, the simulated parameter values are $f_c = 10$, $f_s = 20$, $v_s = 0.1$, and $f_v = 10$ (same values as used in [5]). The control gains in (15)-(16) were selected as: $\alpha = 50$, $\lambda = 20$ and $\eta = 20$; and $g(v)$ was designed as $40v$ to satisfy (20). In (18), the periodic adaptation gain K was selected as 1000. To represent the high order Fourier expansion, the following cogging force was modelled:

$$F_{cogging} \quad = \quad A_1\sin(\omega x) + A_2\sin(3\omega x) + A_3\sin(5\omega x), \tag{44}$$

where $A_1 = 8.5$, $\omega = 314\ rad/m$ (same values as used in [5]), $A_2 = 4.25$, and $A_3 = 2.0$. Recall that reference [5] only used first order Fourier expansion in simulation, but our cogging force model includes higher order Fourier expansions.

Top of Fig. 1 is the desired position on the time axis and bottom is the desired velocity. Note that we used the same desired trajectory as used in [5]. Figure 2 shows the position tracking error where the Y-axis of the top sub-figure ranges from 0.02 meters to -0.02 meters while the Y-axis of the bottom sub-figure ranges from 0.002 meters to -0.002 meters. As shown in the top sub-figure of Fig. 2, after the first repetitive trajectory, the tracking error was significantly reduced. When the bottom sub-figure of Fig. 2 is compared with Figure 3 of [5], it is observed that our result is slightly better than the result of [5] even though we have included high order Fourier terms of the cogging force. Figure 3 shows our adaptive control input signal. Comparing Fig. 3 with Figure 3 in [5], we found that the required control input of our system is slightly less than the required control input of [5] even though we have included high order Fourier terms of the cogging force. The top sub-figure Fig. 4 is the cogging force from the simulation model (44) with respect to the x_d. The bottom sub-figure of Fig. 4 is the friction force signal using the model (43). These signals are the "true" actual forces experienced by the PMLM. For a comparison, with our adaptive control method, the estimated cogging force and friction force time histories are shown in the top and bottom sub-figure of Fig. 5, respectively. From Figs. 4 and 5, we can observe a good agreement except in the initial transient phase.

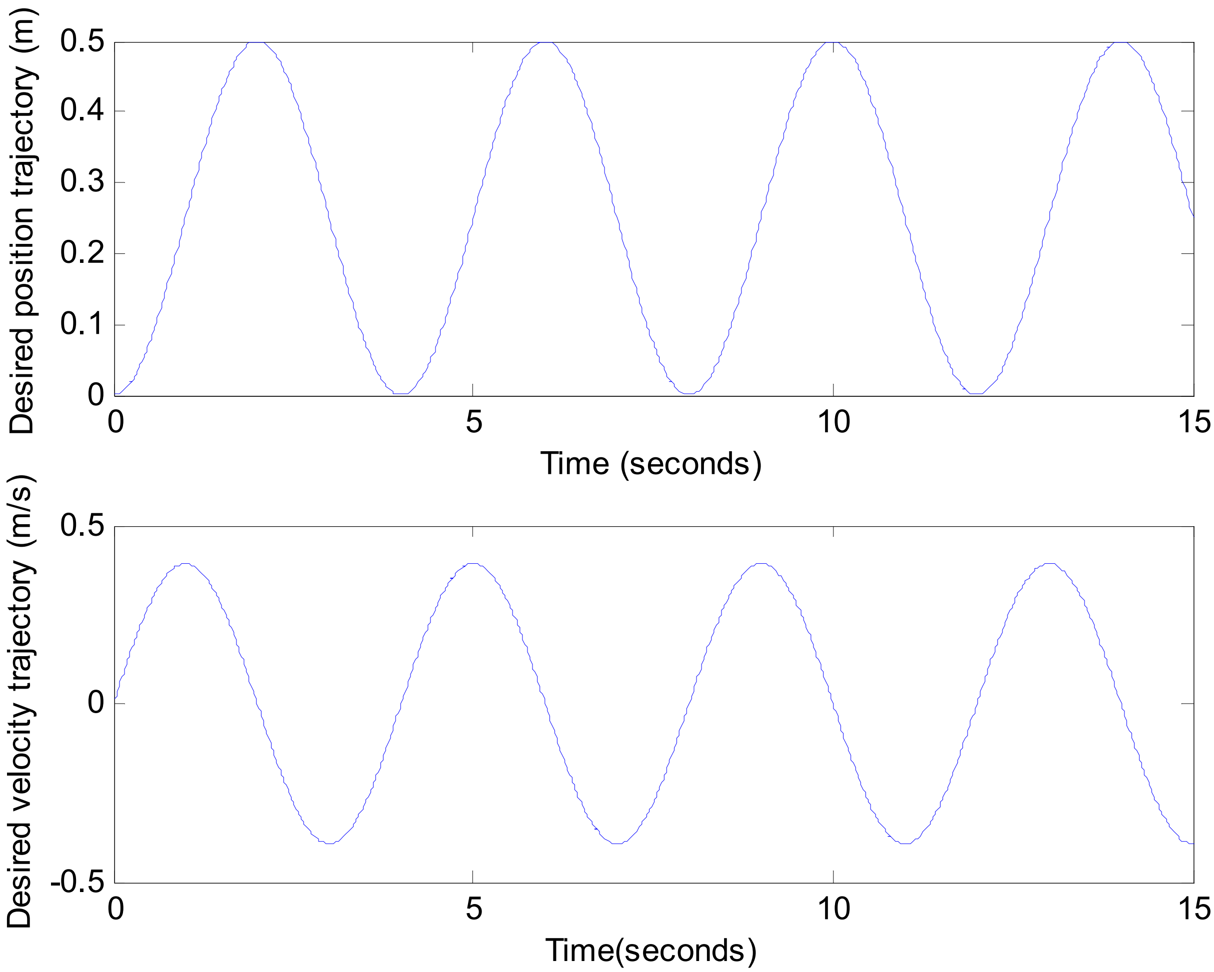

Fig. 1. Top: The desired position trajectory. Bottom: The desired velocity trajectory.

IV. CONCLUSION REMARKS

In this paper, a new cogging and friction force compensation method of the permanent linear magnet motors (PMLM) has been proposed when the PMLM is command to execute a given task repeatedly. The key idea of our method is to use the periodicity of the cogging disturbance, which is dependent on the position. From the

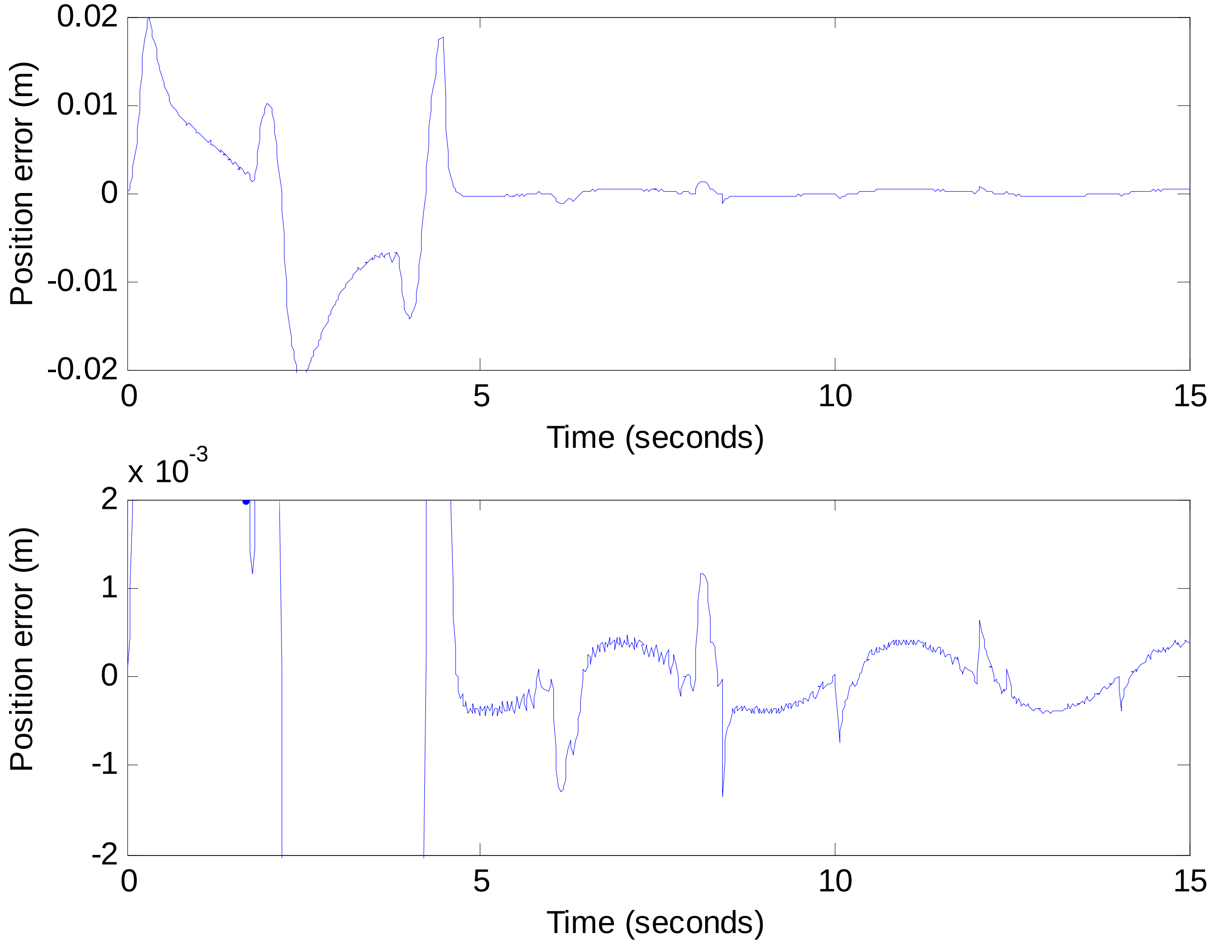

Fig. 2. Top: The position tracking error. Bottom: The position tracking error zoomed.

one past trajectory information, the current adaptation law was updated. Even though the stability analysis was performed on the time axis, the position-dependent cogging disturbance can be successfully compensated on the state-axis. It is believed that the suggested method can be effectively used in many real applications such as satellite, trail system, factory process control, and etc. Note that although the state-periodic adaptive control method was developed for compensating cogging disturbance, the key ideas of our method can be modified to compensate other state-dependent nonlinear disturbances of a general shape. From the simulation results, we can conclude that our proposed control method works effectively when the cogging force is represented in the form of high order Fourier expansion. Furthermore, compared to the reported results, we observe that, even with extra order Fourier expansion terms in the cogging force, our method requires less control effort and achieves smaller position tracking error. In summary, the position-dependent external disturbance such as cogging force can be successfully compensated by using the trajectory periodicity of the state-dependent disturbance.

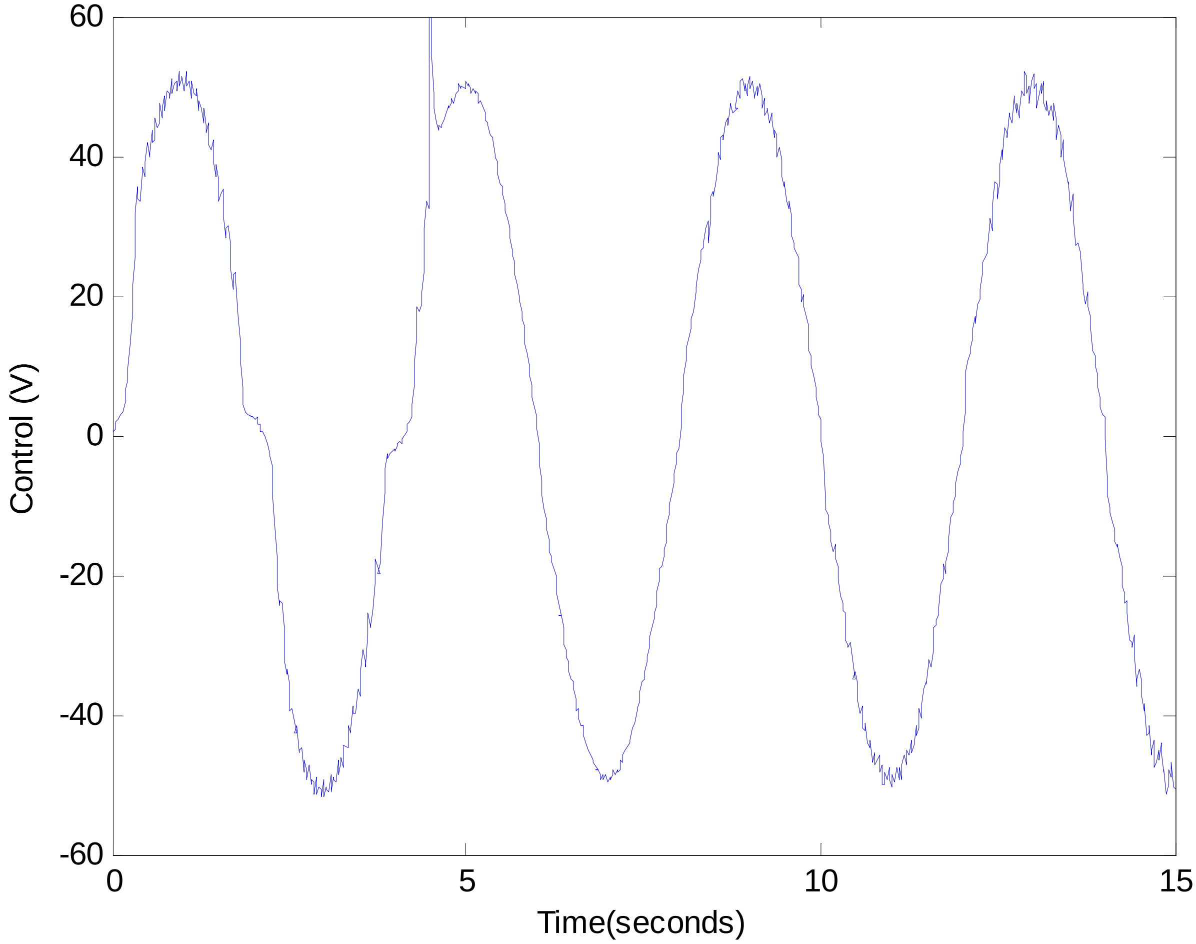

Fig. 3. The adaptive control input signal

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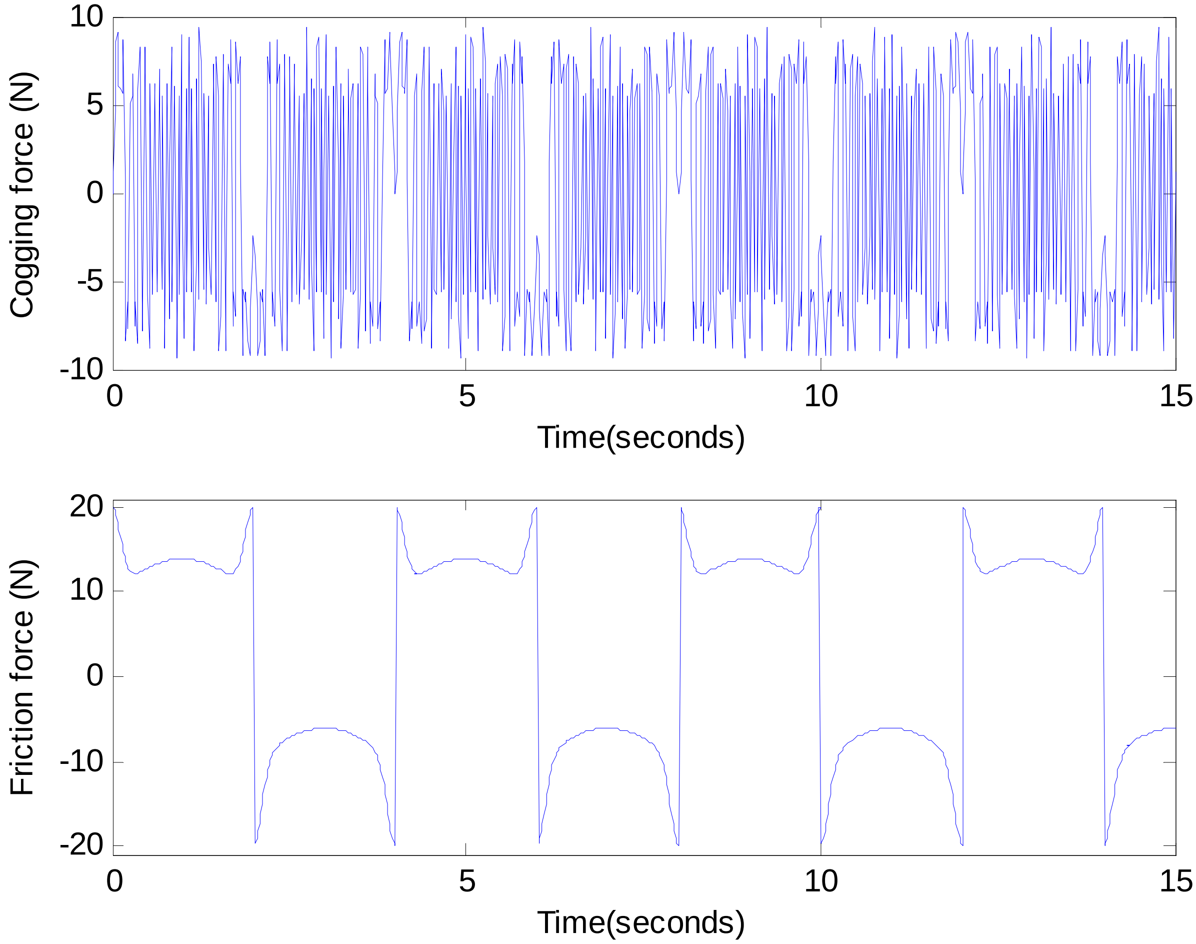

Fig. 4. Top: The true cogging force from the model (44). Bottom: The true friction force from the model (43).

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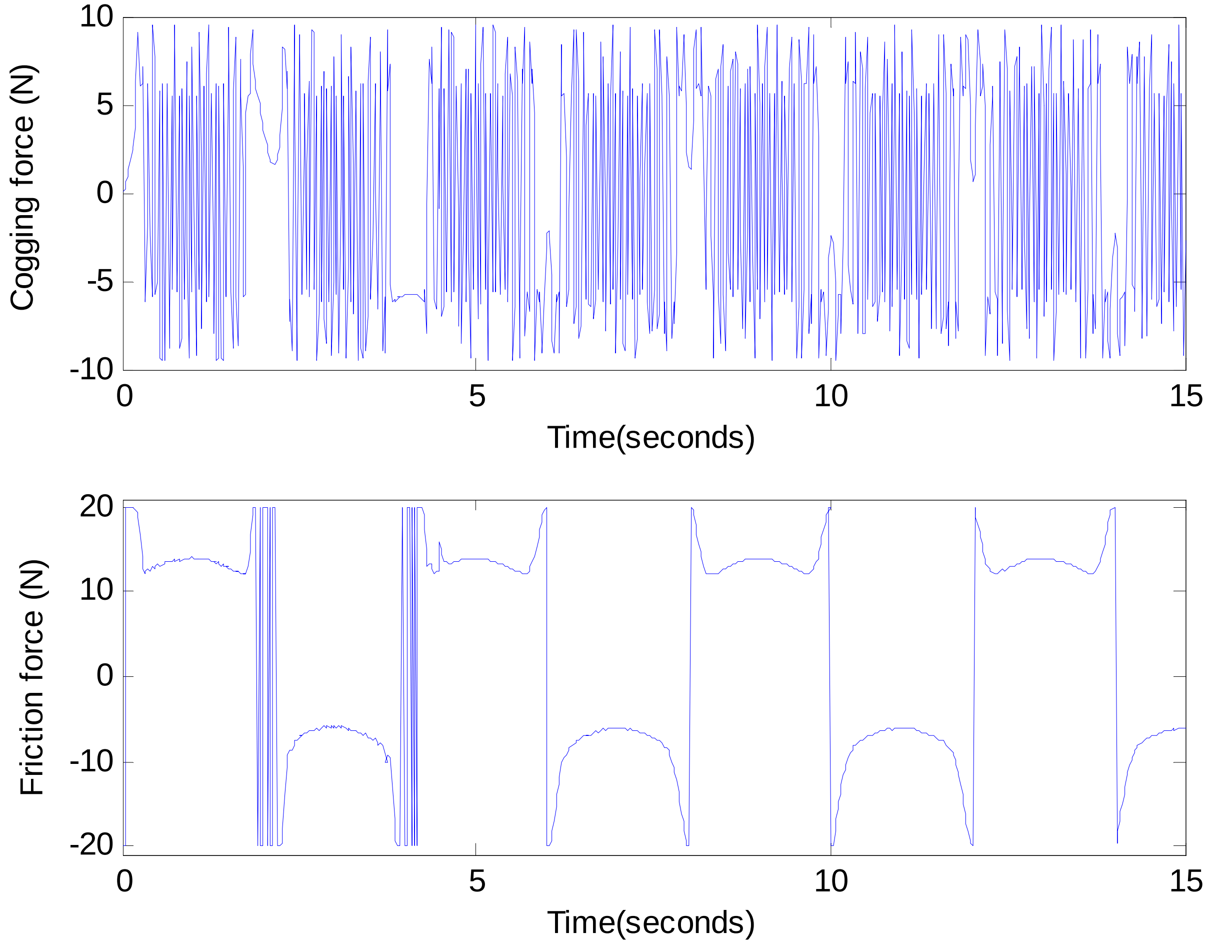

Fig. 5. Top: The estimated cogging force. Bottom: The estimated friction force.

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