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## State-Periodic Adaptive Friction Compensation

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# STATE-PERIODIC ADAPTIVE FRICTION COMPENSATION

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**Abstract:** This paper focuses on the adaptive friction compensation, where the friction is considered as a position-dependent disturbance. We consider the case when the desired trajectory is state (e.g., position) periodical which is of course also time periodical. The key idea of our approach is to use one trajectory past information along the state axis to update the current adaptation since the friction is state-periodic. The new method consists of two main steps: Firstly, in the first repetitive trajectory, an adaptive compensator is designed to guarantee the  $\ell_2$ -stability of the overall system; and secondly, from the second repetitive trajectory and onwards, a state-periodic adaptive compensator is designed based on stored past state-dependent information. Rigorous stability analysis is presented with a simulation example.

**Keywords:** Friction compensation, state-dependent disturbance, adaptive control, periodic update, tracking control.

## 1. INTRODUCTION

In many electromechanical control system, friction disturbance is everywhere. That is why friction is constantly a hot topic in control community. Since the early works in friction compensation (Kubo et al., 1986; de Wit et al., 1989; David A. Haessig and Friedland, 1990), adaptive friction compensation controllers have been designed in (Friedland and Mentzelopoulou, 1992; Yazdizadeh and Khorasani, 1996; Liao and Chien, 2000; Zhang and Guay, 2001; Ahn and Chen, 2004). The pioneering works in (Friedland and Mentzelopoulou, 1992) provided the possibilities for the adaptive compensation of the non-Lipschitz disturbance. Since their works in 1991 and 1992, several modifications were introduced in (Yazdizadeh and Khorasani, 1996; Liao and Chien, 2000; Zhang and Guay, 2001) with a focus on designing new adaptive friction update laws by proposing new forms of the tuning function  $g(|v|)$  where  $v$  is the velocity. Mainly, the considerations were to design a more stable nonlinear adaptive controller.

However, in all the existing efforts in attacking friction effects, the Coulomb friction coefficient is assumed to be constant. In other words, the existing adaptive friction compensations are all restricted to the simplest Coulomb friction coefficient problem.

However, like other hard nonlinearities such as deadzone, hysteresis, and saturation, the friction force is also related with the state. In Du and Nair (1998), the friction force was defined as the disturbance, which is state-dependent parasitic effect. In practice, the state-dependent external disturbances exist in many engineering problems. For example, in Zaremba et al. (1998), the engine crankshaft speed pulsation was expressed as Fourier series expansion as a function of position; in de Wit (1999), the tire/road contact friction was represented as a function of the system state variable; and in David A. Haessig and Friedland (1990), the magnitude of friction coefficient depends on velocity which practically is not a constant. For examples and more detailed explanation about the position-dependent disturbance, refer to de Wit and Praly (2000). As another practical example of the position-dependent friction force, let us consider a mobile robot moving on the

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floor composed of different materials. The friction coefficients of each material are different from each other. Thus, the mobile robot experiences the different friction forces depending on position<sup>2</sup>. Hence, as shown in above examples, the main argument of this paper is that the friction force can be position-dependent disturbance. The scenario is as follows: The mobile vehicle is continuously moving on the fixed trajectory, which could be the roller-coaster rail, floor composed of different materials, or any kind of orbit-systems. The vehicle is moving forward and backward repeatedly; so the sign of the friction force is changing at zero velocity, but with position-dependent variation.

The paper is organized as follows: In Section 2, the stability analysis is performed on the time domain; in Section 3, simulation tests are performed; and conclusions are given in Section 4.

## 2. STATE DOMAIN ANALYSIS ON THE TIME AXIS

In this section, the position-dependent information is matched to the discrete time points. In this paper, the external state-dependent friction is denoted as  $a(x)$ . Similar to the system considered in de Wit and Praly (2000), without loss of generality, the following simple servo control problem is considered:

$$\dot{x}(t) = v(t) \quad (1)$$

$$\dot{v}(t) = -a(x)\text{sgn}(v) + u, \quad (2)$$

where  $x$  is the position;  $a(x)\text{sgn}(v)$  is the unknown position-dependent friction;  $v$  is the velocity; and  $u$  is the control input. First, before proceeding our main results, following definitions and assumptions are necessary.

*Definition 2.1.* The total passed trajectory is given as:

$$s = \int_0^t \frac{|dx|}{d\tau} d\tau = \int_0^t |v(\tau)| d\tau,$$

where  $x$  is the position, and  $v$  is the velocity. In de Wit and Praly (1998), it was defined as the curvilinear abscissa associated with the trajectory of the relative motion. In our definition, since  $s$  is the summation of absolute position increasing along the time axis,  $s$  is a monotonous growing signal. Physically, it is the total passed trajectory, hence it has the following property:  $s(t_1) \geq s(t_2)$ , iff  $t_1 \geq t_2$ . With the notation  $s$ , the position corresponding to  $s(t)$  is denoted by  $x(s)$  and the friction force corresponding to  $s(t)$  is denoted by  $a(s)$ .

<sup>2</sup> See the article “Robot ‘vac’ is naughty and nice” on page 9 of the International Herald Tribune, July 17, 2004 (<http://www.iht.com/articles/529764.html>). In this article, it was reported that the mobile robot encounters trouble in the corners that presented them with rug and hardwood floor. Also, it was reported that the mobile robot prefers materials such as hard, even surface, but shag carpeting is forbidden. Mobile robot was tested on different material floors such as linoleum tiles, hardwood floors, Oriental rugs, carpeting, and etc; but different performance was observed on different materials.

*Definition 2.2.* Since the friction force appears as a function of the position and the desired trajectory to be followed is assumed to be repetitive which is true for many practical applications, the friction force is also periodic with respect to position. So, based on Definition 2.1, following relationship is true:

$$a(s) = a(s - s_p), \text{ and } x(s) = x(s - s_p), \quad (3)$$

where  $s_p$  is called the trajectory period.

*Definition 2.3.* It can be defined that  $s_p$  of Definition 2.2 is a periodic trajectory. Therefore,  $x(t) - s_p$  is one trajectory past position from  $x(t)$ . The time corresponding to  $x(t) - s_p$  is denoted as  $T_t$ . Then,  $t - T_t$  is the time-elapse to complete one periodic trajectory from the time  $T_t$  to time  $t$ . This time-elapse is termed as “cycle”, and it can be called “trajectory cycle” at time  $t$  and is denoted as  $P_t$ . So,  $P_t = t - T_t$ . It is called “the search process” to find  $P_t$  at time instant  $t$  (note: the search process can be performed by interpolation).

Furthermore, the time is always monotonically increasing, and the discrete time controller is used. So, the monotonically increasing time variable is denoted as:  $t_i, i = 0, \dots, \infty$ , where  $t_0$  is the initial time. Thus, following relationship is true:  $s(t_{i+1}) \geq s(t_i)$ . From now on, for accurate notation, the position corresponding to time  $t_i$  is denoted as:  $x(t_i)$  and its total passed trajectory by the time  $t_i$  is denoted as:  $s(t_i)$ . Henceforward, one trajectory past time from the time instant  $t_i$  is denoted as  $T_{t_i}$ , and its corresponding cycle is denoted as  $P_{t_i}$  (i.e.,  $P_{t_i} = t_i - T_{t_i}$ ).

*Assumption 2.1.* Throughout the paper, it is assumed that the current position and time instant of the mobile robot are measured. Let us denote the current position at time  $t_i$  as  $x(t_i)$ , where  $x$  is the position corresponding to  $t_i$ . Then,  $T_{t_i}$  can always be calculated, hence  $P_{t_i}$  is calculated at the time instant  $t_i$ .

With the above definitions and assumption, the following property is observed.

*Property 2.1.* The following relationship is derived:

$$x(t_i) = s(t_i) - ms_p, \quad (4)$$

where  $m$  is the integer part of  $s(t_i)/s_p$ .

*Remark 2.1.* As will be shown in the following theorem, the actual state-dependent friction force  $a(s(t_i))$  is not estimated on the state axis. In our adaptation law,  $a(t_i)$  is estimated on the time axis. So, to find  $a(s(t_i) - s_p)$ , the following formula is used:

$$a(s(t_i) - s_p) = a(t_i - P_{t_i}) \quad (5)$$

Here,  $P_{t_i}$  is calculated in Assumption 2.1 (recall that  $P_{t_i}$  can be used to indicate exactly one-trajectory past position).

From (4) and (5), following properties can also be derived:

*Property 2.2.* The current friction force is equal to one-trajectory past friction force. From the relationship:

$$\begin{aligned} a(s(t_i) - s_p) &= a(x(t_i) + ms_p - s_p) \\ &= a(x(t_i)) = a(t_i - P_{t_i}) \end{aligned} \quad (6)$$

the following equality can be derived:  $a(x(t_i)) = a(t_i - P_{t_i})$ .

Now, based on the above discussions, the following stability analysis is performed. Our compensation approach is summarized as follows:

- When  $s(t_i) < s_p$ , the system is controlled to be bounded input bounded output (in  $\ell_2$ -norm).
- When  $s(t_i) \geq s_p$ , the system is stabilized to track the desired speed at the desired position. By state-dependent periodic adaptation, the unknown  $a(x(t_i))$  is also estimated.

For convenience, the following notations are used:  $e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i))$ ;  $e_v = v(t_i) - v_d(t_i)$ , where  $\hat{a}(s(t_i)) = \hat{a}(t_i)$  (note:  $t_i$  is the current time corresponding to the current total passed trajectory  $s(t_i)$ ). Here, let us change  $e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i))$  into time domain as:

$$\begin{aligned} e_a(s(t_i)) &= a(s(t_i)) - \hat{a}(s(t_i)) \\ &= a(t_i) - \hat{a}(t_i) = e_a(t_i). \end{aligned} \quad (7)$$

In the same way, the following relationships are also true:

$$\begin{aligned} x(s(t_i)) &= x(t_i); \quad x_d(s(t_i)) = x_d(t_i) \\ v(s(t_i)) &= v(t_i); \quad v_d(s(t_i)) = v_d(t_i) \end{aligned}$$

The control objective is to track or servo the given desired position  $x_d(t_i)$  and the corresponding desired velocity  $v_d(t_i)$  with tracking errors as small as possible. In practice, it is reasonable to assume that  $x_d(t_i)$ ,  $v_d(t_i)$  and  $\dot{v}_d(t_i)$  are all bounded. From now on, let us omit subscript  $i$  from  $t_i$  and  $P_{t_i}$ . Our feedback control law is designed as:

$$u = \hat{a}(t)\text{sgn}(v(t)) + \dot{v}_d(t) - \alpha S(t) - \lambda e_v(t), \quad (8)$$

with

$$S(t) = e_v(t) + \lambda e_x(t), \quad (9)$$

where  $\alpha$  and  $\lambda$  are positive gains;  $\hat{a}(t)$  is an estimated friction force from an adaptation mechanism to be specified later;  $\dot{v}_d(t)$  is the desired acceleration; and  $e_x(t) = x(t) - x_d(t)$  is the position tracking error. Also be reminded that  $e_x(s(t)) = e_x(t)$ ; and  $S(s(t)) = S(t)$ .

Our adaptation law is designed as follows:

$$\hat{a}(t) = \begin{cases} \hat{a}(t - P_t) - K\text{sgn}(v)S(t) & \text{if } s \geq s_p \\ z - g(|v|) & \text{if } s < s_p \end{cases} \quad (10)$$

where  $\hat{a}(t - P_t) = \hat{a}(t^s - P_t) = \hat{a}(s - s_p)$  (Note that  $P_t$  is the trajectory cycle defined in Definition 2.3);

$P_t$  is the first trajectory cycle specified in the following definition;  $K$  is a positive design parameter called the ‘‘periodic adaptation gain’’;  $z$  will be defined in the following paragraph; and  $g(|v|)$  is a tuning function to be selected later based on certain guidelines.

*Definition 2.4.* The first trajectory cycle  $P_1$  is the elapsed time to complete the first one repetitive trajectory from the initial starting time  $t_0$ . In other words,  $P_1$  is the time corresponding to the total passed trajectory when  $s(t_i) = s_p$ .

In our analysis part, following inequality condition is required for  $g(|v|)$ :

$$\frac{1}{4} < g'(|v|) < \infty, \quad (11)$$

where  $g'(\cdot) = \frac{\partial g(\cdot)}{\partial \cdot}$ . This can be satisfied by properly selecting a  $g(|v|)$  as in Remark 2.3.

Now, consider two cases in our stability analysis: 1) when  $0 \leq t < P_1$  ( $0 \leq s \leq s_p$ ) and 2) when  $t \geq P_1$  ( $s \geq s_p$ ). The key idea is that, for case 1), it is necessary to show the finite time boundedness of all signals. For case 2), it is required to show the stability or asymptotic stability in the sense of Lyapunov.

*Remark 2.2.* Even if  $a(x)$  is state-dependent disturbance,  $a(x)$  can be analyzed on the time-axis. From (7), using  $e_a(s) = e_a(t)$ , if  $e_a(t)$  is stabilized on the time-axis, then it is interpreted that  $e_a(s)$  is stable on the state-axis ( $s$  domain). In the following Lyapunov analysis, the analysis is performed on the time-axis corresponding to the state-axis.

Let us investigate the case 2) first. Our major results are summarized in the following theorems.

*Theorem 1.* When  $t \geq P_1$  ( $s \geq s_p$ ), the control law (8) and the periodic adaptation law (10) guarantee the stability of the equilibrium points  $e_x(s(t))$ ,  $e_v(s(t))$ , and  $e_a(s(t))$  as  $t \rightarrow \infty$  ( $s \rightarrow \infty$ ).

**Proof:** Consider the following Lyapunov-like function at  $s(t)$ , whose corresponding time is  $t$ :

$$V(t) = \frac{1}{2}S^2(t) + \frac{1}{2K} \int_{t-P_t}^t e_a^2(\tau) d\tau, \quad (12)$$

$$(13)$$

where  $P_t$  is calculated by a search process as commented in Definition 2.3. Then, from (12), the difference of the positive Lyapunov-like functions at two discrete time points (Note 1: the stability analysis can be done along the state-axis also. Note 2: the time difference is  $P_t$ ) can be calculated as:

$$\Delta V(t) = V(t) - V(t - P_t)$$

$$\begin{aligned}
&= \frac{1}{2}S^2(t) - \frac{1}{2}S^2(t - P_t) \\
&\quad + \frac{1}{2K} \int_{t-P_t}^t [e_a^2(\tau) - e_a^2(\tau - P_t)]d\tau \\
&= \int_{t-P_t}^t S(t)\dot{S}(t)d\tau \\
&\quad + \frac{1}{2K} \int_{t-P_t}^t [e_a^2(\tau) - e_a^2(\tau - P_t)]d\tau. \quad (14)
\end{aligned}$$

To simplify our presentation, let the first integral term on the right-hand side be denoted by  $A$  and the second integral term by  $B$ . Here, from  $a(s - s_p) = a(t - P_t)$  in Remark 2.1, following equalities are satisfied:

$$a(s - s_p) = a(t - P_t) = a(t) = a(s)$$

Then, by several algebraic calculations and using  $a(t - P_t) = a(t)$ ,  $B$  can be changed as

$$\begin{aligned}
B &= \frac{1}{2K} \int_{t-P_t}^t \{[a(\tau) - \hat{a}(\tau)]^2 - [a(\tau - P_t) \\
&\quad - \hat{a}(\tau - P_t)]^2\}d\tau \\
&= \frac{1}{2K} \int_{t-P_t}^t [\hat{a}(\tau - P_t) - \hat{a}(\tau)][2\{a(\tau) - \hat{a}(\tau)\} \\
&\quad + \{\hat{a}(\tau) - \hat{a}(\tau - P_t)\}]d\tau \\
&= \frac{1}{2K} \int_{t-P_t}^t \beta(\tau)[2\{a(\tau) - \hat{a}(\tau)\} - \beta(\tau)]d\tau. \quad (15)
\end{aligned}$$

where

$$\beta(\tau) = \hat{a}(\tau - P_t) - \hat{a}(\tau).$$

Using the following

$$\begin{aligned}
\dot{e}_x &= \dot{x} - \dot{x}_d = e_v, \\
\dot{e}_v &= \dot{v} - \dot{v}_d = -a(t)\text{sgn}(v) + u - \dot{v}_d
\end{aligned}$$

we have

$$\dot{S} = \dot{e}_v + \lambda \dot{e}_x = -a(t)\text{sgn}(v) - \dot{v}_d + u + \lambda e_v. \quad (16)$$

Then, from (8),

$$\dot{S} = -\text{sgn}(v)e_a - \alpha S,$$

and  $A$  can be expressed as

$$A = \int_{t-P_t}^t [-\alpha S^2 - \text{sgn}(v)e_a S]d\tau. \quad (17)$$

Thus,  $\Delta V$  becomes

$$\Delta V = A + B$$

$$\begin{aligned}
&= \int_{t-P_t}^t [-\alpha S^2 - \text{sgn}(v)e_a S]d\tau \\
&\quad + \frac{1}{2K} \int_{t-P_t}^t \beta[2\{a(\tau) - \hat{a}(\tau)\} - \beta]d\tau \\
&= \int_{t-P_t}^t [-\alpha S^2 - \frac{1}{2K}\beta^2]d\tau \\
&\quad + \int_{t-P_t}^t \frac{e_a}{K}[\beta - K\text{sgn}(v)S]d\tau, \quad (18)
\end{aligned}$$

where the first integral term on the right-hand side is denoted by  $C$  and the second integral term is denoted by  $D$ . Then, from (10),  $D = 0$ . So, we have

$$\begin{aligned}
\Delta V &= A + B = \int_{t-P_t}^t -\alpha S^2 - \frac{1}{2K}\beta^2 d\tau \\
&= \int_{t-P_t}^t -(\alpha + \frac{K}{2})S^2(\tau)d\tau. \quad (19)
\end{aligned}$$

Since  $\alpha + \frac{K}{2} > 0$ ,  $\Delta V(s) = \Delta V(t) \leq 0$ , which completes the proof of this theorem. ■

The above theorem only guarantees the stability property in the sense of Lyapunov. The asymptotical stability can be explored as follows.

*Theorem 2.* If the initial position ( $x_0$ ) is at the desired initial position ( $x_d(0)$ ), i.e.,  $e_x(0) = 0$ , the control law (8) and the periodic adaptation law (10) guarantee the asymptotically stability of the equilibrium points as  $t \rightarrow \infty$  ( $t \geq P_1$ , or  $s \geq s_p$ ).

**Proof:** The proof can be completed by LaSalle's invariant set theorem. Due to the page limitation, the proof is omitted. ■

Now, let us consider the case 1) when  $t < P_1$  ( $s \leq s_p$ ) and the overall stability when  $t \geq 0$  ( $s \geq 0$ ).

*Theorem 3.* If  $|\dot{a}|$  is bounded and  $g'(|v|) > \frac{1}{4}$ , the equilibrium points of  $e_x$ ,  $e_v$ , and  $e_a$  are stable (or asymptotically stable) as  $t \rightarrow \infty$  ( $s \rightarrow \infty$ ).

**Proof:** In this case, let us use the following Lyapunov function:

$$\begin{aligned}
V(s) &= \frac{1}{2}\alpha\lambda e_x^2(s) + \frac{1}{2}e_v^2(s) + \frac{1}{2}e_a^2(s) \\
&= \frac{1}{2}\alpha\lambda e_x^2(t) + \frac{1}{2}e_v^2(t) + \frac{1}{2}e_a^2(t) \\
&= V(t) \quad (20)
\end{aligned}$$

Then, the derivative of  $V$  is expressed as:

$$\dot{V}(t) = \alpha\lambda e_x e_v + e_v(\dot{v} - \dot{v}_d) + e_a[\dot{a} - \dot{z}]$$

$$\begin{aligned}
& +g'(|v|)\dot{v}\text{sgn}(v)] \\
& = \alpha\lambda e_x e_v + e_v[-a\text{sgn}(v) + u - \dot{v}_d] \\
& \quad + e_a[\dot{a} - \dot{z} + g'(|v|)\dot{v}\text{sgn}(v)], \quad (21)
\end{aligned}$$

where the following substitution was used:

$$\dot{e}_a = \dot{a} - \dot{\hat{a}} = \dot{a} - \dot{z} + g'(|v|)\dot{v}\text{sgn}(v). \quad (22)$$

By inserting the control input, which is given in (8), to the above equation, the derivative of Lyapunov function can be re-written as:

$$\begin{aligned}
\dot{V} & = -e_v e_a \text{sgn}(v) - (\alpha + \lambda)e_v^2 + e_a \dot{a} \\
& \quad + e_a [g'(|v|)\dot{v}\text{sgn}(v) - \dot{z}].
\end{aligned}$$

Then, using one more adaptation law as follows:

$$\dot{z} = g'(|v|)[u - \hat{a}\text{sgn}(v)]\text{sgn}(v) \quad (23)$$

and after several algebraic calculations,  $\dot{V}$  can be changed to

$$\dot{V} = -e_v e_a \text{sgn}(v) - (\alpha + \lambda)e_v^2 + e_a \dot{a} - e_a^2 g'(|v|).$$

Finally, using Young's inequality like  $a^2 + \frac{b^2}{4} \geq ab$ , if  $(\alpha + \lambda) > 1$  and  $g'(|v|) > \frac{1}{4}$ , the following inequality is always true regardless of  $\text{sgn}(v)$ :

$$-e_v e_a \text{sgn}(v) - (\alpha + \lambda)e_v^2 - e_a^2 g'(|v|) < 0. \quad (24)$$

At this moment,  $\dot{V}$  is upper bounded by

$$\begin{aligned}
\dot{V} & = -[e_v \pm 0.5\text{sgn}(v)e_a]^2 - (\alpha + \lambda - 1)e_v^2 \\
& \quad - [g'(|v|) - \frac{1}{4}]e_a^2 + e_a \dot{a}. \quad (25)
\end{aligned}$$

Our argument here is to ensure that  $\dot{V}$  is upper bounded. Denote  $\eta \equiv g'(|v|) - \frac{1}{4} > 0$ . From the above equation, it is easy to see

$$\dot{V} \leq -\eta e_a^2 + e_a \dot{a}. \quad (26)$$

If  $|\dot{a}| < \Theta$ , where  $\Theta$  is the upper bound of  $\dot{a}$ , then,

$$\begin{aligned}
\dot{V} & \leq -\eta e_a^2 + e_a \Theta \\
& \leq -\eta \left\{ e_a - \frac{\Theta}{2\eta} \right\}^2 + \frac{\Theta^2}{4\eta}. \quad (27)
\end{aligned}$$

Clearly, if  $\eta > 0$  and  $|\dot{a}|$  is bounded, then

$$\dot{V} \leq \frac{\Theta^2}{4\eta}. \quad (28)$$

Thus, it can be concluded that if  $g'(|v|) > \frac{1}{4}$ ,  $\dot{V}$  is bounded when  $t < P_1$  ( $s < s_p$ ). Consequently, when  $V$  is bounded at  $t < P_1$ ,  $e_x$ ,  $e_v$ , and  $e_a$  are also bounded in  $l_2$  vector norm topology at  $t < P_1$  ( $s < s_p$ ).

Furthermore, when  $t \geq P_1$  ( $s \geq s_p$ ), the equilibrium points of  $e_x$ ,  $e_v$ , and  $e_a$  are all (asymptotically with  $e_x(0) = 0$ ) stable from equation

(19); so we conclude that the system (1)-(2) can be (asymptotically with  $e_x(0) = 0$ ) stabilized by the control law (8) and the adaptation law (10) as  $t \rightarrow \infty$ . This completes the proof.  $\blacksquare$

The following remark is given to design the adaptation function  $g(|v|)$ .

*Remark 2.3.* On design of the adaptation function  $g(|v|)$ . In designing  $g(|v|)$ , the following function is suggested in order to satisfy the required condition  $\frac{1}{4} < g'(|v|) < \infty$ :

$$g(|v|) = \xi|v| + e^{-\mu|v|}, \quad \xi > \mu + \frac{1}{4}, \quad (29)$$

where  $\xi$  and  $\mu$  are design parameters for the adaptation law. The derivative of  $g(|v|)$  is expressed as:

$$g'(|v|) = [\xi - \mu e^{-\mu|v|}]. \quad (30)$$

Finally,  $\hat{a}(t)$  in (10) and  $\dot{z}$  in (23) are designed as:

$$\hat{a}(t) = z - \xi|v| - e^{-\mu|v|} \quad (31)$$

$$\dot{z} = [\xi - \mu e^{-\mu|v|}][u - \hat{a}\text{sgn}(v)]\text{sgn}(v). \quad (32)$$

### 3. SIMULATION ILLUSTRATIONS

For simulation test, the following reference position and velocity signals are used:

$$\begin{aligned}
x_r(t) & = \cos(2\pi f_s t) \\
v_r(t) & = -2\pi f_s \sin(2\pi f_s t) \\
\dot{v}_r(t) & = -(2\pi f_s)^2 \cos(2\pi f_s t) \quad (33)
\end{aligned}$$

where  $f_s = \frac{1}{Q_s}$ , and  $Q_s = 2$  sec. Note that unlike the  $v_r(t)$  used in the literature that is always positive, in this paper, we can consider any form of bounded  $v_r(t)$ . The control gains for  $e_x$  and  $e_v$  used in this simulation are that  $\alpha = 10$  and  $\beta = 10$ . In (10), the periodic adaptation gain  $K$  was selected as 10, and, in (29),  $\xi$  was selected as 10 and  $\mu$  was selected as 5. The friction force is  $[50 + 5 \sin(2\pi x) + 2 \sin(4\pi x) + \sin(6\pi x)]\text{sgn}(v)$ . Figure 1 shows the state tracking results where the top-left subplot is the desired position and actual position; the bottom-left is the desired velocity and actual velocity; the top-right is the position tracking error and the bottom-right is the velocity tracking error, all w.r.t. time. In the first trajectory repetition, the maximum position tracking error is about  $-0.2$  and the initial velocity tracking error is about  $-1.25$ . As time passes, the position error becomes less than 0.01 and the velocity error becomes less than 0.05. The top-left subplot in Fig. 2 is the true and estimated friction forces without considering the velocity sign. The middle-left subplot in Fig. 2 shows the true and estimated friction forces with considering the velocity sign. When the velocity direction changes, the true/estimated friction value also changes discontinuously. The bottom-left subplot in Fig. 2 is the friction estimation error. Initially, the error was

-50, because all initial values were set to zeros. As time passes, the estimated value becomes close to the true value. The top-right subplot in Fig. 2 is the true and estimated friction force according to position. The up-curve line is the result when the velocity is negative, and the bottom-curve line is the result when the velocity is positive. The bottom-right subplot in Fig. 2 shows the adaptive control input signal which looks acceptable.

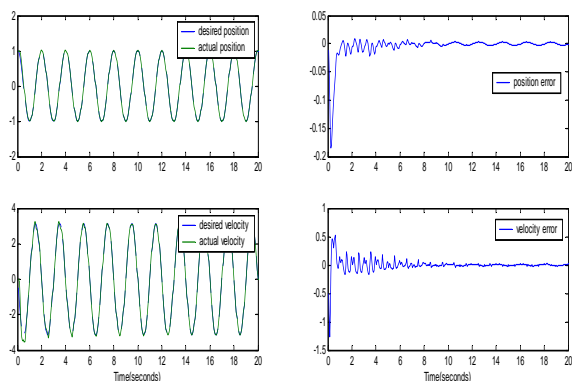


Fig. 1. Tracking performances.

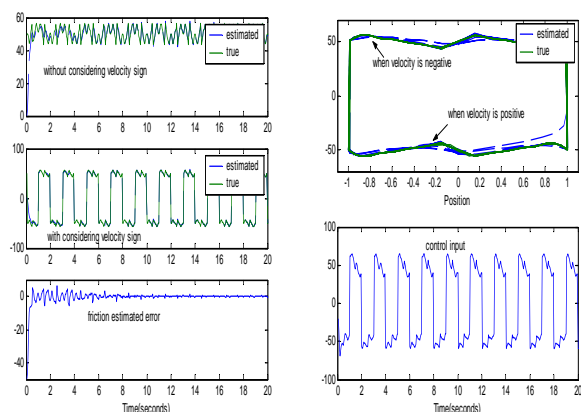


Fig. 2. Friction estimation and control input.

#### 4. CONCLUSION REMARKS

In this paper, a state-dependent friction force compensation method was suggested. The key idea of our method was to use the periodic trajectory of the friction disturbance. From one trajectory past information, the current adaptation law was updated. Even though the stability analysis was performed on the time axis, the position-dependent disturbance was successfully compensated on the state-axis. It is believed that the suggested method can be effectively used in many real applications such as satellite, trail system, factory process control, and etc, which have state-dependent disturbances. Note that even if the new method was developed for compensating the friction disturbance, the key idea of our method can be applied to compensate other nonlinear disturbances which is state dependent. To summarize

in brief, the position-dependent external disturbance can be effectively compensated by using the trajectory periodicity of the state-dependent disturbance.

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