

# Analysis and design of iterative learning controllers for interval uncertain systems

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## Rough Outline of the presentation

- Iterative learning control (ILC)
- Interval Approach and Robust Interval ILC
- Issues in Interval ILC
- Achievements
- Future research direction and proposals
- Conclusions

## Iterative learning control (ILC)

Goals of control system: stabilization and performance.

- Stabilization: BIBO, Asymptotical convergence, Exponentially convergence, Monotonic convergence, etc. Generally, BIBO is termed as stability.
- Performance: Force output response of a system to follow a desired trajectory as close as possible, where “close” is typically defined relative to a specific norm or some other measure of optimality.

It is not always possible to achieve a desired set of performance design requirement. This is due to the presence of unmodelled dynamics or parametric uncertainties that are exhibited during actual system operation or to the lack of suitable design techniques for a particular class of systems.

Iterative learning control: A technique for improving the transient response and tracking performance of processes, machines, equipment, or systems that execute the same trajectory, motion, or operation over and over (KLM, 98 IEEE CDC Workshop).

So, ILC can be used to overcome some of the traditional difficulties associated with performance design of control systems.

ILC applications:

- Robotic manipulator performing spot welding in a manufacturing assembly line.
- Antenna tracking, factory process, aerodynamic drag coefficients, and etc.

## ILC: Early Researches and Research groups

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- Japan: Uchiyama, 1978.
- Japan: Arimoto, Kawamuar, Miyazaki, middle to the late 1980's: Pioneering works.
- Kevin L. Moore, late 1980's to 1990'. The first book in ILC: Iterative Learning Control for Deterministic Systems (1993).
- Italy: Bondi, Lucibello, De Luca, et al.
- Horowitz, Sadegh, Saab.
- KAIST: Bien's group. Columbia University: Longman's group. Princeton: Phan. England: Amann, Ownes, and Rogers. Singapore: Jian-Xin Xu and YangQuan Chen.

## Books and Publications in ILC

- Iterative Learning Control for deterministic systems, Kevin L. Moore, 1993.
- Iterative Learning Control: Analysis, Design, Integration, and Application, edited by Z. Bien, Jian-Xin Xu, 1998.
- Iterative Learning Control: Convergence, Robustness and Applications, YangQuan Chen and Changyun Wen, 1999.
- Linear and Nonlinear Iterative Learning Control, Jian-Xin Xu and Ying Tan, 2003.
- Publications: IEEE Xplore: Your search matched 314 of 1069805 documents. A maximum of 500 results are displayed, 15 to a page, sorted by Relevance in Descending order.
- From Web of science: TS=(iterative learning control) Doc-Type=All document types; Language=All languages; Databases=SCI-EXPANDED, SSCI, A HCI; Timespan=1987-2004 230 results found.

## Recent Ph.D Thesis in ILC

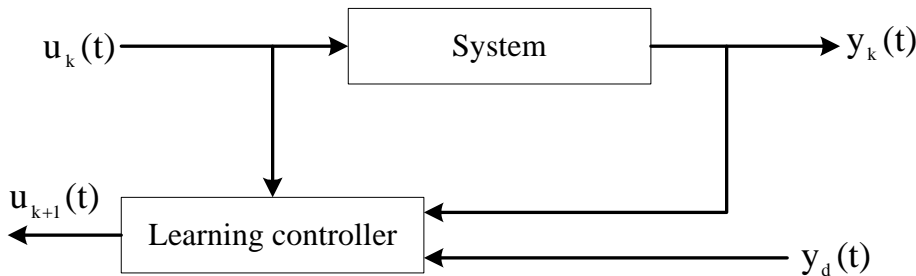
- Iterative Learning Control: Analysis, Design, and Experiments by Mikael Norrlöf, Dept of Electrical Engineering, Linköping Studies in Science and Technology, 2000.
- Iterative learning control with basis functions by Frueh, James Arthur, Princeton University, 2000.
- Model and System Inversion with Applications in Nonlinear System Identification and Control by Ola Markkussun, School of Electrical Engineering, Royal Institute of Technology, 2002.
- Iterative learning control of an electrohydraulic injection molding machine with smoothed fill-to-pack transition and adaptive filtering by Zheng, Danian, University of Illinois at Urbana-Champaign, 2002.
- Iterative Learning Control with Applications to a wafer-stage by Branko Dijkstra, Technische Univeriteit Delft, 2004.
- Vision-Based Measurements for Dynamic Systems and Control by Lili Ma, Utah State University, 2004.

## ILC: By areas

- Super-vector ILC: To guarantee the monotone convergence.
- Optimal ILC: To improve the performance.
- Adaptive ILC: With parameter estimation.
- Nonlinear ILC: Nonlinear system.
- High-order ILC: Convergence speed, improved performance.
- Robust and stochastic ILC: To improve the performance and meet the design requirement.

**Unanswered problem: How to connect the monotone convergence issue with the robustness issue of ILC ?**

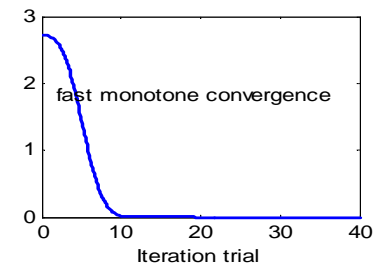
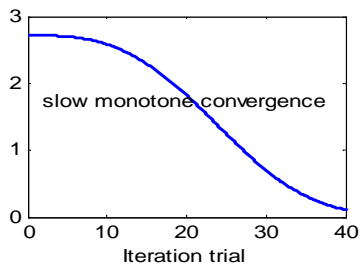
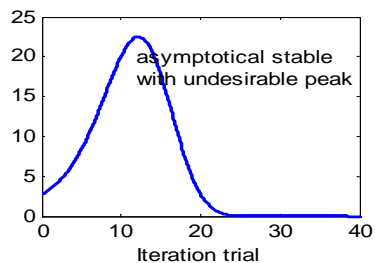
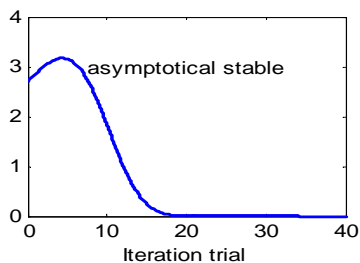
## ILC: How it works ?



$u_{k+1}(t) = f(u_k(t), y_k(t), y_d(t))$ , where  $k$  represents the iteration trial,  $t$  is the discrete time point,  $u_k$  is the control input,  $y_k$  is the current trial output, and  $y_d$  is the desired trajectory.

## Super-vector ILC

What is difference between the asymptotical stability and monotone convergence ?



**Optimal ILC, Adaptive ILC, Nonlinear ILC, High-order ILC, Robust and stochastic ILC can not guarantee the monotone convergence.**

Consider SISO discrete-time LTI system with relative degree 1:

$$Y(z) = H(z)U(z) = (h_1z^{-1} + h_2z^{-2} + \dots)U(z)$$

Assume the standard ILC reset condition:  $y_k(0) = y_d(0) = y_0$  for all  $k$ .

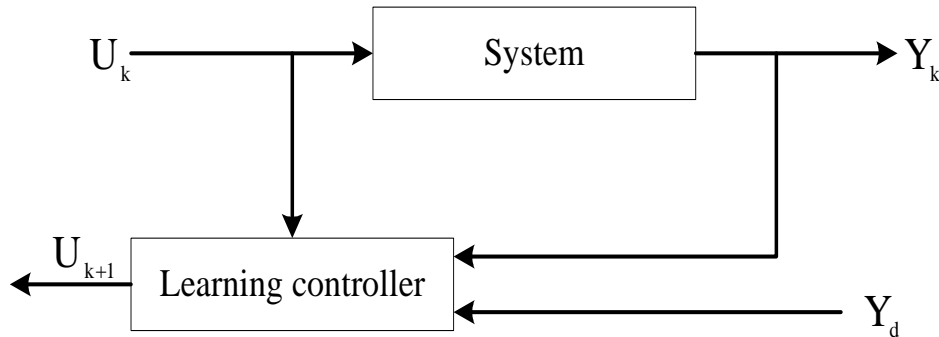
Define the “super-vectors:”

$$U_k = [u_k(0), u_k(1), \dots, u_k(n-1)]^T$$

$$Y_k = [y_k(1), y_k(1), \dots, y_k(n)]^T$$

$$Y_d = [y_d(1), y_d(1), \dots, y_d(n)]^T$$

$$E_k = [e_k(1), e_k(1), \dots, e_k(n)]^T$$



Using super-vectors, 2 dimensional problem has been changed as 1 dimensional MIMO system. Then, the system is described by:

$$Y_k = HU_k,$$

where Markov matrix is defined as:

$$H = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_n & h_{n-1} & \cdots & h_1 \end{bmatrix}$$

If the control vector is updated by:

$$U_{k+1} = U_k + \Gamma E_k,$$

where  $\Gamma$  is the learning gain matrix. Then, the error vector is propagated by:

$$E_{k+1} = (I - H\Gamma)E_k$$

The monotone convergence condition is derived as:

$$\|E_{k+1}\| = \|(I - H\Gamma)E_k\| \leq \|(I - H\Gamma)\| \|E_k\|$$

Thus, if  $\|(I - H\Gamma)\| < 1$ , then  $\|E_{k+1}\| < \|E_k\|$ . So, error monotonically converges to zero as  $k$  increases. Asymptotical stability condition is defined as:  $\rho(I - H\Gamma) < 1$ .

## Robustness issue in Super-vector ILC

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- The robustness is related with the model uncertainty of the system plant.
- Let the nominal plant be given as:

$$x_{k+1} = Ax_k + BU$$

$$Y_k = Cx_k$$

- Assume that the model uncertainty exist in  $A$ ,  $B$ , and  $C$ . Then, from following relationship:

$$h_i = CA^{i-1}B$$

the model uncertainties in  $A$ ,  $B$ , and  $C$  are conversed into  $h_i$ . So, the model uncertainty of  $H$  should be determined from  $A$ ,  $B$ , and  $C$ .

- Interval matrix method and interval matrix polynomial method are utilized for this purpose.

## Interval robust control

- Parametric interval approach: Interval polynomial problem, Russian journal by Kharitonov, 1978
- Mid 80's: Integrated to  $H_2$  and  $H_\infty$  theories.
- Interval matrix: Stability analysis. Active researches in 80's. Main portion: Hurwitz stability.
- Interval matrix: Interval conversion. Very rare research. Only Mayer and Chin-Tzong Pang.
- Interval matrix polynomial: Studied on the base on Kharitonov theorem and LMI based stability analysis.
- Interval parameter and matrix are defined as:

$$a^I \in [\underline{a}, \bar{a}]; A^I \in [\underline{A}, \bar{A}]$$

## Interval problems of first order ILC

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- Q1: How to calculate the interval boundaries of  $h_i \in [\underline{h}_i, \overline{h}_i]$  from  $C^I(A^I)^{i-1}B^I$  ?
- Q2: How to find maximum spectral radius of  $(I - H^I\Gamma)$  ?
- Q3: How to find the maximum induced operator norm of  $(I - H^I\Gamma)$  ?
- Q4: How to design  $\Gamma$  such that  $\rho(I - H^I\Gamma) < 1$  ?
- Q5: How to design  $\Gamma$  such that  $\|I - H^I\Gamma\| < 1$  ?
- Q6: How to maximize  $\|\Delta H\|$  ( $\Delta H := H^I - H$ ) by designing  $\Gamma$  under the stability condition and monotone convergence condition ?

## High order Interval ILC

- The control input vector is upgraded by:

$$U_{k+1} = \Lambda_k U_k + \Gamma_k E_k + \cdots + \Lambda_1 U_1 + \Gamma_1 E_1$$

- If  $\Lambda_i$  are lower triangular Toeplitz matrix, the error vector is propagated in high order ILC:

$$E_{k+1} = (\Lambda_k - H^I \Gamma_k) E_k + \cdots + (\Lambda_1 - H^I \Gamma_1) E_1$$

So, since  $H^I$  is interval matrix, the high order interval ILC is an interval matrix polynomial.

## Interval problems of high order ILC

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- Q7: How to analyze the stability of interval matrix polynomial ?
- Q8: How to design  $\Lambda_i$  and  $\Gamma_i$  to guarantee the asymptotical stability of interval high order ILC ?
- Q9: What is the monotone convergence condition of interval matrix polynomial ?
- Q10: How to design  $\Lambda_i$  and  $\Gamma_i$  to guarantee the monotone convergence of interval high order ILC ?

# Achievements

- The learning gain matrix was designed using LMI.
- Q1: The interval conversion method was developed based on first order perturbation theory. This method uses eigenpair-decomposition method.
- Q2: The maximum spectral radius of  $(I - H^I\Gamma)$  is less than maximum spectral radius of vertex matrices:  $(I - H^v\Gamma)$ . Interval matrix stability checking method is used.
- Q3: The maximum 1 and  $\infty$  norms of  $\|I - H^I\Gamma\|$  occur at  $\|I - H^v\Gamma\|$ .
- Q4-Q5: The optimization methods have been suggested to design  $\Gamma$  such that  $\rho(I - H^I\Gamma) < 1$  and  $\|I - H^I\Gamma\| < 1$  based on interval matrix stability checking method.
- Q6: Lyapunov stability analysis method was used to maximize  $\|\Delta H\|$  ( $\Delta H := H^I - H$ ) under the stability condition and monotone convergence condition.
- Q7: Stability analysis method of interval matrix polynomial was developed.

## Publication plan

- The LMI based ILC: under modification.
- The results of Q1, Q4, and Q5 will be submitted to IEEE CNCNS 05 (Under modification. Agreement needed.).
- The results of Q2-Q3 will be submitted to IFAC 05.
- The result of Q6 was submitted to ACC 05.
- The results of Q7 will be submitted to IFAC 05.

## Current and Future research directions

- Q8: Currently, optimization method is under development to design  $\Lambda_i$  and  $\Gamma_i$  to guarantee the asymptotical stability of interval high order ILC.
- Q9: Under research !
- Q10: This problem could be solved if Q9 is answered.
- To solve Q9 and Q10, the interval polynomial problem should be solved first. However, even this problem is considered tough problem in control society.

## Research proposal of Q8

Let the control input vector be calculated as:

$$\begin{aligned} U_{k+1} = & \Lambda_k U_k + \Lambda_{k-1} U_{k-1} + \cdots + \Lambda_1 U_1 \\ & + \Gamma_k E_k + \Gamma_{k-1} E_{k-1} + \cdots + \Gamma_1 E_1, \end{aligned} \quad (1)$$

where  $k$  denotes the iteration trial;  $\Lambda_i, i = 1, \dots, k$  are learning gain matrices from pervious control input vectors; and  $\Gamma_i, i = 1, \dots, k$  are learning gain matrices from previous error vectors. From  $E_k = Y_d - Y_k$ , we have

$$Y_k = Y_d - Ek. \quad (2)$$

After changing  $Y_k = HU_k$  to  $U_k = H^{-1}Y_k$ , we substitute  $U_k = H^{-1}Y_k$  into (1). Then, we obtain following relationship:

$$H^{-1}Y_{k+1} = \Lambda_k H^{-1}Y_k + \cdots + \Lambda_1 H^{-1}Y_1 + \Gamma_k E_k + \cdots + \Gamma_1 E_1 \quad (3)$$

By inserting (2) into (3),

$$\begin{aligned} H^{-1}(Y_d - E_{k+1}) &= \Lambda_k H^{-1}(Y_d - E_k) + \dots \\ &+ \Lambda_1 H^{-1}(Y_d - E_1) + \Gamma_k E_k + \dots + \Gamma_1 E_1. \end{aligned} \quad (4)$$

We change the right-hand side such as

$$\begin{aligned} (\Lambda_k + \dots + \Lambda_1)H^{-1}Y_d - \Lambda_k H^{-1}E_k - \dots \\ - \Lambda_1 H^{-1}E_1 + \Gamma_k E_k + \dots + \Gamma_1 E_1. \end{aligned} \quad (5)$$

Then, we make following relationship to make the steady-state error go to zero:

$$\Lambda_k + \dots + \Lambda_1 = I$$

Then (4) is changed as:

$$E_{k+1} = (H\Lambda_k H^{-1} - H\Gamma_k)E_k + \dots + (H\Lambda_1 H^{-1} - H\Gamma_1)E_1 \quad (6)$$

In (6), the Markov matrix  $H$  is an interval matrix such as  $H^I$ . So, if we denote  $H^I \Lambda_i (H^I)^{-1} - H^I \Gamma_i = A_i^I$ , then  $A_i^I$  are interval coefficient matrices. If  $A_i$  are nominal matrices without intervals, we define the interval radius matrices such as:  $\Delta A_i = A_i^I - A_i$ . If we rewrite (6) such as:

$$E_{k+1} = A_k^I E_k + \cdots + A_1^I E_1, \quad (7)$$

then we find that (7) is a discrete interval matrix polynomial.

- Is there advantage if  $H^I \Lambda_i (H^I)^{-1} - H^I \Gamma_i$  is simplified as  $\Lambda_i - H^I \Gamma_i$  ?
- How to guarantee the asymptotical stability of (7) ? The results of Q7 can be used to solve this problem. Optimization should be used here.
- It is not easy work to design  $\Lambda_i$  and  $\Gamma_i$  such that (7) is stable against the interval uncertainty in  $H^I$ .

## Research proposal of Q9

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- What is the monotone convergence condition of (7) ?
- One possible answer: From

$$\begin{aligned}\|E_{k+1}\| &= \|A_k^I E_k + \cdots + A_1^I E_1\|, \\ &\leq \|A_k^I E_k\| + \cdots + \|A_1^I E_1\| \\ &\leq \|A_k^I\| \|E_k\| + \cdots + \|A_1^I\| \|E_1\|\end{aligned}\quad (8)$$

if  $\|A_k^I\| + \cdots + \|A_1^I\| < 1$ , then the system is monotonic convergent ? Conservatism issue ? and only is this monotone convergence condition ? . Is this sufficient condition or also necessary condition ? . Should be answered !. From matrix companion form,  $\|A_k^I\| + \cdots + \|A_1^I\| < 1$  could be if and only if condition (should be answered).

- Is there other solution ?

# An interesting discovery of Q9

Let us consider simple case:

$$E_{k+1} = (\Lambda_k - H^I \Gamma_k) E_k + \cdots + (\Lambda_1 - H^I \Gamma_1) E_1$$

- Following condition is required to make zero steady state error:

$$\Lambda_k + \cdots + \Lambda_1 = I$$

- Let us assume that the interval ILC is not monotonic convergent in first order ILC, i.e.:

$$\|I - H^I \Gamma\| \geq 1$$

- Then, if and only if  $\sum_{i=k}^{i=1} \|\Lambda_i - H^I \Gamma_i\| < 1$  is monotone convergence condition, the high-order ILC does never achieve the monotone convergence with zero steady-state error due to the following relationships:

$$\begin{aligned} \|\Lambda_k - H^I \Gamma_k\| + \cdots + \|\Lambda_k - H^I \Gamma_k\| &\geq \|\Lambda_k - H^I \Gamma_k + \cdots + \Lambda_1 - H^I \Gamma_1\| \\ &= \|I - H^I \Gamma_k - \cdots - H^I \Gamma_1\| \\ &= \|I - H^I (\Gamma_k + \cdots + \Gamma_1)\| \\ &= \|I - H^I \Gamma\| \geq 1 \end{aligned}$$

## Preliminary question of Q9

- Let us consider just interval polynomial such as:

$$p^I(z) = a_n^I z^n + \cdots + a_1^I z + a_0^I \quad (9)$$

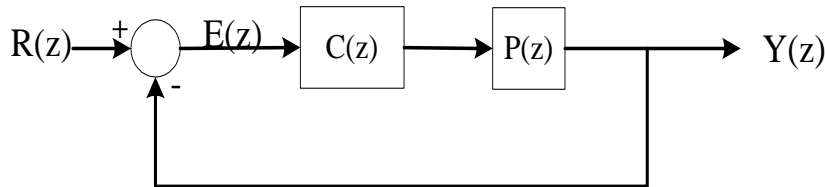
- In interval algebra and robust control area, only the asymptotical stability of (9) has been studied. No appropriate result can be found for checking the monotone convergence of (9).
- Thus, the monotone convergence condition of (9) should be found as preliminary work of Q9.

# Preliminary (or advanced) question of Q9

- In following plant:

$$P(z) = \frac{a_m z^m + \cdots + a_1 z + a_0}{b_n z^n + \cdots + b_1 z + b_0}, \quad n > m \quad (10)$$

how can we guarantee the non-overshooting of the step response ?  
How to design controller  $C(z)$  ?



- Reference papers: Four IEEE TAC papers: Dahleh and Pearson 88; Bhattacharyya, Keel and Howze 88; Moore and Bhattacharyya 90; Darbha and Bhattacharyya 03. One CDC paper: Deodhare and Vidyasagar 90.

- If (10) is non-overshoot by controller  $C(z)$ , we might make (9) to be monotonic convergent using feedback controller. Then, if (8) is transformed into a scalar interval polynomial (as done in the solution of Q7; or, still matrix polynomial without interval uncertainty, by Smith form), then we may guarantee the monotonic convergence of (8). However, the feedback controller should be added to ILC (note: ILC is feedforward controller).
- This approach is considered quite tough and will require long term research.

## Research proposal of Q10

- The solution of Q10 can be found after Q9 is solved.
- Some existing works related with Q10: None.

## Summary: future research topic

- What is the monotonic convergent condition of (interval) (matrix) polynomial ?
- How to design the controller to make the non-overshoot output ?
- How to effectively design learning gain matrices in high order interval ILC ?
- Ultimate question ? Is there any advantage using high order ILC, when Markov matrix is interval matrix ?. Currently, definitely it looks like that there exist some advantages. However, analytically we have to find it !!.

## Conclusions

- By this time, the first order interval ILC has been analyzed and the learning gain matrix design techniques have been developed.
- However, the high-order interval ILC has not been analyzed properly, particularly the monotone convergence property.
- In ILC, the high order approach is one of the powerful ILC advantages. So, for developing a complete new framework of interval robust ILC, it is indispensable to devote more efforts to the high order interval ILC system.
- This problem should be attacked gradually by solving the preliminary problems as explained.
- The suggested research has two main contributions in control area. (1) ILC system can be designed with considering both monotone convergence and robustness and (2) the results of this research can be used in robust control area, because we are handling a few unsolved interval polynomial problem and unanswered linear system problems.

Thank you!

Q/A session