PATH TRACKING OF A FIXED-WING AUTONOMOUS AERIAL VEHICLE BY HIGH ORDER SLIDING MODE CONTROL

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ABSTRACT
In this paper, a robust control algorithm, based on sliding modes techniques, designed for trajectory tracking is applied to a fixed-wing small Autonomous Aerial Vehicle (AAV). Based on a mathematical modeling, parameters identification, and aerodynamics of both the AAV fuselage and its mobile surfaces a full dynamical model is obtained, where the control is proven. However, for control design, simplified versions of the motion models are studied resulting in simplified independent controller for the roll, pitch and yaw trajectories. Due to the nature of such controller, time derivative of control variables are needed but not available. Numerical differentiators also based on sliding modes are used in order to estimate those derivatives. Simulation results are given to illustrate the performance of the proposed tracking controller under parametric and unmodeled dynamics.

Keywords: UAVs, Path tracking, high order sliding mode control, Dynamics of an UAV.

NOMENCLATURE
\( u, v, w \) Velocity components of the aircraft.
\( p, q, r \) Angular rates components.
\( \theta, \phi, \psi \) Roll, pitch and yaw angles
\( F_X, F_Y, F_Z \) Forces applied.
\( F_L, F_M, F_N \) Moments applied.
\( \beta \) Sideslip angle.
\( \alpha \) Angle of attack.
\( \bar{c} \) Mean aerodynamic chord.
\( c_D \) Drag coefficient.
\( C_L \) Lift coefficient.

INTRODUCTION
Unmanned Aerial Vehicles (UAVs) are vehicles not provided by human pilot in the aircraft, although some times they are provided via remote control. When the flight control is done automatically by an autopilot or small controller card these vehicles are said to be Autonomous Aerial Vehicles (AAVs). There is a wide range of UAVs whose characteristics vary in size, weight, configuration and geometry. Each characteristic affects the aerodynamics of the aircraft. Due to their control performance it is possible to distinguish 2 types of UAVs: first controlled from away location via remote control and the autonomous flight based on pre-programmed flight. UAVs are affected by different environmental factors such as altitude, wind, pressure, temperature and other atmospheric factors that interfere with the programmed flight plan. Therefore, the design of robust controllers for AAVs must not only take account of the dynamics that rep-
represents the movement of vehicle (three-dimensional translational and rotational speed) but also external factors affecting the fulfillment of its mission. Notice that the automatic control of the AAV presents several challenges ranging from mathematical modeling to the aerodynamic configuration of the UAV, both parts are necessary to understand the characteristics of the vehicle and therefore limit the adequate implementation.

Recently, sliding mode control technique has attracted the attention of a lot of research groups. This technique allows to design robust control laws that are insensitive to the parametric uncertainties and unmodeled dynamics (see for more details [10]). Furthermore, the stability properties of the closed loop system can be guaranteed in finite time. The analysis for the control of an autonomous unmanned aircraft is very extensive, and thus breaks down into several steps, that are already reported in the literature. The mathematical model of a aircraft is a fundamental part of the study that express the vehicle dynamic behavior (see for more details [2, 3]). While the kinematics and inertial analysis are basically the same for any aircraft, the aerodynamic is not due to its dependence on the vehicle geometry. Besides, the aerodynamics are complex non-linear functions that need to be measured in costly fluid laboratory facilities or estimated using software (see [6, 8]). Even when the resulting model is extremely complicated, the control design for ensuring the stability of the air vehicle during flight, should reduce and decouple the moving control surfaces of the airplane to allow for guidance (see [7]). Moreover, the problem of path tracking is also a major one for which some results have been published (see for more details [13–15]), where different schemes have been considered. The main issue of this problem is that the aircraft cannot perform all the admissible movements in the space without loosing control (for example lateral displacement, or turning in a static location).

In this work we proposed the use of a robust control law, with the particular application of the path tracking of a small AAV. The overall control law is composed of simple decoupled controllers for each independent control surface on the aircraft, that guarantees closed loop stability of the overall system even under parametric uncertainties and coupled dynamics.

This paper is organized as follows: In the first section the mathematical model with the particular aerodynamic analysis is presented. Subsequently, control design for two complementary flight modes is explained. Finally, the path tracking problem is solved and validated by numerical simulation.

**DESCRIPTION OF THE UAV**

The particular model of the UAV to be used along this work is a scale model of a B-25 Mitchell, shown in Figure 1, whose main parameters are also shown in Table 1. The differences due to control surfaces between our scale model and a manned version of this airplane are: 1) the scale UAV does not have flaps, but its does have the same ailerons, rudder and elevator; 2) the cruiser velocity of the prototype does not reach more than 30% of Mach number.

Then the prototype behaves mostly as the real one, although it is more sensible to the environment (wind currents and turbulence).

**EQUATIONS OF MOTION**

The derivation of the motion equations for a rigid object (fixed-wing aerial vehicle) moving freely in the 3D space, using the Newton’s laws, leads to 6 scalar equations (see for more details [4]). Most of the time these equations although expressed in the airplane frame are deduced in an inertial one due to Newton’s laws. In most aeronautical literature they are expressed as the full set of 6 equations. In the following section we present a compact, linear algebra based expression that reduce the written equations.
that are no unique but represent the 3 orientation degrees of freedom. These parameters can be given in any set of euler angles (3 variables) or other non minimal sets of attitude representation such quaternions. In this work we use the roll-pitch-yaw euler angles convention (see [4]), expressed as \( \mathbf{\hat{\Theta}} = [\phi, \theta, \psi]^T \). Then for the particular set of roll-pitch-yaw angles, the Rotation matrix is given as

\[
R(\hat{\Theta}) = \begin{bmatrix}
C_{\psi}C_{\theta} - S_{\psi}S_{\phi} & C_{\psi}S_{\phi} & S_{\psi}S_{\theta} + C_{\psi}C_{\theta}C_{\phi} \\
S_{\psi}C_{\theta} + C_{\psi}S_{\phi}S_{\theta} & C_{\phi}C_{\theta} & -S_{\psi}S_{\phi} + C_{\psi}S_{\theta}C_{\phi} \\
-S_{\theta} & 0 & C_{\theta}
\end{bmatrix}
\]

where \( S_x \) and \( C_x \) stand for the \( \sin(x) \) and \( \cos(y) \) function with their corresponding arguments. The angular velocity is given by the rate change of the attitude and in consequence by the time derivative of the rotation matrix by the next expression

\[
\dot{R} = [\hat{\omega}(0) \times] R \tag{1}
\]

where the matrix term \([\hat{\omega} \times]\) stands for the matrix expression of the vector cross product and is defined as

\[
[\hat{\omega} \times] \triangleq \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix} \in \mathbb{R}^{3 \times 3} \tag{2}
\]

A vector expression for the angular velocity can be computed as \( \hat{\omega}(0) = \frac{1}{2} (\vec{r}_1 \times \vec{r}_1 + \vec{r}_2 \times \vec{r}_2 + \vec{r}_3 \times \vec{r}_3) \) where vectors \( \vec{r}_i \) are the column vectors of matrix \( R \) (see for more details [5]). From last expression in can be deduced that the inertial expression of the angular velocity can be computed as \( \hat{\omega}(0) = J_{0}(\hat{\Theta})\hat{\omega} \), where the linear operator is given explicitly as

\[
J_{0}(\hat{\Theta}) = \begin{bmatrix}
C_{\theta}C_{\psi} & -S_{\psi} & C_{\psi}S_{\theta} \\
C_{\phi}S_{\theta} & C_{\phi}C_{\theta} & -S_{\phi}S_{\theta}C_{\phi} \\
-S_{\theta} & 0 & C_{\theta}
\end{bmatrix}
\]

Finally, the computation of the linear velocity \( \vec{v} = \vec{v}^{(1)} = [u, v, w]^T \) and angular velocity \( \vec{\omega} = \vec{\omega}^{(1)} = [p, q, r]^T \) of the aircraft, expressed in it non inertial frame, is given by the next set of equations, named here as the aircraft kinematics

\[
\dot{\vec{d}} = R(\hat{\Theta})\vec{v} \tag{3}
\]

\[
\hat{\Theta} = [J_{0}^{-1}(\hat{\Theta})R(\hat{\Theta})]\hat{\omega} \tag{4}
\]

where \( \vec{d} = [x, y, z]^T \) is the inertial expression in cartesian component, of the aircraft position.

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1This is is due to the orthonormal properties of this kind of matrices where the nine components can be fully parameterized by only 3 independent variables.
Dynamics
If the body (aircraft) is considered to be rigid, i.e. that there is no deformation due to the nature of the material it is built of, then distances between each particle remains constant. Under this assumption, the body can be characterized with lumped parameters, which happens to be constant when expressed in the non inertial body frame. Then from linear and angular momentum conservation laws, both dynamics expressions, for forces and moments, are given as follows [5], which represent the aircraft dynamics:

\[
\ddot{\mathbf{f}} = m(\ddot{\mathbf{v}} + \mathbf{\omega} \times \mathbf{v}) - m \mathbf{R}^T (\mathbf{\Theta}) \ddot{\mathbf{g}}
\]

\[
\ddot{\mathbf{M}} = I \ddot{\mathbf{\omega}} + [\mathbf{\omega} \times I \mathbf{\omega}]
\]

The force vector \(\ddot{\mathbf{f}} = [F_x, F_y, F_z]^T\) in the dynamic equation, is composed with the cartesian components of the external forces expressed in the aircraft non inertial frame, applied at the center of mass. The moment vector \(\ddot{\mathbf{M}} = [M_x, M_y, M_z]^T\) is likewise expressed in the local aircraft frame and applied at the center of mass. The gravity vector \(\ddot{\mathbf{g}} = [0, 0, g]^T\) expresses the direction of gravity acceleration. The dynamic lumped parameters are the aircraft mass \(m\) and the inertia tensor \(I \in \mathbb{R}^{3 \times 3}\) (also known as the moment of inertia matrix). This inertia tensor is a symmetric positive definite square matrix \((I = I^T > 0)\) of the form:

\[
I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}
\]

The set of equations (3)-(4)-(5)-(6) express the 6 DOF motion of any rigid object. These are the same equations presented in the literature (see [4]), but written in a compact (linear algebra) form. The external forces and moment components on dynamics equations (5)-(6) in an aircraft are given by two natures. One is due to the engines that input energy to the system and the other is to aerodynamics forces and moments, known as the aerodynamics of the aircraft.

Aerodynamics
The aerodynamics forces and moments in (5)-(6) can be calculated via aerodynamic coefficients as follows (see [4]):

\[
\ddot{\mathbf{f}} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + \bar{q} S [\bar{C}_{b-w}(\alpha, \beta)]^{-1} \begin{bmatrix} -C_D(\cdot) \\ C_y(\cdot) \\ -C_L(\cdot) \end{bmatrix} \tag{7}
\]

\[
\ddot{\mathbf{M}} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \bar{q} S \begin{bmatrix} bC_l(\cdot) \\ \bar{c}C_m(\cdot) \\ bC_n(\cdot) \end{bmatrix} \tag{8}
\]

where \(\alpha = \arctan(\frac{w}{V})\) is the angle of attack (if the chord line is aligned the fuselage’s) and the sideslip angle \(\beta = \arcsin(\frac{w}{V})\) expresses the lateral slip’s. The transformation matrix \(\bar{C}_{b-w}(\alpha, \beta)\) maps the aerodynamic forces along a relative velocity frame to the aircraft non inertial frame, and is given explicitly as [4]:

\[
\bar{C}_{b-w}(\alpha, \beta) = \begin{bmatrix} C_aC_\beta & S_\beta & S_aC_\beta \\ -C_aC_\beta & C_\beta & -S_aS_\beta \\ -S_\alpha & 0 & C_\alpha \end{bmatrix}
\]

The terms \(L = \bar{q} SC_L(\cdot)\) and \(D = \bar{q} SC_D(\cdot)\) are the Lift and Drag forces acting along the airplane. The dynamic pressure \(\bar{q} = \frac{1}{2} \rho V^2\) is a scalar dynamic inner variable where \(V = \sqrt{u^2 + v^2 + w^2}\) is the relative airspeed magnitude. The aerodynamic constant parameters are: wing surface area \(S\), the wingspan \(b\), the mean aerodynamic cord \(\bar{c}\), and the air density \(\rho\). The dimensionless coefficients in the force/moment expressions can be decomposed furthermore in the following set\(^2\) [4], with the control surface angles \(\delta e, \delta a\) and \(\delta r\) corresponding to the moving surfaces: elevator, ailerons\(^3\) and rudder respectively.

\(^2\)The subscripts represent the influence coefficient. For instance \(C_{\alpha a}\) represents the contribution of the angle of attack with the lift coefficient of the airplane.

\(^3\)The aileron angle is considered to be angle of the right section aileron, positive upward, that produce a positive banking. The left section aileron is the negative of the former.
(see Figure 4):

\[ C_L = C_{L0} + C_{La} \alpha + C_{L\delta f} \delta f + C_{L\delta e} \delta e \]  
\[ + \frac{c}{2V} (C_{L\alpha} \alpha + C_{Lq} q) + C_{LM} M \]  
\[ C_D = C_{D0} + \left( \frac{C_{L0} - C_{L0}}{\pi eAR} \right) + C_{D\delta f} \delta f + C_{D\delta e} \delta e \]  
\[ + C_{D\delta a} \delta a + C_{D\delta r} \delta r + C_{LM} M \]  
\[ C_Y = C_{Y\beta} + (C_{Yp} p + C_{Yr} r) \frac{b}{2V} + C_{Y\delta a} \delta a + C_{Y\delta e} \delta e \]  
\[ C_l = C_{l\beta} + (C_{lp} p + C_{lr} r) \frac{b}{2V} + C_{l\delta a} \delta a + C_{l\delta e} \delta e \]  
\[ C_M = C_{m\beta} + C_{ma} \alpha + C_{m\delta f} \delta f + C_{m\delta e} \delta e \]  
\[ + (C_{mq} q + C_{ma} \alpha) \frac{c}{2V} \]  
\[ C_n = C_{n\beta} + (C_{np} p + C_{nr} r) \frac{b}{2V} + C_{n\delta a} \delta a + C_{n\delta r} \delta r \]  

Last expressions use also the dimensionless numbers: Oswalds efficient number \( e \), the Mach number \( M \), and the aspect ratio \( AR = b^2/S \). The Tornado software (see \[8, 9\]) has been used to identify the coefficients of our Case Study using the “vortex lattice method” (See Table 2). The analysis assumes incompressible\(^5\) and potential\(^6\) flow acting on any airfoils and wings in an airplane. The lack of viscosity induce some minor errors in the final results, because skin friction at the laminar layer is neglected. However turbulent flows are taken into account via the drag coefficients. The geometry is input in the software via the panel method, which for our Case Study is shown in Figure 3.

### CONTROL DESIGN

The physical elements of control in the airplane are (see Figure 4) the control surfaces. They are three: a) the elevator that produce an angle \( \delta e \) which in turns generates a pitching motion around the local \( y \)-axis, at a pitch-rate \( q \); b) the rudder that produce an angle \( \delta r \) which in turns generates a heading motion around the local \( z \)-axis, at a yaw-rate \( r \); and c) the ailerons that produce an angle \( \delta a \) which in turns generates a rolling motion around the local \( x \)-axis, at a roll-rate \( p \). Because these variables are the control inputs of the aircraft, the control technique is focus on the attitude control of the engine: roll, pitch and yaw. Since the parameter estimation has been performed with hypothetical assumptions, possibility of having parametric uncertainty is highly possible. Also the real system (5)-(6) is coupled in almost all the velocity variables which induce parasitic dynamic phenomena form one variable to another.

Therefore, a control algorithm has been chosen to be a high-order sliding-mode controller proposed in \[10\], based on simplified versions of the motion modeling. This algorithm has been chosen due to its properties of robustness against uncertainties (in this case for both parametric and coupled dynamic).

### TABLE 2. Characteristics of the UAV B-25 Mitchell.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{La} )</td>
<td>5.1923</td>
<td>( C_{lp} )</td>
<td>-0.5018</td>
</tr>
<tr>
<td>( C_{L0} )</td>
<td>0.45454</td>
<td>( C_{Y\beta} )</td>
<td>1.7649</td>
</tr>
<tr>
<td>( C_{La} )</td>
<td>11.2138</td>
<td>( C_{Yr} )</td>
<td>-1.8558</td>
</tr>
<tr>
<td>( C_{D0} )</td>
<td>0.011903</td>
<td>( C_{Yp} )</td>
<td>0.028185</td>
</tr>
<tr>
<td>( C_{Da} )</td>
<td>0.27227</td>
<td>( C_{n\delta a} )</td>
<td>0.014258</td>
</tr>
<tr>
<td>( C_{Da} )</td>
<td>0.48183</td>
<td>( C_{n\delta r} )</td>
<td>-0.06825</td>
</tr>
<tr>
<td>( C_{mq} )</td>
<td>-17.9711</td>
<td>( C_{l\delta a} )</td>
<td>0.20688</td>
</tr>
<tr>
<td>( C_{ma} )</td>
<td>-3.1037</td>
<td>( C_{l\delta a} )</td>
<td>0.20688</td>
</tr>
<tr>
<td>( C_{ng} )</td>
<td>-0.03077</td>
<td>( C_{l\delta r} )</td>
<td>0.012136</td>
</tr>
<tr>
<td>( C_{nr} )</td>
<td>-0.20076</td>
<td>( C_{Y\delta a} )</td>
<td>0.015862</td>
</tr>
<tr>
<td>( C_{np} )</td>
<td>-0.015294</td>
<td>( C_{Y\delta r} )</td>
<td>-0.15502</td>
</tr>
<tr>
<td>( C_{I\beta} )</td>
<td>0.15875</td>
<td>( C_{m\delta e} )</td>
<td>-1.6459</td>
</tr>
<tr>
<td>( C_{Ir} )</td>
<td>0.020368</td>
<td>( C_{L\delta e} )</td>
<td>0.56974</td>
</tr>
<tr>
<td>( C_{D\delta e} )</td>
<td>0.026848</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^4\)Because of the velocity range of a scale airplane, this factor is neglected for this AUV.

\(^5\)This is appropriate for the flight regime of a UAV being under 0.3M

\(^6\)With no viscosity
Simplified Motion Modes

Two motion modes are studied in this work. They are complementary with each other and the separation allows a more simple representation of the overall movement of the AAV, as well as simplified models on which the above mention control technique would be applied. These are the **lateral motion mode** and the **longitudinal motion mode**.

**Longitudinal motion**  This mode represents the motion in the sagittal plane of the aircraft (See Figure 5.a) with only three degrees of freedom: horizontal displacement (x-direction), altitude (z-direction) and the pitch. To get the expression of this mode, we introduced the following assumptions to the full model in (5)-(6):

**Assumption A1:** The roll and yaw angles are zero, i.e., $\phi = \psi = 0$, then the lateral angle $\beta$ does not exist and the angular velocities $p = r = 0$. Thus, the number of aerodynamic equations are reduced and only the vertical plane motions are needed.

**Assumption A2:** If displacement cruise speed and altitude are constant, then the $\dot{x}$ and $\dot{z}$ dynamics can be neglected. Therefore, the equation set representing the longitudinal motion mode of an airplane can be simplified\(^7\) to

\[\begin{align*}
\dot{u} &= \frac{F_x}{m} - qw - g \sin \theta \quad (15) \\
\dot{w} &= \frac{F_z}{m} + qu + g \cos \theta \quad (16) \\
\dot{q} &= \frac{F_M}{I_z} \quad (17) \\
\dot{\theta} &= q \quad (18)
\end{align*}\]

**Lateral motion**  This mode represents the remanent motion of the aircraft (See Figure 5.b) with the remaining three degrees of freedom: sideslip displacement (y-direction), the roll and the heading (also referred as the yaw angle). To get the expression of this mode, again the following assumptions to the full model are introduced:

**Assumption A3:** If longitudinal motion is controlled and steady, the pitch angle $\theta$ and translation speeds $u$ and $w$ can be considered to be constant. Then the pitch rate $q = 0$, and the horizontal and vertical dynamics can be neglected.

**Assumption A4:** Also lateral movement can be neglected because it is not important for control purposes.

Then the set of equations that represents the dynamics in the lateral motion mode of an airplane are given by:

\[\begin{align*}
\dot{v} &= \frac{F_y}{m} + pw - ru + g \sin \phi \cos \theta \quad (19) \\
\dot{p} &= \left(\frac{I_z F_L + I_z F_N}{I_x I_z} - \frac{I_z^2}{I_{zz}}\right) \\
\dot{r} &= \left(\frac{I_x F_L + I_x F_N}{I_x I_z} - \frac{I_z^2}{I_{zz}}\right) \\
\dot{\psi} &= r \cos \phi \\
\dot{\phi} &= p
\end{align*}\]

**Pitch control**

The pitch control is designed to stabilize the longitudinal dynamics of the aircraft. To do so the aerodynamic pitch moment coefficient (13) is used under the following conditions: Since there are no flaps in the scale model and the Mach influence is not significant we have that $C_m \delta f = C_m M = 0$. Moreover, the symmetry of the wings and fuselage leads to null coefficients: $C_{m0} = C_{ma} = 0$.

Then the simplified dynamic equation for the pitch movement can be expressed in state-space using equations (17)-(18) together with (13) as

\[\begin{align*}
\dot{\theta} &= q \quad (24) \\
\dot{q} &= \frac{\rho V^2 S e}{2 I_{yy}} \left[ C_{m\alpha} \alpha + C_m \delta e + C_{mq} \frac{\dot{\phi}}{V} \right] \quad (25)
\end{align*}\]

Since this is a non-linear system with and a relative degree

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\(^7\) According to Table 1, for this Case Study, and due to the symmetry of the aircraft some of the specific off-diagonal terms in the inertia tensor are null: $I_{y\phi} = I_{y\psi} = 0$.
posed be as follows

\[ \delta e = \frac{v - \frac{PV^2 \delta e}{2 \rho} [C_{m\alpha} \alpha + C_{ma} \dot{q} \frac{c}{V}]}{\frac{PV^2 \delta e}{2 \rho} C_{m\delta e}} \]  

(26)

\[ v = \frac{q + | \theta - \theta_{ref} |^{1/2} \text{sign}(\theta - \theta_{ref})}{| q | + | \theta - \theta_{ref} |^{1/2}} \]  

(27)

The controller (26) is an inverse dynamics loop for the simplified system (25), which renders the closed-loop to \( \ddot{\theta} = \dot{q} = v \), where \( v \) is a secondary controller. However, the real closed-loop dynamics would become something like \( \dot{q} = v + \eta(\vec{\Theta}, \vec{v}, \vec{\omega}) \), where the function \( \eta \) is a disturbance due to parametric uncertainties, non-modeled dynamics or oversimplification on the previous procedure. To cope with that disturbance, the second loop (27), based on a high order sliding mode is used here.

Figure 6 shows simulation results when the pitch controller is applied to the full six degree of freedom model, and the coefficient \( C_m \) is changed (\( \pm 9\% \)) of its original value at time \( t = 25s \). Figure 6.a shows the linear velocity, inertial positions and attitude variables. The vertical component of the velocity presents variations due to the change of the pitch angle. Notice that the oscillations in theta curve (third plot), start at \( t = 0 \) because of the controller action and these oscillation become bigger when the parameter uncertainty increases. Figure 6.b shows the detail for the pitch angle \( \theta \) tracking its reference (\( \theta_{ref} \)) and the corresponding angular velocity. The pitch angle quickly converges to the desired attitude, while the angular rate \( q \), also converges to zero. Figure 6.c shows the control signal, i.e the deflection of elevator \( \delta_e \), and the tracking error. The robustness of the controller has been demonstrated since the controller get a good performance under coefficient variation.

**Roll control**

Roll control is designed to lateral-directional dynamic model. To do so, the aerodynamic pitch moment coefficient (12) is used under the following conditions: Lateral speed is very small compared to front displacement, then the sideslip angle \( \beta \cong 0 \). Suppose the angular velocity in the direction of rotation \( r = 0 \). Finally, if the position of the deflection is in neutral position, i.e. \( \delta r = 0 \).

Then the simplified dynamic equation for the roll movement can be expressed in state-space using equations (20) and (23) together with (12) as

\[ \dot{\phi} = p \]  

(28)

\[ \dot{p} = \frac{I_{\phi} PV^2 S b}{4 \Gamma} \left[ C_{l\phi} p \frac{b \phi}{2V} + C_{l\delta\alpha} \delta \alpha \right] \]  

(29)
With $\Gamma = I_{xx} h_y - I_{zz}$. Since this a non-linear system with relative degree equal to 2, and $\delta \alpha$ is the input control, the control function is proposed be as (see [10]).

$$\delta \alpha = \frac{v - \frac{L_{xx} \rho V^2 S b}{4\Gamma} \left[ C_{p} \frac{b \phi}{2V} \right]}{\frac{L_{xx} \rho V^2 S b}{4\Gamma} (C_{d\alpha})}$$

$$v = \frac{p + |\phi - \phi_{ref}|^{1/2} \text{sign}(\phi - \phi_{ref})}{|p| + |\phi - \phi_{ref}|^{1/2}}$$

Figure 7 shows simulation results when the roll controller is applied to the full six degree of freedom model, and the coefficient $C_l$ is changed ($\pm 50\%$) of its original value, at time $t = 25\,s$.

Figure 7.a shows the linear velocity, inertial position and attitude variables, when the roll controls applied to the full 6dof aircraft. Note a small perturbation in the pitch angel due to the coupled dynamics. From Figure 7.b shows how the roll angle $\phi$ follows a square wave $\phi_{ref}$, and its angle speed $p$. It can be seen that the roll angle converges quickly to the desired position and velocity converges to zero. The deflection of the ailerons $\delta \alpha$ and the trackings error can be seen in the Figure 7.c. The robustness of the controller has been demonstrated since the controller get a good performance under coefficient variation.

**Yaw control**

Finally, to get the heading control the yaw control is designed. To do so the aerodynamic yaw moment coefficient (14) is used under the following conditions: Suppose the angular velocity in the direction of rotation $p = 0$. Also, if the position of the deflection is in neutral position, i.e., $\delta \alpha = 0$. Then the simplified dynamic equation for the yaw movment can be expressed in state-space using equations (21) and (22) together with (12) as

$$\dot{\psi} = r$$

$$\dot{r} = \frac{I_{xx} \rho V^2 S b}{4\Gamma} \left[ C_{n\beta} \psi + \frac{C_{np} b \psi}{2V} + C_{n\delta_r} \delta r \right]$$

With $\Gamma = I_{xx} h_y - I_{zz}$. Since this a non-linear system with relative degree equal to 2, and $\delta r$ is the input control, the control function is proposed be as (see [10]).

$$\delta r = \frac{v - \frac{L_{xx} \rho V^2 S b}{4\Gamma} \left[ C_{n\beta} \psi + \frac{C_{np} b \psi}{2V} \right]}{\frac{L_{xx} \rho V^2 S b}{4\Gamma} (C_{d\delta_r})}$$

$$v = \frac{r + |\psi - \psi_{ref}|^{1/2} \text{sign}(\psi - \psi_{ref})}{|r| + |\psi - \psi_{ref}|^{1/2}}$$

Figure 8 shows simulation results when the yaw controller is applied to the full six degree of freedom model, and the coefficient $C_n$ is changed ($\pm 50\%$) of its original value at $t = 25\,s$. 

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Figure 8.a shows the linear velocity, inertial position and attitude variables. Notice a slight deflection in the lateral displacement and high perturbation in banking and pitch, due to coupled dynamics. Figure 8.b shows that the yaw angle $\psi$ follows a square wave reference $\psi_{ref}$, and its angular rate $r$. The yaw angle rapidly converges to the desired position, while the velocity converges to zero. Figure 8.c represents the rudders deflection $\delta r$, and the convergence error. The robustness of the controller has been demonstrated since the controller get a good performance under coefficient variation.

**PATH TRACKING**

The kinematic model is represented by:

$$\dot{\bar{x}} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} V \cos \psi \\ V \sin \psi \\ u \end{bmatrix}$$

Where $x$ and $y$ are the cartesian coordinates, $V$ is the aircraft velocity and $\psi$ is the yaw angle, ie, the real input of the system. The control objective is to control the plane from a initial position to a desired trajectory $y = f(x)$. Defined $\sigma = y - f(x)$, let $V = 10m/s$ be, $x = y = 0.1$ and $\psi = 0$ in $t = 0$, $f(x) = 100\sin(0.05x) + 10$.

For this system uses the technique of differentiation and control by sliding mode output feedback (see [12]). The system relative degree is 2 and is sufficient with a relative degree controller for 2. However, to ensure they do not interfere the chattering phenomenon we upgrade the degree of the controller up to 3 for SISO systems (see for more details [11]). Then, the resulting controller is given by:

$$u = -\frac{[z_2 + 2(|z_1| + |z_0|^{2/3})^{-1/2}(z_1 + |z_0|^{2/3}\text{sign}(z_0))] - [z_2]}{[z_2] + 2(|z_1| + |z_0|^{2/3})^{1/2}}$$ (36)

Where the differentiator that estimates the time derivatives of the variables used in the sliding surface, which are not measurable and that are necessary for control implementation, is as follows:

$$z_0 = v_0, \ldots, v_0 = -14.7361|z_0 - \sigma|^{2/3}\text{sign}(z_0 - \sigma) + z_1$$
$$z_1 = v_1, \ldots, v_1 = -30|z_1 - v_0|^{1/2}\text{sign}(z_1 - v_0) + z_2$$
$$z_2 = -440\text{sign}(z_2 - v_1)$$ (37)

Figures 9 and 10 show simulation results of the scheme consisting on both the Controller and the differentiator. Figure 9 show the convergence path, the path tracking error and the behavior of the angle $\psi$. In addition, a simulation of the video image is shown in Figure 10, showing in the flight simulator the dynamic behavior of unmanned vehicle along a specific path.
CONCLUSIONS

In this paper, a robust control scheme for tracking of autonomous aerial vehicles has been presented. Based on assumptions of flight two simple mathematical models, representing the longitudinal and lateral dynamics of an airplane flight, can be used as a basis for the synthesis of 3 independent controllers, for each control surfaces. The proposed control scheme based on sliding mode techniques exhibits good performance on coupled trajectory tracking when used in the full nonlinear coupled aircraft dynamics, showing robustness against parametric uncertainties and coupled dynamics.

REFERENCES