Express Letters

Novel IIR Differentiator from the Simpson Integration Rule

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***Abstract*— A novel digital differentiator is introduced. The proposed differentiator is a stable second-order recursive differentiator suitable for applications that require fast differentiation methods. It is obtained from the Simpson integration rule. The accuracy and the range of the magnitude response of the proposed differentiator is the same as that of the Simpson integrator. Thus, it is comparable to that obtained by higher order algorithms. In addition, the resulting differentiator has an almost linear phase at low frequencies.**

I. Introduction

The Simpson integration rule is known to be much more accurate than the trapezoidal rule. Although it is a second-order interpolator, its accuracy is comparable to third-order interpolators [1]. In this letter, an IIR differentiator is obtained from the Simpson integrator by inverting the transfer function of the integrator using the approach outlined in [2]. The resulting stable differentiator is a second-order IIR differentiator with high accuracy at low frequencies. The low order and the high accuracy of the differentiator makes it attractive for real-time applications.

II. The Proposed Differentiator

The Simpson integrator approximates the ideal integrator for low frequencies while it amplifies the higher frequencies [3]–[4]. Thus, the differentiator obtained by inverting the transfer function of the Simpson's integrator would yield a filter that has high accuracy differentiation capabilities at low frequencies. It also combines a low pass function. The simplicity of the filter makes it practical and usable in some real-time biomedical [4], control [5], and other applications where the implementation platform is either a slow general purpose microprocessor or an application-specific integrated circuit (ASIC) [6].

The transfer function of the Simpson's integrator is [4]

$$G(z) = \frac{\{T(z^2 + 4z + 1)\}}{\{3(z^2 - 1)\}}. \tag{1}$$

Applying the approach in [2], the inverse of (1) is obtained, stabilized, and its magnitude is compensated. In taking the inverse of (1), a pole that lies outside the unit circle at $z = -3.7321$ is obtained. Substituting for the unstable pole a pole at $z = -1/3.7321 = -0.2679$, and multiplying the magnitude by $1/3.7321$, the following transfer function is obtained:

$$H(z) = \frac{\{3(z^2 - 1)\}}{\{T(3.7321)(z^2 + 0.5358z + 0.0718)\}}. \tag{2}$$

Thus, the new differentiator has two real zeros located at $z = +1$ and $z = -1$. It also has a double pole located at $z = -0.2679$.

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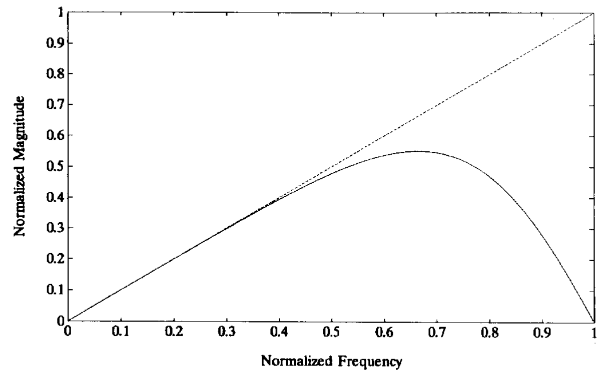

Fig. 1. Amplitude response of new differentiator (solid line) and ideal differentiator (dashed line).

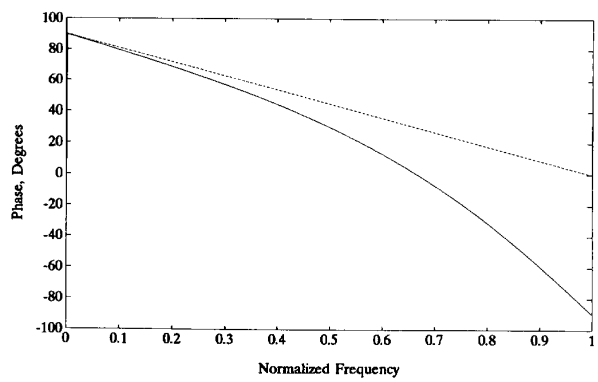

Fig. 2. Phase response of new differentiator (solid line) and ideal half-sample delay differentiator (dashed line).

III. The Magnitude and Phase Responses of the Proposed Differentiator

In the remainder of the letter, the Nyquist frequency will be normalized to 1. Fig. 1 shows the normalized magnitudes of the new and the ideal differentiators. It is seen that the new differentiator approximates the magnitude of the ideal differentiator up to 0.4 of full band.

Fig. 2 shows the phases of the new and the ideal half-sample delay differentiators. The phase of the ideal differentiator without delay is 90° from zero to the Nyquist frequency, where it jumps to −90°. Adding a delay of half a sample to the differentiator shifts the discontinuity from the Nyquist frequency to zero frequency, where the magnitude is zero, thus alleviating the approximation difficulties [7]. From Fig. 2 it is seen that the phase of the new differentiator approximates the phase of the ideal half-sample delay differentiator for low frequencies.

Fig. 3 shows the percent relative error in the magnitude response of the new differentiator. At 0.4 of full band the new differentiator has a relative error of 1.7%, which corresponds to −35 dB. More significant is the fact that the relative error of the new differentiator

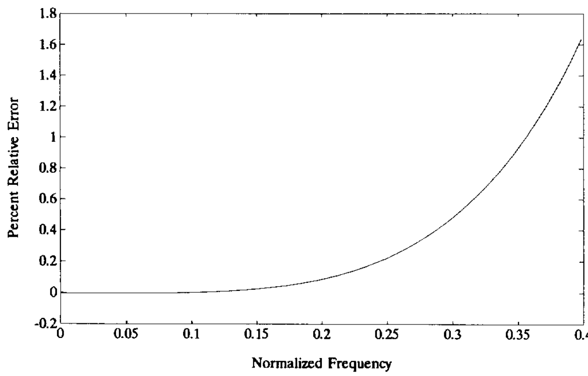

Fig. 3. Percent relative error of amplitude response of new differentiator.

over the frequency range up to 0.1 of full band is −87 dB. The proposed differentiator compares favorably with the state-of-the-art differentiators of Kumar and Dutta-Roy [7].

IV. Conclusion

A novel second-order recursive digital differentiator is presented. The low order of the differentiator makes it suitable for real-time applications. It approximates an ideal differentiator in the passband region with an accuracy comparable to that obtained by higher order filters. The proposed differentiator has an almost linear phase at low frequencies.

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Generalized Cellular Neural Network for Novelty Detection

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***Abstract*— A cellular neural network (CNN) for novelty detection is proposed. Each cell is connected to its neighboring inputs via an adaptive control operator, and interacts with neighboring cells via nonlinear feedback. In the learning mode, the control operator is modified in correspondence of a given set of patterns applied at the input. In the application mode, the CNN behaves like a memoryless system, which evidences those components of the input pattern that cannot be explained as a linear combination of the learned patterns.**

I. Introduction

The novelty filter is one of the more interesting models of adaptive systems that are able to mimic neurobiological functions [1], [2]. The characteristic property of a novelty filter can be summarized as follows. Consider a set of real-valued vectors $\{\mathbf{u}_k \in \Re^n, k = 1, \ldots, m\}$ that span a subspace $\mathcal{L} \subset \Re^n$. The complement space of $\mathcal{L}$ is denoted $\mathcal{L}^{\perp}$, and is defined as the set of vectors orthogonal to $\mathcal{L}$. An arbitrary vector $\mathbf{u}$ can be uniquely decomposed into the sum of a vector $\hat{\mathbf{u}} \in \mathcal{L}$, and a vector $\tilde{\mathbf{u}} \in \mathcal{L}^{\perp}$ [1]. The component $\hat{\mathbf{u}}$ represents the orthogonal projection of $\mathbf{u}$ on $\mathcal{L}$. The component $\tilde{\mathbf{u}}$ represents the orthogonal projection of $\mathbf{u}$ on $\mathcal{L}^{\perp}$. The component $\tilde{\mathbf{u}}$, which cannot be expressed as a linear combination of $\mathbf{u}_1 \ldots \mathbf{u}_m$, represents the *novelty* of $\mathbf{u}$ with respect to the *old* vectors. A novelty filter stores in its memory the set of vectors $\mathbf{u}_1 \ldots \mathbf{u}_m$ during a learning phase. Then, if a new input vector $\mathbf{u}$ is presented, the filter output represents the novelty component $\tilde{\mathbf{u}}$, orthogonal to the subspace $\mathcal{L}$[1]. Novelty detection can be useful in industrial and medical applications, when it is of interest to discover anomalies or imperfections.

The most known novelty filter model was deeply investigated by Kohonen and Oja in the 1970's [1]. It consists of a full-connection feedback system. If the input pattern is organized as an $N \times N$ array, the filter requires N^4 interconnections, making the practical realization very difficult when processing real images. In this paper, a cellular neural network (CNN) for novelty detection is proposed. CNN's are characterized by an interconnecting structure that is, at the same time, sparse and regular, allowing the physical realization of large-scale networks [3]–[4]. It is worth noting that the proposed neural network is a generalized CNN [5]. Generalization regards the following aspects: (1) Linear input-output characteristic of the cell; (2) adaptive control operator, which can vary from cell to cell; and (3) two distinct modes of operation.

In the following section, the proposed CNN is defined regarding both the topology and the structure of a single cell. In Section 3 it is shown that the proposed system model, in the full-connection case, exhibits an equivalent long-term behavior as that of the classical Kohonen's model. Finally, Section 4 presents some simulation results that point out the satisfactory behavior of the proposed CNN novelty detector.

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