TABLE II
COMPARISON OF REGULARIZATION PARAMETERS

SNR (dB)	α_{opt} min. $\|\hat{x} - x_{orig}\|^2$	$\alpha_{ST,0}$ $(\epsilon^2/\|Cx^*\|^2)$	α_{ST_orig} $(\epsilon^2/\|Cx_{orig}\|^2)$	α_{ST} $(\epsilon^2/\|Cx^*\|^2 + \delta)$	α_{CV} min. CV function
10	15.9	15.8	7.03	3.44	0.571
20	1.35	1.09	0.703	0.310	0.0128
30	0.120	0.0804	0.0703	0.0280	0.00112

$\Delta_{SNR}^{k=78}$ = 6.01 dB), 4(c) (SNR = 20 dB, $\Delta_{SNR}^{k=73}$ = 7.72 dB) and 4(d) (SNR = 30 dB, $\Delta_{SNR}^{k=84}$ = 10.14 dB). In all cases the restored images are very satisfactory, based on the improvement in SNR and visual inspection. We finally compare the values of α obtained by the proposed approach at convergence (denoted by $\alpha_{ST} = (\epsilon^2/\|Cx^*\|^2 + \delta)$, where x^* is the restored image at convergence) with the values obtained by the cross-validation method (α_{CV}) and from the minimization of $\|\hat{x}(\alpha) - x_{orig}\|^2$, denoted by α_{opt} (x_{orig} represents the original image which is available in a simulation experiment). We further compare these values of $\alpha_{ST\_orig} = \epsilon^2/(\|Cx_{orig}\|^2)$ and $\alpha_{ST,0} = (\epsilon^2/\|Cx^*\|^2)$, that is the value of α obtained by using the restored image obtained by iteration (6) but setting $\delta = 0$. All these values are shown in Table II for three different SNR's. From this table it is clear that $\alpha_{opt} \approx \alpha_{ST,0} > \alpha_{ST\_orig} > \alpha_{ST} > \alpha_{CV}$. This implies that the restored images obtained with the use of these values of the regularization parameter will be increasingly smoother as α increases from α_{CV} to α_{opt}. We also observe that $\alpha_{ST,0} \approx \alpha_{opt}$, making the restored image obtained with the use of $\alpha_{ST,0}$ optimal in the sense that $\|\hat{x} - x_{orig}\|^2$ is minimized. At the same time, as discussed in [6] and the references therein, the restored image with the use of α_{opt} is oversmooth, making the image obtained with the use of α_{ST} subjectively more preferable.

V. CONCLUSION

In this correspondence we have proposed a regularized iterative image restoration algorithm according to which a restored image and an estimate of the regularization parameter are provided simultaneously at each iteration step. Sufficient conditions for the convergence of the algorithm have been derived. The algorithm assumes the same amount of prior knowledge as the CLS algorithm, namely, knowledge of the value of the variance of the noise, but requires considerably fewer computations. Linear constraints can also be incorporated into the iteration. When nonlinear constraints are used the linearization step in the analysis of the algorithm can not be applied, although the algorithm has been shown to converge experimentally. Although the analysis is carried out for the case that D and C represent spatially invariant filters and the autocorrelation matrix of the image is block circulant, iteration (6) can be run for the more general case when D and C represent spatially varying filters and the image is nonstationary. The convergence analysis holds true in the general case as well, although it becomes more computationally expensive to verify the sufficient conditions for convergence at each iteration step. In other words, no explicit bounds for β and δ can be reached in this case.

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On the Design of FIR Digital Differentiators which Are Maximally Linear at the Frequency π/p, $p \in$ {Positive Integers}

Balbir Kumar, S. C. Dutta Roy, and Hitendra Shah

***Abstract*—In a number of signal processing applications, a digital differentiator (DD) performing over a narrow band of frequencies is required. The minimax relative error DD's are especially suitable for broad-band frequencies and become inefficient when adapted for narrow-band signals. The maximally linear DD's are, therefore, preferred for the latter situations. This correspondence proposes digital differentiators which are maximally linear at the spot frequency: $\omega = \pi/p$, $p \in$ {positive integers}. The suggested DD's, besides giving zero phase error over the entire band of frequencies ($-\pi \leq \omega \leq \pi$), can achieve very high accuracy in the magnitude response, over a given frequency**

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range, with attractively low order of the structure. For example, for $p = 3$, magnitude accuracy better than 99.999% can be achieved over the passband $0.26\pi \le \omega \le 0.41\pi$ with an order 21 of the structure. Mathematical formulas for the weighting coefficients required in the design have also been given.

I. Introduction

The digital differentiator (DD) constitutes an important unit in many information processing systems. The frequency response of an ideal DD is

$$\tilde{H}(\omega) = j\omega \triangleq j\tilde{H}(\omega), \qquad -\pi \le \omega \le \pi \tag{1}$$

where $\tilde{H}(\omega) = \omega$ and is purely real. Various FIR approximations of $\tilde{H}(\omega)$ have been reported in the literature based on the minimax relative error (MRE) criterion [1] as well as the maximally linear (ML) criterion [2]–[4]. The MRE designs are highly suitable for wide-band DD's while the ML designs are particularly adaptable for narrow-band operations centered around $\omega = 0$ [2], $\pi/2$ [3], or π [4]. Applications of MRE designs to narrow-band situations would be computationally inefficient and uneconomical as compared to ML designs.

In this correspondence, we extend the ML designs for narrow-band operation around $\omega = \pi/p$, where p is a positive integer. These designs would be useful in a number of applications. For radar systems using Doppler tracking [5], [6], for example, we use "speed gates" (also called "Doppler tracking filters") which perform differentiation over a narrow band of frequencies centered at different positions of the frequency spectrum. Also, in airborne Doppler navigation systems [5], [6], it is necessary to perform differentiation around the frequency ranges $\pi/12$ to $\pi/6$, typically, with extremely high accuracy (relative error[1] ≤ -140 dB, typically). Similar requirements are also encountered in underwater navigation [7], beam-forming [8], and a host of communication problems [9], [10].

II. The Design

Consider the following maximally linear, FIR approximations of $\tilde{H}(\omega)$, of order N, suggested by us earlier:

A) Digital differentiators, maximally linear at $\omega = \pi$, are approximated by [4]

$$H_1(\omega) = \pi \sum_{i=1}^{m} a_i \sin (i - 1/2)\omega + \sum_{i=1}^{m} b_i \sin i\omega, \qquad n = (N-1)/2,\ m = n/2 \tag{2}$$

where n gives the total number of weights (a_i's and b_i's). The weighting coefficients are given by the recursive relations[2]

$$a_k = \left[\binom{2k-3}{k-1}\Big/2^{(4k-5+\delta_{k,1})}\right] + \sum_{q=k}^{m-1} (-1)^{q+k} \binom{q+k-1}{q-k+1} a_{q+1} \tag{3a}$$

and

$$b_k = \frac{-1}{\left[k\binom{2k-1}{k-1}\right]} + \sum_{r=1}^{m-k} (-1)^{r-1} \binom{2k+r-1}{2k-1} b_{k+r} \tag{3b}$$

with

$$k = m, m-1, m-2, \cdots, 2, 1 \qquad \text{(descending order)}$$

and

$$\delta_{m,n} \triangleq \begin{cases} 1, & m = n \\ 0, & \text{otherwise.} \end{cases} \tag{3c}$$

By simple algebraic manipulation, we can write (2) as

$$H_1(\omega) = \pi \sum_{\substack{i=1\\ i\,\mathrm{odd}}}^{2m-1} a_{(i+1)/2} \sin \frac{i\omega}{2} + \sum_{\substack{i=2\\ i\,\mathrm{even}}}^{2m} b_{i/2} \sin \frac{i\omega}{2}, \qquad m = \frac{n}{2}. \tag{4}$$

B) Digital differentiators, maximally linear at $\omega = \pi/2$, are approximated by [3]

$$H_2(\omega) = \left(\frac{\pi}{2}\right) \sum_{\substack{i=1\\ i\,\mathrm{odd}}}^{n-1} d_i \sin i\omega - \left(\frac{1}{2}\right) \sum_{\substack{i=2\\ i\,\mathrm{even}}}^{n} d_i \sin i\omega, \qquad n \text{ even} \tag{5}$$

where $n = (N-1)/2$ is the total number of weights and N is the order of the approximation. The weights d_i's are given by [3]

$$d_i = \left[\binom{i-2}{(i-1)/2}\Big/2^{2i-3+\delta_{i,1}}\right] + \sum_{k=(i+1)/2}^{(n-2)/2} (-1)^{(2k+i+1)/2} \cdot \binom{(2k+i-1)/2}{(2k-i+1)/2} d_{2k+1}$$

$$i = n-1, n-3, n-5, \cdots, 5, 3, 1 \qquad \text{(descending order)}, \quad i \text{ odd} \tag{6a}$$

and

$$d_i = \frac{2}{\left[i\binom{i-1}{i/2}\right]} + \sum_{r=1}^{(n-i)/2} (-1)^{r+1} \binom{i+r-1}{r} d_{i+2r}$$

$$i = n, n-2, n-4, \cdots, 6, 4, 2 \qquad \text{(descending order)}, \quad i \text{ even.} \tag{6b}$$

If we let $k = (i+1)/2$ and $m = n/2$ in (3a); $k = i/2$ and $m = n/2$ in (3b), then after considerable algebraic manipulations and by using some combinatorial identities, it can be shown that

$$a_{(i+1)/2} = d_i, \qquad i \text{ odd} \tag{7a}$$

$$b_{i/2} = -d_i, \qquad i \text{ even} \quad (i \ne 0). \tag{7b}$$

Using the results of (7) in (4), we obtain

$$H_1(\omega) = \pi \sum_{\substack{i=1\\ i\,\mathrm{odd}}}^{n-1} d_i \sin \frac{i\omega}{2} - \sum_{\substack{i=2\\ i\,\mathrm{even}}}^{n} d_i \sin \frac{i\omega}{2}, \qquad n \text{ even.} \tag{8}$$

It is readily seen that (5) and (8) can be combined into the following composite equation:

$$H_p(\omega) \triangleq \frac{1}{p}\left[\pi \sum_{\substack{i=1\\ i\,\mathrm{odd}}}^{n-1} d_i \sin \frac{ip\omega}{2} - \sum_{\substack{i=2\\ i\,\mathrm{even}}}^{n} d_i \sin \frac{ip\omega}{2}\right], \qquad p = 1, 2, \quad n \text{ even} \tag{9}$$

where $p = 1$ and 2 correspond to maximal linearity at π and $\pi/2$, respectively.

We now show that (9) also represents an approximation of $\tilde{H}(\omega)$ such that $H_p(\omega)$ is maximally linear at the spot frequencies ω

[1]Relative error (RE) is defined by [1]

$$\mathrm{RE} = \left|\frac{|H_p(\omega)| - |\omega|}{\omega}\right|, \qquad -\pi \le \omega \le \pi.$$

[2]Here, we have designated the weights a_i and b_i instead of c_i and d_i used in [4] to avoid confusion with "d_i" also used in [3].

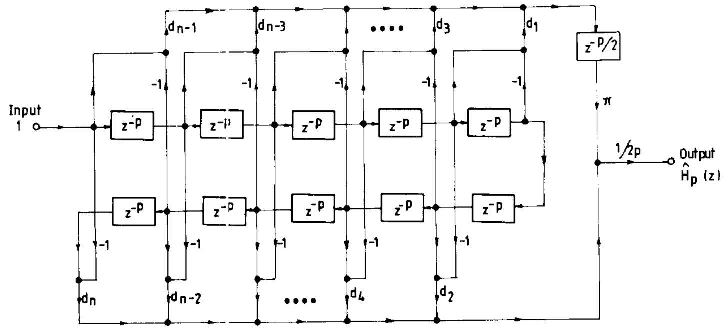

Fig. 1. Realization of transfer function $\hat{H}_p(z) = jH_p(\omega)|_{e^{j\omega}=z}$.

TABLE I
THE WEIGHTING COEFFICIENTS d_i'S FOR SELECTED VALUES OF n = 2 TO 12 COMPUTED BY USING FORMULAS (6a) AND (6b)

n	NF1	d_1	d_3	d_5	d_7	d_9	d_{11}	NF2	d_2	d_4	d_6	d_8	d_{10}	d_{12}
2	1	1						1	2/2					
4	2^3	9	1					3	8/2	2/4				
6	2^7	150	25	3				10	30/2	12/4	2/6			
8	2^{11}	2450	490	98	10			35	112/2	56/4	16/6	2/8		
10	2^{15}	39690	8820	2268	405	35		126	420/2	240/4	90/6	20/8	2/10	
12	2^{19}	640332	152460	45738	10890	1694	126	462	1584/2	990/4	440/6	132/8	24/10	2/12

Note 1
NF1 is normalizing factor for d_i, i odd
$\text{NF1} \triangleq 2^{[2n-5+\delta_{n,2}]}$

Note 2
NF2 is normalizing factor for d_i, i even
$\text{NF2} \triangleq \binom{n-1}{n/2-1}$

$= \pi/p$, where *p is any positive integer* (and not necessarily 1 or 2 only). For $H_p(\omega)$ to be maximally linear at $\omega = \pi/p$, we must have

$$H_p(\omega)|_{\omega=\pi/p} = \frac{\pi}{p} \tag{10a}$$

$$\left.\frac{dH_p(\omega)}{d\omega}\right|_{\omega=\pi/p} = 1 \tag{10b}$$

$$\vdots$$

$$\left.\frac{d^u H_p(\omega)}{d\omega^u}\right|_{\omega=\pi/p} = 0, \qquad u = 2, 3, \cdots, n-1. \tag{10c}$$

Forcing conditions (10) on (9) and simplifying, we obtain the following two sets of linear equations[3]:

$$\left.\begin{aligned}
&d_1 - d_3 + d_5 - d_7 + - \cdots d_{n-1} &&= 1\\
&d_1 - 3^2 d_3 + 5^2 d_5 - 7^2 d_7 + - \cdots (n-1)^2 d_{n-1} &&= 0\\
&d_1 - 3^4 d_3 + 5^4 d_5 - 7^4 d_7 + - \cdots (n-1)^4 d_{n-1} &&= 0\\
&\vdots &&\vdots\\
&d_1 - 3^{n-2} d_3 + 5^{n-2} d_5 - 7^{n-2} d_7 + - \cdots (n-1)^{n-2} d_{n-1} &&= 0
\end{aligned}\right] \tag{11}$$

and

$$\left.\begin{aligned}
&d_2 - 2\, d_4 + 3\, d_6 - 4\, d_8 + - \cdots (n/2)\, d_n &&= 1\\
&d_2 - 2^3 d_4 + 3^3 d_6 - 4^3 d_8 + - \cdots (n/2)^3 d_n &&= 0\\
&d_2 - 2^5 d_4 + 3^5 d_6 - 4^5 d_8 + - \cdots (n/2)^5 d_n &&= 0\\
&\vdots &&\vdots\\
&d_2 - 2^{n-1} d_4 + 3^{n-1} d_6 - 4^{n-1} d_8 + - \cdots (n/2)^{n-1} d_n &&= 0
\end{aligned}\right]. \tag{12}$$

[3]Note that the first line of (12) has +1 on the right-hand side instead of −1 appearing in [4, eq. (7b)]. This is due to our substitution: $b_{i/2} = d_i$, i even, according to (7b) of this correspondence.

Equations (11) and (12) above are precisely the same as (6) and (7), respectively, of [3] and are, therefore, satisfied. This proves that the approximation $H_p(\omega)$ is maximally linear at spot frequencies $\omega = \pi/p$, for $p = 1, 2, 3, \cdots$.

Fig. 1 gives a possible realization for the transfer function (TF), $\hat{H}_p(z) \triangleq jH_p(\omega)|_{e^{j\omega}=z}$. Note that for $p = 2, 4, 6, \cdots$ (an even value), the realization $\hat{H}_p(z)$ would involve integral delays and for $p = 1, 3, 5, \cdots$ (an odd value), a half delay ($z^{-1/2}$) is necessary for the structure. For $p = 1$, for example, a canonic realization of $\hat{H}_1(z)$ would be the same as [4, fig. 1].

III. PERFORMANCE

The values of the weights d_i, used in (9), are given in [3] and reproduced here, in Table I, for completeness. The transfer function $\hat{H}_p(z)$ gives zero phase error over the entire frequency range of operation on the unit circle ($z = e^{j\omega}$).

Figs. 2 and 3 show the frequency response, $H_p(\omega)$, for $p = 3$ and 4, respectively, for selected orders N ($= 2n + 1$) of the structure $\hat{H}_p(z)$. The frequency response is periodic in ω with period $4\pi/p$ rad. Since we are interested in its passband performance around $\omega = \pi/p$, the unwanted portion of the response is, of course, to be blanked by appropriate filters following the differentiators. As expected, extremely low relative errors are indeed available in the narrow bands of frequencies centered around $\omega = \pi/p$. This may be clearly seen from the "zoomed" part of frequency response curves, shown in insets in Figs. 2 and 3. Table II

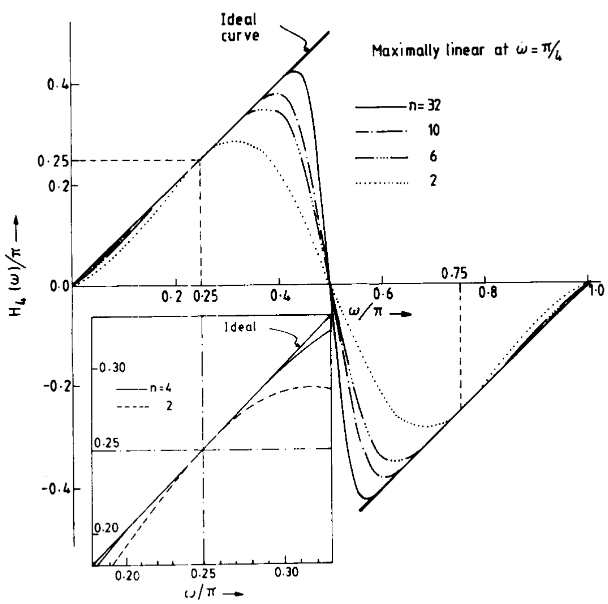

Fig. 2. The frequency response, $H_4(\omega)$, of the proposed digital differentiators, maximally linear at $\omega = \pi/4$ for $n = 2, 6, 10$, and 32. The inset shows the "zoomed" view of the frequency response around $\omega = \pi/4$.

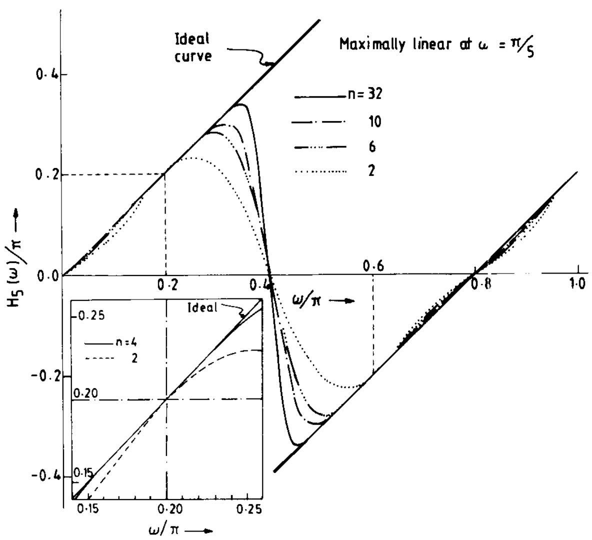

Fig. 3. The frequency response, $H_5(\omega)$, of the proposed digital differentiators, maximally linear at $\omega = \pi/5$, for $n = 2, 6, 10$, and 32. The inset shows the "zoomed" view of the frequency response around $\omega = \pi/5$.

gives a comparison of the number of multiplications, n_1, of the proposed design (ML-DD) with the number of multiplications, n_2, required in a minimax relative error broad-band design (MRE-DD) [1], for RE's less than or equal to -100 dB and -160 dB and selected bandwidths of operation. As an example, for $p = 4$, an ML-DD (i.e., $H_4(\omega)$) meant to be maximally linear at $\omega = \pi/4$, for performing over the frequency range $(0.25 \pm 0.1)\ \pi$ and RE ≤ -160 dB, requires 30 multiplications as compared to 34 required in the case of MRE-DD, per input sample of the signal.

The realization of the TF $\hat{H}_p(z)$, with $p = 2, 4, 6, \cdots$, requires integral delays. The structure, shown in Fig. 1, has an interesting feature that the TF $\hat{H}_p(z)$, for all even values of p ($p \neq 0$), can be easily achieved from the same structure *without changing any of the coefficients*, d_i's, by simply choosing the delay blocks (z^{-p}'s and $z^{-p/2}$) commensurate with the value of the p chosen. In a digital processor, it is rather easy to obtain integral delays (by step down counters, for example) at no extra cost of the softwares. The multiplier $1/(2p)$, shown just before the OUTPUT node, is only a "scaling factor."

The realization of the TF $\hat{H}_p(z)$, $p = 1, 3, 5, \cdots$, requires a half delay ($z^{-1/2}$). The proposed design would, therefore, be particularly suitable in a multirate system where, at times, the intermediate data values may be available without extra cost, for example, in decimation of sampling rate by a factor of two [4].

To allow compensation for clutter-Doppler shift and also for avoiding blind speeds (i.e., target speeds which preclude detection), in the moving target indication (MTI) radars, various systems, including two-frequency MTI, are in practical use [5]. The two-frequency MTI uses two carriers, f_{o1} and f_{o2} to obtain, respectively, the Doppler shifts, say, f_{d1} and f_{d2} from a moving target. As

$$\frac{f_{d_1}}{f_{d_2}} \propto \frac{f_{o_1}}{f_{o_2}} \triangleq \frac{l}{m} \quad \text{(say)} \tag{13}$$

where l and m are integers, the detection of f_{d1} and f_{d2} necessitates equal sensitivities of the two-frequency MTI for reception in uncorrelated clutter-Doppler frequency spread conditions [5]. It can be shown that the use of the proposed differentiators, maximally linear at $\omega = \pi/l$ and $\omega = \pi/m$, for detection of f_{d1} and f_{d2}, respectively, would make the MTI receiver sensitivities exactly equal.

Besides the aforementioned possible applications of the suggested design, the mathematical formulas, given by (6a) and (6b), for computation of the exact values of the weights d_i's, required in the design, is an attractive feature as compared to the optimization algorithms required in the minimax design [1].

TABLE II
THE RATIO n_1/n_2 SHOWING COMPARISON OF THE MULTIPLICATIONS REQUIRED PER INPUT SAMPLE OF THE SIGNAL FOR THE DIGITAL DIFFERENTIATORS PERFORMING AROUND THE FREQUENCIES: $\omega = \pi/p$ FOR p = 3, 4, 5, AND 6 AND RELATIVE ERRORS (RE'S) LESS THAN OR EQUAL TO −100 dB AND −160 dB, WHERE n_1 = MULTIPLICATIONS FOR THE ML DESIGN (PROPOSED), n_2 = MULTIPLICATIONS FOR THE MRE DESIGN (BROAD-BAND DESIGN) [1]

	Ratio n_1/n_2 for							
	$p = 3$		$p = 4$		$p = 5$		$p = 6$	
(Frequency Range)/π	RE ≤ −100 dB	RE ≤ −160 dB	RE ≤ −100 dB	RE ≤ −160 dB	RE ≤ −100 dB	RE ≤ −160 dB	RE ≤ −100 dB	RE ≤ −160 dB
$\frac{1}{p} + 0.025$	4/7	8/12	6/8	10/14	8/16	10/18	8/20	12/28
$\frac{1}{p} + 0.050$	8/9	12/15	8/12	14/18	12/20	18/23	14/28	22/31
$\frac{1}{p} + 0.075$	10/14	16/19	14/17	20/24	16/28	26/36	20/38	36/50
$\frac{1}{p} + 0.100$	14/28	20/24	26/30	30/34	30/34	*	*	*

*Data not available.

IV. CONCLUSIONS

Efficient FIR digital differentiators, suitable for maximally linear operation around spot frequencies, have been proposed. Mathematical formulas for calculation of the exact values of the weighting coefficients needed in the design have been derived. Possible applications of the suggested structures have also been given.

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On the Cascade Realization of 2-D FIR Filters Designed by McClellan Transformation

Brian K. Lien

Abstract—**The multidimensional finite impulse response (FIR) filters designed by McClellan transform can be implemented efficiently by the direct transformed structure, transpose direct transformed structure, cascade structure, Chebyshev structure, and reversed Chebyshev structure. In this correspondence, peak scaling and section reconfiguration are provided to modify the cascade structure realization. The modifications result in a reduction in the output roundoff noise power and the number of operations required.**

I. INTRODUCTION

The McClellan transform is by far the most popular and successful method to design multidimensional FIR filters by transforming 1-D FIR filters [1]-[3]. The filters designed by McClellan transform can be implemented efficiently by the direct transformed structure, transpose direct transformed structure, cascade structure [4], Chebyshev [5], and reversed Chebyshev [6] structure with the number of operations required proportional to N. The direct transformed structure is not suitable for fixed-point implementation because of its very large output roundoff noise. The Chebyshev and reversed Chebyshev structure have the best roundoff noise performance but they have the drawback of more storage required and more complicated architecture [7]. Though the roundoff noise of cascade structure is larger than that of Chebyshev structure it has the advantage of inherent pipelineability and low sensitivity.

The output roundoff noise of filters in cascade form depends mainly on scaling, ordering, and section configuration [8], [9]. The peak scaling was applied to the cascade structure in [5], but it was not applied to the internal node of each section. So by driving the

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