A New Application of High Order Learning Scheme

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Abstract

High order learning schemes can be used to improve the convergence property and the transient learning behavior along the learning iteration number direction. As a new application, high order learning schemes are applied to handle reference trajectories which may change fast along the iteration direction. Some sufficient conditions are derived for the convergence of the learning system.

Keywords: Iterative Learning Control, High Order, Varying Reference Trajectories, Convergence

1 Introduction

Learning is to grasp knowledge about an object through some processes. An effective learning method is to keep studying the object repeatedly, where some useful information is passed during successive operations. This type of learning is said to be iterative learning. It can be used to obtain a better control of repetitive systems [1], which leads to iterative learning control.

Generally, four aspects of learning learning control should be considered. The first one is to find more learning method. The second one is to ensure learning speed as fast as possible. The third one is to make class of learned plants as wide as possible. The last one is to ensure studied knowledge, which can be characterized by reference trajectories, as more as possible.

The first three aspects of iterative learning control have been studied by many researchers. [1] proposed a first-order iterative control law and this law has been used widely. [2] studied the robustness of an iterative learning controller with a same non-zero initial condition at each iteration. [3] proposed an initial state learning law to automatically initialize the system at each iteration. [4] used iterative learning control to improve the dynamic accuracy of the system. [5] considered the iterative learning control of deterministic systems. Most of the existing iterative learning control scheme mentioned above are based on the first order updating laws, i.e. only the information of the immediate previous iteration is employed. In [6], a high order iterative learning control algorithm is used to control time-varying linear systems where information of several previous learning iterations is used to construct the control in the current iteration. It was demonstrated in [6] that high order schemes did have potential to give a better convergence performance than the first order schemes. In fact, high order schemes can also be used to improve the transient behaviour along the learning iteration number direction [7]. The class of plants is continuous. [8] proposed an effective method to study repetitive linear piecewise constant systems. However, the reference trajectory, i.e. the studied knowledge, is unchanged for all iterative operations, In other words, the studied knowledge is very narrow.

It is important to extend the studied knowledge. Because in practice, the necessarily studied knowledge can be very wide, such as, the reference trajectories may vary from one iteration to another. A typical example is that a robot performing pick and place task can have different ending points to place objects and this will result in different reference trajectories at different iterations. There are some other practical examples given in [9] where reference trajectories may change. Thus, some learning algorithms were proposed to handle slowly varying trajectories in [9]. Note that the trajectories may change quickly, that is, the studied knowledge can be wider. So it is necessary and important to consider the tracking of possibly fast varying trajectories. Furthermore, in the existing results, system disturbances and/or uncertainties or their differences between two successive iterations are assumed to be bounded by some constants when the robustness of iterative learning control is studied. The bound of the final tracking error depends on these constants. To ensure that the tracking error tends to zero, these constants are required to be zero. These requirements are still quite demanding. It is therefore important to relax these conditions.

The main goal of this paper is to extend the studied knowledge. We consider possibly fast changing reference trajectories along the iteration axis by using high order iterative learning control scheme. We formulate our problem as a convex optimization problems in terms of linear matrix inequalities and this makes the problem computationally tractable by some existing tools [10]. The iterative control laws can be obtained by solving the optimization problems. We also relax certain restrictive requirements on system disturbances and/or uncertainties. To ensure that the final tracking error tends to zero, we only require the norms of the system disturbances and/or uncertainties to be non-increasing along the iteration numbers. Moreover, by proposing some new high order D-type iterative learning laws, we can handle a wider class of time varying trajectories. Some sufficient conditions are derived for the convergence of the learning system. Although these approaches are only proposed for linear systems described by (1), it can be used to study nonlinear iterative learning control in a similar way.

The rest of the paper is organized as follows. Section 2 considers the formulation of the problems. Section 3 derives the main results of this paper. The concluding remarks are given in section 4.

2 Problem Formulation

In this paper, we consider the class of repetitive linear systems modeled as

$$\begin{cases} \dot{X}_i(t) = AX_i(t) + BU_i(t) + w_i(t) \\ Y_i(t) = CX_i(t) + v_i(t) \end{cases} \tag{1}$$

where i denote the ith repetitive operation of the system; $X_i \in R^n$, $U_i \in R^r$, $Y_i \in R^p$ are the state vector, the input vector and the output vector, respectively; w_i and v_i are uncertainties or disturbances to the system; $t \in [t_0, T] \subseteq [0, T]$ is the time with t_0 and T given; A, B and C are system matrices with appropriate dimensions.

We let $Y_{d,i}(t)$, $X_{d,i}(t_0)$, $X_{d,i}(t)$ and $U_{d,i}(t)$ denote respectively the desired output, the desired initial state, the desired state and the corresponding input to achieve $Y_{d,i}(t)$ and $X_{d,i}(t)$ at the ith iteration. The norms used in this paper are defined as follows:

$$\begin{aligned} \|b\| &= \sqrt{\sum_{i=1}^{n} b_i^2} \; ; \; b \in R^n \\ \|A\| &= \sqrt{\lambda_{max}(A^T A)} \; ; \; A \in R^{n \times n} \\ \|h(t)\|_\lambda &= \sup_{t \in [0,T]} e^{-\lambda t} \|h(t)\| \; ; \; h : [0, T] \to R \end{aligned}$$

Given a system described by (1), and a varying desired output trajectory $Y_{d,i}(t)$ in some given compact set, the tracking error $e_i(t)$ at the ith repetition is given by $e_i(t) = Y_{d,i}(t) - Y_i(t)$. Then the problem is formulated as follows. Starting from an arbitrary continuous initial input $U_0(t)$ and initial state $X_0(t_0)$, obtain the subsequence $\{U_i(t), X_i(t_0) | i = 1, 2, 3, \cdots\}$ for system (1) iteratively such that when $i \to \infty$, $Y_i(t) - Y_{d,i}(t) \to 0$ [9].

To solve the above problem, we make the following assumption about the learning control system (1).

A1). CB is full rank.

A2). The desired trajectory $Y_{d,i}(t)$ is differentiable with respect to t $\forall i$ and for all $t \in [t_0, T]$.

A3). In each iteration the desired trajectory differs from the previous ones by

$$Y_{d,i+1}(t) = \sum_{j=1}^{N} a_{i,j} Y_{d,i-j+1}(t) + \eta_i(t) \tag{2}$$

where N is an integer, $a_{i,j} \in [\hat{a}_1, \hat{a}_2]$, and

$$\begin{aligned} \|\eta_{i+1}(t)\| &\leq r_0 \|\eta_i(t)\| && (3) \\ 0 &\leq r_0 < 1 && (4) \end{aligned}$$

A4). The uncertainty and disturbance terms $w_i(t)$ and $v_i(t)$ are bounded as follows, $\forall t \in [t_0, T]$ and $\forall i$

$$\|w_{i+1}(t)\| \leq r_0 \|w_i(t)\| \; ; \; \|v_{i+1}(t)\| \leq r_0 \|v_i(t)\| \tag{5}$$

Remark 1. *Note that in [9] $\eta_i(t)$ is required to satisfy the following inequality*

$$\|\eta_i\| \leq \xi_1 \tag{6}$$

for a constant ξ_1. To achieve zero or small tracking error, ξ_1 must be zero or small. It can be noted from (3) that η_i is not necessary to satisfy (6). This implies that $\|\eta_i\|$ may be very large and this allows the reference trajectories can vary quickly.

Remark 2. *Note that $\|w_i(t)\|$, $\|v_i(t)\|$ and $\|\eta_i(t)\|$ are only required to be non-increasing along the iteration numbers. It will be shown that the final tracking error can still tend to zero under this condition.*

Remark 3. *For a practical system, equation (2) can be obtained by solving the following minimal least square problem.*

$$\min_{a_{i,j}} \| \int_{t_0}^{T} Y_{d,i+1}(t) dt - \sum_{j=1}^{N} a_{i,j} \int_{t_0}^{T} Y_{d,i-j+1}(t) dt \|^2 \tag{7}$$

Remark 4. *Equation (2) can also be regarded as a class of rules. In other words, besides approaching a given curve, the iterative learning control can also be used to approach a class of rules.*

3 Main Results

To establish our main results, we now present some preliminaries.

Lemma 1. *Suppose that a real positive series* $\{e_i\}_1^\infty$ *satisfies*

$$e_n \le \rho_1 e_{n-1} + \rho_2 e_{n-2} + \cdots + \rho_N e_{n-N} + r_0^{n-N} M_0$$

where $\rho_i > 0 (i = 1, 2, \cdots, N)$, *and*

$$0 \le r_0 < 1 \tag{8}$$

$$\rho = \sum_{i=1}^{N} \rho_i < 1 \tag{9}$$

$$M_0 > 0 \tag{10}$$

then

$$\lim_{n\to\infty} e_n = 0 \tag{11}$$

holds.

Proof: The proof is straightforward. □

Lemma 2. *The following three conditions are equivalent.*

1. *There exists a* Γ *such that*
$$\|I - CB\Gamma\| < 1$$
2. *There exists an* $a < 1$ *and* Γ *such that*
$$\begin{bmatrix} aI & (I - CB\Gamma)^T \\ I - CB\Gamma & I \end{bmatrix} \ge 0 \tag{12}$$
3. CB *is full rank.*

Proof: The proof is similar to that of Lemma 1 and Lemma 2 in [8] □

Based on Lemma 2, the iterative control problem can be formulated as solving the following convex optimization problems.

$$\min\{a\} \tag{13}$$

subject to

$$\begin{bmatrix} aI & (I - CB\Gamma)^T \\ I - CB\Gamma & I \end{bmatrix} \ge 0 \tag{14}$$

It can be shown that the optimal solutions are as follows.

$$a^* = 0 \tag{15}$$

$$\Gamma^* = (CB)^+[CB(CB)^+]^{-1} \tag{16}$$

Thus, our learning control laws are given as follows.

$$U_{i+1}(t) = \sum_{j=1}^{N}[a_{i,j}(U_{i-j+1}(t) + \Gamma^* \dot{e}_{i-j+1}(t))] \tag{17}$$

$$X_{i+1}(t_0) = \sum_{j=1}^{N}[a_{i,j}(X_{i-j+1}(t_0) + B\Gamma^* e_{i-j+1}(t_0))] \tag{18}$$

where N and a_{ij} are defined in (2).

In the remaining part of this section, we consider the following two cases.

Case 1. $w_i(t) = 0$, $v_i(t) = 0$ and $\eta_i(t) = 0$ $(i = 0, 1, 2, \cdots)$. In other words, there is no uncertainty in system (1).

In this case, we have the following result.

Theorem 1. *Consider system (1) under assumptions A1)-A3). The iterative learning control laws (17) and (18) can ensure that* $Y_i(t) \to Y_{d,i}(t)$ *when* $i \to \infty$.

Proof: The proof is omitted due to space limitation. □

Case 2. $w_i(t) \ne 0$, $v_i(t) \ne 0$ and $\eta_i(t) \ne 0$ hold for $i = 0, 1, 2, \cdots$.

Our result in this case can be stated as follows

Theorem 2. *Consider system (1) under assumptions A1)-A4). The iterative learning control laws (17) and (18) can ensure that* $Y_i(t) \to Y_{d,i}(t)$ *when* $i \to \infty$.

Proof: From (1), we have

$$\begin{aligned}
& e_{i+1}(t) \\
= \; & Y_{d,i+1}(t) - Y_{i+1}(t) \\
= \; & Y_{d,i+1}(t) - v_{i+1}(t) - Ce^{A(t-t_0)}X_{i+1}(t_0) \\
& -C\int_{t_0}^{t} e^{A(t-\tau)}(BU_{i+1}(\tau) + w_{i+1}(\tau))d\tau \\
= \; & \sum_{j=1}^{N} a_{i,j} Y_{d,i-j+1}(t) - \sum_{j=1}^{N} a_{i,j} v_{i-j+1}(t) \\
& -\sum_{j=1}^{N} a_{i,j} Ce^{A(t-t_0)} X_{i-j+1}(t_0) \\
& -\sum_{j=1}^{N} a_{i,j} C\int_{t_0}^{t} e^{A(t-\tau)}(BU_{i-j+1}(\tau) + w_{i-j+1}(\tau))d\tau \\
& -\sum_{j=1}^{N} a_{i,j} C\int_{t_0}^{t} e^{A(t-\tau)} B\Gamma^* \dot{e}_{i-j+1}(\tau)d\tau
\end{aligned}$$

$$-C\int_{t_0}^{t} e^{A(t-\tau)}(w_{i+1}(\tau)-\sum_{j=1}^{N} a_{i,j}w_{i-j+1}(\tau))d\tau$$

$$-\sum_{j=1}^{N} a_{i,j}Ce^{A(t-t_0)}B\Gamma^* e_{i-j+1}(t_0)$$

$$+\eta_i(t)-(v_{i+1}(t)-\sum_{j=1}^{N} a_{i,j}v_{i-j+1}(t))$$

Note that

$$\begin{aligned} & -C\int_{t_0}^{t} e^{A(t-\tau)}B\Gamma^* \dot{e}_{i+1-j}(\tau)d\tau \\ = & -CB\Gamma^* e_{i+1-j}(t)+Ce^{A(t-t_0)}B\Gamma^* e_{i+1-j}(t_0) \\ & -C\int_{t_0}^{t} e^{A(t-\tau)}AB\Gamma^* e_{i+1-j}(\tau)d\tau \end{aligned}$$

It follows that

$$\begin{aligned} & ei+1(t) \\ = & \eta_i(t)-(v_{i+1}(t)-\sum_{j=1}^{N} a_{i,j}v_{i+1-j}(t)) \\ & -C\int_{t_0}^{t} e^{A(t-\tau)}(w_{i+1}(\tau)-\sum_{j=1}^{N} a_{i,j}w_{i+1-j}(\tau))d\tau \\ & -\sum_{j=1}^{N} a_{i,j}C\int_{t_0}^{t} e^{A(t-\tau)}AB\Gamma^* e_{i+1-j}(\tau)d\tau \end{aligned} \quad (19)$$

Taking the norm operation on both sides of (19), we have

$$\begin{aligned} & \|e_{i+1}(t)\| \\ \leq & \|v_{i+1}(t)\|+\|\sum_{j=1}^{N} a_{i,j}v_{i+1-j}(t)\|+\|\eta_i(t)\|+\|C\|\bullet \\ & (\sum_{j=1}^{N}|a_{i,j}|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|A\|\|B\|\|\Gamma^*\|\|e_{i+1-j}(\tau)\|d\tau + \\ & \int_{t_0}^{t} e^{\|A\|(t-\tau)}(\|w_{i+1}(\tau)\|+\sum_{j=1}^{N}\|a_{i,j}w_{i+1-j}(\tau)\|)d\tau) \\ \leq & \|v_{i+1}(t)\|+\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|v_{i+1-j}(t)\|+\|\eta_i(t)\| \\ & +\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|C\|\bullet \\ & \int_{t_0}^{t} e^{\|A\|(t-\tau)}\|A\|\|B\|\|\Gamma^*\|\|e_{i+1-j}(\tau)\|d\tau + \\ & \max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{i+1-j}(\tau)\|d\tau \\ & +\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{i+1}(\tau)\|d\tau \end{aligned} \quad (20)$$

Let

$$\begin{aligned} & \theta_0(i,t) \\ = & \|v_{i+1}(t)\|+\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|v_{i+1-j}(t)\|+\|\eta_i(t)\| \\ & +\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{i+1}(\tau)\|d\tau \\ & +\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{i+1-j}(\tau)\|d\tau \\ & M(t) \\ = & \|v_{N+1}(t)\|+\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|v_{N+1-j}(t)\|+\|\eta_N(t)\| \\ & +\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{N+1}(\tau)\|d\tau \\ & +\max\{|a_1|,|a_2|\}\sum_{j=1}^{N}\|C\|\int_{t_0}^{t} e^{\|A\|(t-\tau)}\|w_{N+1-j}(\tau)\|d\tau \end{aligned}$$

Then, we get

$$\begin{aligned} \theta_0(i,t) & \leq r_0\theta_0(i-1,t) \\ & \leq r_0^{i-N}M(t) \end{aligned} \quad (21)$$

Multiplying $e^{-\lambda t}$ on the both side of (20) and taking the λ-norm $\|\cdot\|_\lambda$, we have

$$\begin{aligned} & \|e_{i+1}(t)\|_\lambda \\ \leq & \sum_{j=1}^{N}\frac{|a_{i,j}|\|C\|\|A\|\|B\|\|\Gamma^*\|}{\lambda-\|A\|}\|e_{i+1-1}(t)\|_\lambda+\theta_0(i,T) \\ \leq & \sum_{j=1}^{N}\frac{|a_{i,j}|\|C\|\|A\|\|B\|\|\Gamma^*\|}{\lambda-\|A\|}\|e_{i+1-1}(t)\|_\lambda+r_0^{i-N}M(22) \end{aligned}$$

Clearly, there exists a λ^* such that

$$\hat{\rho} \triangleq \frac{\max\{|a_1|,|a_2|\}\|C\|\|A\|\|B\|\|\Gamma^*\|}{\lambda^*-\|A\|} < \frac{1}{N+1}$$

Using Lemma 1, we have

$$\lim_{i\to\infty}\|e_{i+1}(t)\|_\lambda = 0$$

From the definition of $\|\cdot\|_\lambda$, this implies

$$\lim_{i\to\infty}(Y_i(t)-Y_{d,i}(t)) = 0$$

□

Remark 5. *The results can be extended to the nonlinear case directly by using general methods for nonlinear iterative learning control.*

4 Conclusion

In this paper, we have proposed some new high order D-type iterative learning control laws to design iterative learning controllers for repetitive linear systems with fast varying reference trajectories. This extends the application of iterative learning control. We formulated our approach as a convex optimization problems in terms of linear matrix inequalities and this makes the problem computationally tractable by using some existing tools. Some sufficient conditions are also derived to ensure the convergence of the learning system.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki. Bettering operation of robots by learning. *Journal of Robotic Systems.*, 1:123–140, 1984.

[2] H. K. Lee and Z. Bien. Study on robustness of iterative learning control with non-zero initial error. *International Journal of Control*, 64:345–359, 1996.

[3] Y. Chen, C. Wen, Z. Gong, and M. Sun. An iterative learning controller with initial state learning. *IEEE Transactions on Automatic Control*, 44:371–376, 1998.

[4] R. H. Middleton, G. C. Goodwin, and R. W. Longman. A method for improving the dynamic accuracy of a robot performing a reprtitive task. *Int. J. of Robotics Research.*, 8:67–74, 1989.

[5] K. L. Moore. *Iterative Learning Control of Deterministic Systems.* Springer-Verlag, New York, 1993.

[6] Z. Bien and K. M. Huh. High-order iterative learning control algorithm. *IEE Proc. Pt.-D.*, 136:105–112, 1989.

[7] Y. Chen, Z. Gong, and C. Wen. Analysis of a high-order iterative learning control algorithm for uncertain nonlinear systems with state delay. *Automatica*, 34:345–353, 1998.

[8] Z. G. Li, C. Y. Wen, Y. C. Soh, and Y. Q. Chen. Iterative learning control of linear time varying uncertain systems. In *Proc. of the 2000 ASCC*, pages 1890–1894, 2000.

[9] S. S. Saab, W. G. Vogt, and M. H. Mickle. Learning control algorithms for tracking slowly varying trajectories. *IEEE Trans. Systems Man Cybernetics-Part B: Cybernetics*, 27:657–669, 1997.

[10] S. Boyd, L.EI Ghoui, E. Feron, and V. Balakrishaan. *Linear Matrix Inequalities in system and Control Theory.* SIAM, Boston, 1994.