Proceedings of the 3rd
Asian Control Conference
July 4-7,2000,Shanghai

Iterative Learning Control of Linear Time Varying Uncertain Systems

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Abstract

In this paper, we consider iterative learning control of repetitive linear time varying systems with polytope uncertainties. We formulate it as a group of convex optimization problems. With the proposed approach, an optimal iterative learning control law can be obtained from the solution of these convex optimization problems. Some sufficient conditions are derived for the convergence of the resulting learning system.

1 Introduction

In this paper, we consider the following repetitive linear time varying system

$$\begin{cases} \dot{X}_i(t) = A(t)X_i(t) + B(t)U_i(t) \\ Y_i(t) = C(t)X_i(t) \end{cases} \quad (1)$$

where i denotes the ith repetitive operation of the system; $X_i \in R^n, U_i \in R^r, Y_i \in R^p$ are the state vector; the input vector and the output vector, respectively; $t \in [t_0, T] \subseteq [0, T]$ is the time variable with t_0 and T given; A(t), B(t) and C(t), the system matrices with appropriate dimensions, are given as follows,

$$\begin{aligned} A(t) &= A(1) + \sum_{j=1}^{M_1} A(j+1)t^j \\ &= \sum_{i=1}^{N_1} a_i \hat{A}_i(1) + \sum_{i=1}^{N_1}\sum_{j=1}^{M_1} a_i \hat{A}_i(j+1)t^j \\ & \quad a_i \geq 0; \sum_{i=1}^{N_1} a_i = 1 \end{aligned} \quad (2)$$

$$\begin{aligned} B(t) &= B(1) + \sum_{j=1}^{M_2} B(j+1)t^j \\ &= \sum_{i=1}^{N_2} b_i \hat{B}_i(1) + \sum_{i=1}^{N_2}\sum_{j=1}^{M_2} b_i \hat{B}_i(j+1)t^j \\ & \quad b_i \geq 0 \sum_{i=1}^{N_2} b_i = 1 \end{aligned} \quad (3)$$

$$\begin{aligned} C(t) &= C(1) + \sum_{j=1}^{M_3} C(j+1)t^j \\ &= \sum_{i=1}^{N_3} c_i \hat{C}_i(1) + \sum_{i=1}^{N_3}\sum_{j=1}^{M_3} c_i \hat{C}_i(j+1)t^j \, ; \\ & \quad c_i \geq 0; \sum_{i=1}^{N_3} c_i = 1 \end{aligned} \quad (4)$$

where $A(j)(1 \leq j \leq M_1+1), B(j)(1 \leq j \leq M_2+1)$ and $C(j)(1 \leq j \leq M_3+1)$ are uncertain matrices, and $\hat{A}_i(j)(1 \leq i \leq N_1, 1 \leq j \leq M_1+1)$, $\hat{B}_i(j)(1 \leq i \leq N_2, 1 \leq j \leq M_2+1)$ and $\hat{C}_i(j)(1 \leq i \leq N_3, 1 \leq j \leq M_3+1)$ are known vertex matrices, a_i, b_i and c_i are unknown constants.

Iterative learning control of repetitive linear systems has been studied by many researchers. [1] proposed a first-order iterative control law and this law has been widely used. [2] studied the iterative learning control with the same non-zero initial condition at each iteration. [3] proposed an initial state learning law to automatically initialize the system at each iteration. [4] have considered the iterative learning control design of repetitive linear time invariant system with unknown system matrices A, B and C by using the estimated knowledge of A, B and C. However, the iterative learning control design of linear time varying system uncertain system (1)–(4) has not been considered yet. Generally, we only have some rough knowledge of .-l(t) , $B(t)$ and C(t). For example, we may know that $A(t)$,$B(t)$ and C(t) are in some intervals, but it is very difficult to estimate the actual matrices in practice. Moreover, it is impossible to use the method in [4] to obtain an optimal learning gain with a fast convergence speed. So far, there does not exist any effective approach for the design of the optimal iterative learning controller for repetitive linear time varying systems with parametric uncertainties.

In this paper, we present an effective approach for the design of iterative learning control for linear time varying uncertain system (1)–(4) by using some rough knowledge of $A(t)$, B(t) and C(t). We formulate our problem as a group of convex optimization problems and this makes the problem computationally tractable by some existing tools [5]. Some sufficient conditions are derived for the convergence of the learning system. Our approach extends the potential application of iterative learning control and enlarges the class of systems to include parametric uncertainties. For simplicity, we only consider the case that $M_1 = M_2 = M_3 = 1$. The result obtained in this paper can be easily generalized.

The rest of the paper is organized as follows. Section 2 considers the formulation of the problems. Section 3 derives the main results of this paper. A numerical example is given in Section 4 to illustrate the application of the main result. The concluding remarks are given in section 5.

2 Problem Formulation

In this paper, we let $Y_d(t), X_d(t_0), X_d(t)$ and $U_d(t)$ denote respectively the desired output, the desired initial state, the desired state and the corresponding input to achieve $Y_d(t)$ and X,(t). The norms are defined as follows:

$$\begin{aligned} \|b\| &= \sqrt{\sum_{i=1}^{n} b_i^2} \; ; \; b \in R^n \\ \|A\| &= \sqrt{\lambda_{max}(A^T A)} \; ; \; A \in R^{n\times n} \\ \|h(t)\|_\lambda &= \sup_{t\in[0,T]} e^{-\lambda t}\|h(t)\| \; ; \; h : [0,T] \to R \end{aligned}$$

and $\rho(A)$ is the spectral radius of matrix A and $\lambda_{max}(A)$ the maximum eigenvalue of matrix A.

Given a system described by (1)-(4), and a desired output trajectory $Y_d(t)$, the tracking error $e_i(t)$ at the ith repetition is given by $e_i(t) = Y_d(t) -$ Y(t). Then the iterative learning problem is formulated as follows. Starting from an arbitrary continuous initial input $U_0(t)$ and an arbitrary initial state $X_0(t_0)$, which may be different from $X_d(t_0)$, obtain the subsequence $\{U_i(t), X_i(t_0) | i = 1,2,3,\cdots\}$ for the system (1) iteratively such that when $i \to \infty$, Y(t) $\to Y_d(t)$ and $X_i(t_0) \to X_d(t_0)$. Furthermore, $Y_i(t) - Y_d(t)$ and Xi(t) $- X_d(t)$ are independent of the initialization error $X_i(t_0) - X_d(t_0)$ when $i \to \infty$.

To solve the above problem, the following D-type iterative learning laws can be used [1, 2].

$$U_{i+1}(t) = U_i(t) + \Gamma(t)\dot{e}_i(t) \tag{5}$$

where T(t) denotes the learning gain. The resulting system is convergent if [1, 2]

$$\sup_{t\in[t_0,T]} \|I - C(t)B(t)\Gamma(t)\| \le \epsilon < 1 \tag{6}$$

To ensure the existence of a T(t) satisfying (6), we need the following assumption:

Assumption 1 $\hat{C}_i(1)\hat{B}_j(1), \hat{C}_i(2)\hat{B}_j(2)$ and $\hat{C}_i(1)\hat{B}_j(2) + \hat{C}_i(2)\hat{B}_j(1)$ are full row rank for all $1 \le i \le N_3$ and all $1 \le j \le N_2$.

To handle the mismatch in the initial state, we further assume that

Assumption 2 Repeatability of the initial setting is satisfied within an admissible derivation level, i.e.,

$$\begin{aligned} \|X_d(t_0) - X_i(t_0)\| &\le b_{x0}(i) && (7) \\ b_{x0}(i) &\le r_0 b_{x0}(i-1) && (8) \end{aligned}$$

where $0 \le r_0 < 1$.

Our design objective is to find control gain I t) such that the following cost function is minimized.

$$J_1 = \sup_{t\in[t_0,T]} \|I - C(t)B(t)\Gamma(t)\| \tag{9}$$

Remark 1. *Cost* function J_1 *determines the convergence speed of the* learning process *under* law **(5).** *Cost function* J_1 *was considered by [6] with* different *method for the case* where B and C are *constant* matrices.

3 Main Results

We shall first derive some supporting results.

Lemma 1. *Suppose that* $\|A(t)\| \le \hat{a}$, *then*

$$\|\phi(t, t_0)\| \le e^{\hat{a}(t-t_0)}$$

where

$$\begin{aligned} \frac{d\phi(t,t_0)}{dt} &= A(t)\phi(t,t_0) \\ \phi(t_0,t_0) &= I \end{aligned}$$

Proof: The proof is straightforward. ❑

Lemma 2. *The following three conditions* are *equivalent.*

1. *There exists* a Γ *such that*

$$\|I - CB\Gamma\| \le a < 1$$

2. *There exist* an $a < 1$ and Γ *such that*

$$\begin{bmatrix} aI & I - CB\Gamma \\ (I - CB\Gamma)^T & I \end{bmatrix} \ge 0 \tag{10}$$

3. CB is full row rank.

Proof: The proof is straightforward. □

Lemma 3. *Suppose that there exists a* **constant** *matrix* Γ *such that*

$$||I - C(1)B(1)\Gamma|| < 1$$
$$||I - (C(1)B(2) + C(2)B(1))\Gamma|| < 1$$
$$||I - C(2)B(2)\Gamma|| < 1$$

Then,

$$\sup_{t\in[t_0,T]} ||I - C(t)B(t)\Gamma(t)|| < 1 \qquad (11)$$

where

$$\Gamma(t) = \frac{1}{1+t+t^2}\Gamma \qquad (12)$$

Proof: Note that

$$\begin{aligned}
& ||I - C(t)B(t)\Gamma(t)|| \\
= & ||I - (C(1) + C(2)t)(B(1) + B(2)t)\Gamma \frac{1}{1+t+t^2}|| \\
= & ||\frac{1}{1+t+t^2}[(I - C(1)B(1)\Gamma) \\
& +(I - (C(2)B(1) + C(1)B(2))\Gamma)t + (I - C(2)B(2)\Gamma)t^2]|| \\
\leq & \frac{1}{1+t+t^2}(||I - C(1)B(1)\Gamma|| \\
& +||I - (C(2)B(1) + C(1)B(2))\Gamma||t + ||I - C(2)B(2)\Gamma||t^2)|| \\
\leq & \max\{||I - C(1)B(1)\Gamma||, ||I - (C(2)B(1) + C(1)B(2))\Gamma||, \\
& ||I - C(2)B(2)\Gamma||\} \\
< & 1
\end{aligned}$$

Therefore, (11) holds. □

Lemma 4. *There exists* **a** Γ *such that* $||I - C(1)B(1)\Gamma|| < 1$ *holds for all* C(1) and B(1) *defined in (3) and* (4) *if and only if there exists a* Γ *such that* $||I - \hat{C}_i(1)\hat{B}_j(1)\Gamma|| < 1$ *holds for all* $i(1 \leq i \leq N_3)$ *and all* $j(1 \leq j \leq N_2)$.

Proof:

$\Longleftarrow$ **Note** that

$$\begin{aligned}
& ||I - C(1)B(1)\Gamma|| \\
= & ||I - \sum_{i=1}^{N_3} c_i\hat{C}_i(1)\sum_{j=1}^{N_2} b_j\hat{B}_j(1)\Gamma|| \\
= & ||\sum_{i=1}^{N_3}\sum_{j=1}^{N_2} c_i b_j(I - \hat{C}_i(1)\hat{B}_j(1)\Gamma)|| \\
\leq & \sum_{i=1}^{N_3}\sum_{j=1}^{N_2} c_i b_j ||(I - \hat{C}_i(1)\hat{B}_j(1)\Gamma)|| \\
< & 1
\end{aligned}$$

$\Longrightarrow$ The proof is straightforward.

□

Similarly, a-e have

Lemma 5. *There exists a* Γ *such that* $||I - C(2)B(2)\Gamma|| < 1$ *holds* **for all C(2)** and B(2) *defined* **in** (3) *and (4) if and only if there exists* **a** Γ *such that* $||I - \hat{C}_i(2)\hat{B}_j(2)\Gamma|| < 1$ *holds* **for** *all* $i(1 \leq i \leq N_3)$ *and all* $j(1 \leq j \leq N_2)$.

Lemma 6. *There exists a* Γ *such that* $||I - (C(1)B(2) + C(2)B(1))\Gamma|| < 1$ *holds for all* C(l), C(2), B(1) and *B(2) defined in (3)* and (4) *if* and *only if there exists a* Γ *such that* $||I - (\hat{C}_i(1)\hat{B}_j(2) + \hat{C}_i(2)\hat{B}_j(1))\Gamma|| < 1$ *holds* **for** *all* $i(1 \leq i \leq N_3)$ *and all* $j(1 \leq j \leq N_2)$.

Using Lemmas 2-6, we know that if we can find a Γ such that

$$a_{l,i,j} < 1\ ;\ 1 \leq l \leq 3;\ 1 \leq i \leq N_3;\ 1 \leq j \leq N_2; \quad (13)$$

$$\begin{bmatrix} a_{1,i,j}I & I - \hat{C}_i(1)\hat{B}_j(1)\Gamma \\ (I - \hat{C}_i(1)\hat{B}_j(1)\Gamma)^T & I \end{bmatrix} \geq 0; \quad (14)$$
$$1 \leq i \leq N_3; 1 \leq j \leq N_2$$

$$\begin{bmatrix} a_{2,i,j}I & I - \Psi(1,2)\Gamma \\ (I - \Psi(1,2)\Gamma)^T & I \end{bmatrix} > 0\ ; \quad (15)$$
$$1 \leq i \leq N_3; 1 \leq j \leq N_2$$

$$\begin{bmatrix} a_{3,i,j}I & I - \hat{C}_i(2)\hat{B}_j(2)\Gamma \\ (I - \hat{C}_i(2)\hat{B}_j(2)\Gamma)^T & I \end{bmatrix} \geq 0\ ; \quad (16)$$
$$1 \leq i \leq N_3; 1 \leq j \leq N_2$$

where

$$\Psi(1,2) = (\hat{C}_i(1)\hat{B}_j(2) + \hat{C}_i(2)\hat{B}_j(1)).$$

Then, we can design a controller of the form (5) with $\Gamma(t)$ defined in (12) such that (6) holds for linear time varying uncertain system (1)-(4). Although (13)-(16) can be easily solved by using linear matrix inequalities [5], the convergence speed may be slow if we only find Γ satisfying (13)-(16).

Thus, to improve the convergence speed, we need to find a Γ to minimize J_1 defined in (9). Such a Γ can be found by using the following lemma.

Lemma 7. *There exists a* Γ *such that* J_1 *defined in(9) is minimized* and $J_1 < 1$ if and *only* ***if*** *there exists* **a** *solution to the following* $3N_3N_2$*-objective optimal problem.*

$$\min_\Gamma\{a_{1,1,1}\}, \cdots, \min_\Gamma\{a_{3,N_3,N_2}\} \qquad (17)$$

subject to (13) − (16).

Proof: The proof is straightforward by using Lemmas 2-6. □

This multi-objective optimal problem defined in Lemma 7 is difficult to be solved. So, we only consider its simplified version by converting it into the following $3N_3N_2$ convex optimization problems.

$$\min_{\Gamma}\{a_{l,i,j}\} \tag{18}$$

subject to (13)–(16), and

$$\begin{aligned} &a_{l,i,j} \geq a_{s,p,q}\ ;\ s \neq l, 1 \leq s \leq 3; p \neq i; \\ &1 \leq p \leq N_3; q \neq j; 1 \leq q \leq N_2 \end{aligned} \tag{19}$$

where $1 \leq l \leq 3$, $1 \leq i \leq N_3$ and $1 \leq j \leq N_2$.

These optimal problems can be easily solved by using linear matrix inequalities [5]. From the solution of the above optimization problems, our learning control laws are given as (5) and (12).

Theorem 1. *Suppose that there exists at least one optimal solution, $a^*_{l,i,j} < 1$, to one of the above $3N_3N_2$ convex optimization problems. Then, the iterative* learning *control law (5)* and *(12) ensures that $Y_i(t) \to Y_d(t)$* and *$X_i(t_0) \to X_d(t_0)$ when $i \to \infty$.*

Proof: From Lemmas 226, w-e know- that

$$\sup_{t\in[t_0,T]} \|I - C(t)B(t)\Gamma(t)\| \leq a^*_{l,i,j} < 1$$

From (1), we have

$$\begin{aligned} e_{i+1}(t) &= Y_d(t) - Y_{i+1}(t) \\ &= (I - C(t)B(t)\Gamma(t))e_i(t) \\ &\quad +C(t)\phi(t,t_0)B(t_0)\Gamma(t_0)e_i(t_0) \\ &\quad -C(t)\phi(t,t_0)(X_{i+1}(t_0) - X_i(t_0)) \\ &\quad +C(t)\int_{t_0}^{t} \frac{d\phi(t,\tau)B(\tau)\Gamma(\tau)}{d\tau}e_i(\tau)d\tau \end{aligned} \tag{20}$$

Taking the norm of equation (20), we have

$$\begin{aligned} &e_{i+1}(t) \\ \leq\ & a^*_{l,i,j}\|e_i(t)\| + \|C(t)\phi(t,t_0)B(t_0)\Gamma(t_0)\|\|e_i(t_0)\| \\ &-\|C(t)\phi(t,t_0)\|\|(X_{i+1}(t_0) - X_i(t_0))\| \\ &+ \int_{t_0}^{t} \|C(t)\frac{d\phi(t,\tau)B(\tau)\Gamma(\tau)}{d\tau}\|\|e_i(\tau)\|d\tau \end{aligned} \tag{21}$$

Note that

$$\begin{aligned} &\frac{d\phi(t,\tau)B(\tau)\Gamma(\tau)}{d\tau} \\ =\ & -A(\tau)\phi(t,\tau)B(\tau)\Gamma(\tau) \\ &+\phi(t,\tau)B(2)\Gamma(t) - \phi(t,\tau)B(\tau)\Gamma\frac{1+2\tau}{(1+\tau+\tau^2)^2} \end{aligned}$$

and

$$\begin{aligned} \|A(t)\| &\leq \max_{1\leq i\leq N_1}\{\|A_i(1)\|\} + \max_{1\leq i\leq N_1}\{\|A_i(2)\|\}(T-t_0) \triangleq \hat{a} \\ \|B(t)\| &\leq \max_{1\leq i\leq N_2}\{\|B_i(1)\|\} + \max_{1\leq i\leq N_2}\{\|B_i(2)\|\}(T-t_0) \triangleq \hat{b} \\ \|C(t)\| &\leq \max_{1\leq i\leq N_3}\{\|C_i(1)\|\} + \max_{1\leq i\leq N_3}\{\|C_i(2)\|\}(T-t_0) \triangleq \hat{c} \\ \|B(2)\| &\leq \max_{1\leq i\leq N_2}\{\|B_i(2)\|\} \triangleq \hat{d} \end{aligned}$$

Using Lemma 1, we have

$$\|\phi(t,t_0)\| \leq e^{\hat{a}(t-t_0)} \leq e^{\hat{a}(T-t_0)}$$

Thus, we have

$$\begin{aligned} &\|e_{i+1}(t)\| \\ \leq\ & a^*_{l,i,j}\|e_i(t)\| + \hat{c}(\hat{c}\hat{b}\|\Gamma\| + 1 + r_0)e^{\hat{a}(T-t_0)}b_{x0}(i) \\ &+ \int_{t_0}^{t} \hat{c}e^{\hat{a}(T-t_0)}((\hat{a}+1)\hat{b}\|\Gamma\| + \hat{d}\|\Gamma\|)\|e_i(\tau)\|d\tau \end{aligned} \tag{22}$$

Multiplying $e^{-\lambda t}$ on the both side of (22), we have

$$\begin{aligned} &\|e_{i+1}(t)\|_\lambda \\ \leq\ & (a^*_{l,i,j} + \frac{\hat{c}e^{\hat{a}(T-t_0)}((\hat{a}+1)\hat{b}\|\Gamma\| + \hat{d}\|\Gamma\|)}{\lambda})\|e_i(t)\|_\lambda \\ &+\hat{c}(\hat{c}b\|\Gamma\| + 1 + r_0)e^{\hat{a}(T-t_0)}b_{x0}(i) \end{aligned}$$

Clearly, there exists an λ^* such that

$$a^*_{l,i,j} + \frac{\hat{c}e^{\hat{a}(T-t_0)}((\hat{a}+1)\hat{b}\|\Gamma\| + \hat{d}\|\Gamma\|)}{\lambda^*} < 1 \tag{23}$$

Using Assumption X2, we know that

$$b_{x0}(i) \leq r_0 b_{x0}(i-1)$$

Therefore, when $i \to \infty$, we have $Y_i(t) \to Y_d(t)$ and $X_i(t) \to X_d(t)$. □

4 An Illustrative Example

To illustrate the theoretical result, we consider a two-input tw-o-output system

$$\begin{aligned} \hat{A}_1(1) &= \begin{bmatrix} -0.5 & 0 \\ 0 & -1.2 \end{bmatrix}\ ;\ \hat{A}_2(1) = \begin{bmatrix} -1.5 & 0 \\ 0 & -0.8 \end{bmatrix} \\ \hat{A}_1(2) &= \begin{bmatrix} -0.3 & 0 \\ 0 & -0.2 \end{bmatrix}\ ;\ \hat{A}_2(2) = \begin{bmatrix} -0.7 & 0 \\ 0 & -0.6 \end{bmatrix} \\ \hat{B}_1(1) &= \begin{bmatrix} 16/3 & -4/3 \\ -3 & 1 \end{bmatrix}\ ;\ \hat{B}_2(1) = \begin{bmatrix} 1.926 & -0.5926 \\ -1.1111 & 0.4444 \end{bmatrix} \\ \hat{B}_1(2) &= \begin{bmatrix} 8/3 & -2/3 \\ -1.5 & 0.5 \end{bmatrix}\ ;\ \hat{B}_2(2) = \begin{bmatrix} 1.5408 & -0.47408 \\ -0.88888 & 0.35552 \end{bmatrix} \\ \hat{C}_1(1) &= \begin{bmatrix} 1.5 & 2 \\ 3 & 6 \end{bmatrix}\ ;\ \hat{C}_2(1) = \begin{bmatrix} 2.25 & 3 \\ 4.5 & 9 \end{bmatrix} \\ \hat{C}_1(2) &= \begin{bmatrix} 0.6 & 0.8 \\ 1.2 & 2.4 \end{bmatrix}\ ;\ \hat{C}_2(2) = \begin{bmatrix} 0.9 & 1.2 \\ 1.8 & 3.6 \end{bmatrix} \\ Y_d(t) &= \begin{bmatrix} Y_{d,1}(t) \\ Y_{d,2}(t) \end{bmatrix} = \begin{bmatrix} \sin(t) \\ \sin(t) \end{bmatrix} \end{aligned}$$

Based on Theorem 1, an optimal gain is obtained to be $\Gamma = \begin{bmatrix} 0.5454 & 0 \\ 0.5454 & 0.4924 \end{bmatrix}$ and

$$\Gamma(t) = \frac{1}{1+t+t^2}\Gamma$$

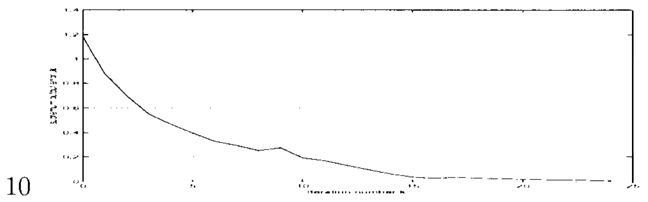

Figure 1: Tracking Errors

Suppose that the actual system is described by

$$\begin{aligned}
A(t) &= \frac{\hat{A}_1(1)+\hat{A}_2(1)}{2} + \frac{\hat{A}_1(2)+\hat{A}_2(2)}{2}t \\
&= \begin{bmatrix} -1-0.5t & 0 \\ 0 & -1-0.4t \end{bmatrix} \quad (24)\\
B(t) &= (0.9\hat{B}_1(1)+0.1\hat{B}_2(1)) + (0.9\hat{B}_1(2)+0.1\hat{B}_2(2))t \\
&= \begin{bmatrix} 4.9926+2.5541t & -1.2593-0.6474t \\ -2.8111-1.4389t & 0.9444+0.48561 \end{bmatrix} \quad (25)\\
C(t) &= (0.9\hat{C}_1(1)+0.1\hat{C}_2(1)) + (0.9\hat{C}_1(2)+0.1\hat{C}_2(2))t \\
&= \begin{bmatrix} 1.575+0.63t & 2.1+0.84t \\ 3.15+1.26t & 6.3+2.52t \end{bmatrix} \quad (26)
\end{aligned}$$

With the proposed iterative learning law (5) applied to (24)–(26), the tracking errors are shown in Figure 1. Clearly, both Y_1 and Y_2 track $Y_{d,1}$ and $Y_{d.2}$ perfectly after the 24th iteration, where the simulation is stopped if $\max\{|Y_1(t)-Y_{d,1}(t)|, |Y_1(t)-Y_{d,1}(t)|\} < 0.01$.

5 Conclusion

In this paper, we have considered the iterative learning control of repetitive linear time varying system with polytope uncertainties. We have formulated our approach as a group of convex optimization problems and this makes the problem computationally tractable by using some existing tools. Some sufficient conditions are also derived to ensure the convergence of the learning system.

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