1 KNOWLEDGE LEARNING TECHNIQUES AND APPLICATIONS IN DISCRETE TIME CONTROL SYSTEMS

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Abstract:

Knowledge learning techniques are playing an increasingly important role in control systems which leads to an active area of research – iterative learning control systems. This chapter focuses on the iterative learning control for discrete-time nonlinear systems. Two learning schemes are presented. In the first learning scheme, an explicit feedback-feedforward configuration for a class of uncertain discrete-time nonlinear systems is considered. A high-order ILC updating law is used as a feedforward controller which includes tracking errors in more than one previous iterations. It is shown that both the feedback and the high-order schemes can speed up learning convergence. Robust convergence conditions for the proposed learning control method are established. The second learning scheme considers the insufficient measurement problem encountered in many real control applications such as Rapid Thermal Processing (RTP). It may happen that only the terminal output tracking error instead of the whole output trajectory tracking error is available. The control profile for the next operation cycle has to be updated using the terminal tracking error alone. A revised ILC method is then proposed to address this terminal output tracking problem by parameterizing the control profile with a piecewise continuous functional basis. The control parameters are then updated by a high-order updating scheme. A convergence condition is obtained for a class of uncertain discrete-time time-varying linear systems including the RTPCVD system as the subset. Simulation results are presented to demonstrate the effectiveness of both learning schemes.

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1.1 INTRODUCTION

Generally speaking, knowledge used for control can be learned in two ways. A fuzzy control rule base is essentially learned by our human beings - either a skilled operator or an experienced expert. The more objective way of acquiring knowledge for control is to learning analytically through patterns (neural network) or through repetitons (iterative learning control). In this chapter, we will concentrate on iterative learning control (ILC) techniques for uncertain discrete-time systems. Here, *learning plays a role as a bridge between knowledge and experience.* The *lack* of system modelling knowledge is bridged by the experience acquired iteratively through control repetitions.

In this chapter, *learning* is the process of achieving a desired result by experience when only partial knowledge about the system to be controlled is available. Roughly speaking, the purpose of introducing the ILC is to utilize the system repetitions as *experience* to improve the system control performance even with incomplete system modelling. It differs from most existing control methods in the sense that, it exploits every possibility to incorporate past control information, such as tracking errors and control input signals, into the construction of the present control action. There are two phases in Iterative Learning Control: first the long term memory components are used to store past control information, then the stored control information is fused in a certain manner so as to ensure that the system meets control specifications such as convergence, robustness, etc. It is worth pointing out that, those control specifications may not be easily satisfied by other control methods as they require more prior knowledge of the process in the stage of the controller design. ILC requires much less information of the system variations to yield the desired dynamic behaviors.

'Iterative learning control' (ILC) was proposed by Arimoto and his associates [1] for improving system control with repetitions. Since early eighties, increasing attention has been drawn from the control community. On one hand, substantial work has been conducted and reported in the core area of developing and analyzing new ILC algorithms. On the other hand, researchers have realized that integration of ILC with other control techniques may give rise to better controllers that exhibit desired performance which is impossible by any individual approach. Integration of adaptive, robust or other learning techniques into ILC for various control problems has been frequently reported to remove usual requirements on conventional ILC algorithms. These can be observed from the latest book [3] and the dedicated ILC web server [4]. Detailed literature review on ILC research can be found in [5; 6].

To implement the ILC algorithm, a discrete-time form of the ILC algorithm and its theoretical analysis are more important. This is because ILC is essentially a memory based scheme where the past control signals and tracking errors are to be stored for later use. Therefore, a computer controller is indispensable. A lot of results were obtained for the linear discrete-time systems by using 2-D system theory [7; 8; 9], by including a parameter estimator [10], by applying the approximated impulse sequence [11; 12] and by Taylor series expansion [13]. This chapter thus focuses on the iterative learning control of discrete-time systems. Two learning schemes are presented. Firstly, a high-order ILC updating law is proposed to include tracking errors in more than one previous iterations. An explicit feedback-feedforward configuration for a class of uncertain discrete-time nonlinear systems is considered. It is shown that both the feedback control and the high-order scheme help speed up learning convergence. Simulation results of a single link robotic manipulator are presented to illustrate the effectiveness of the proposed scheme. Secondly, a terminal iterative learning control scheme is developed to deal with the insufficient measurement problem in real control engineering in which only the terminal output tracking error instead of the whole output trajectory tracking error is available. In the rapid thermal processing (RTP) chemical vapor deposition (CVD) of wafer fab. industry, the ultimate control objective is to control the deposition thickness (DT) at the end of the RTP cycle. The control profile for the next operation cycle has to be updated using the terminal DT tracking error alone. That is, the DT is measured only at the end of the RTP cycle. Terminal ILC method caters well to this kind of terminal output tracking problem. By parameterizing the control profile with a piecewise continuous functional basis, the control parameters are updated by a high-order updating scheme. A convergence condition is obtained for a class of uncertain discrete-time time-varying linear systems including the RTPCVD system as the subset. Simulation results for an RTPCVD thickness control problem are presented to demonstrate the effectiveness of the proposed iterative learning scheme.

The organization of this chapter is as follows: In Sec. 1.2, a class of uncertain discrete-time nonlinear systems are considered in a feedback-feedforward configuration where feedback controller is given in a general form and ILC is applied as the feedforward control using high-order updating law. Sec. 1.3 considers the terminal iterative learning control problem with an application for RTPCVD (rapid thermal processing chemical vapour deposition) application. Finally, some concluding remarks are presented in Sec. 1.4.

The norms used in this chapter are defined as

$$\begin{aligned} \|\bar{v}\| &= \max_{1\le i\le n} \mid \bar{v}_i \mid, \\ \|G\| &= \max_{1\le i\le m} (\sum_{j=1}^{n} \mid g_{i,j} \mid) \end{aligned} \tag{1.1}$$

where $\bar{v} = [\bar{v}_1, \cdots, \bar{v}_n]^T$ is a vector, $G = [g_{i,j}]_{m\times n}$ is a matrix.

1.2 HIGH-ORDER DISCRETE-TIME LEARNING CONTROL FOR UNCERTAIN DISCRETE-TIME NONLINEAR SYSTEMS WITH FEEDBACK

From the robust analysis in [14], it is clear that if the trajectories of each iterations can be adjusted inside the neighborhood of the desired ones, the ILC performance will be better. This can be achieved by introducing a feedback loop. Then the system considered is controlled by an ILC feedforward controller and a feedback controller simultaneously.

1.2.1 Basic Idea of High-order ILC

Uncertain discrete-time nonlinear systems are considered with a high-order learning updating scheme. The introduction of the feedback controller and the high-order scheme is mainly for the ILC performance improvements both in the time t-direction and in the ILC iteration number i-direction. The high-order ILC scheme, first proposed in [15], is actually for the improvement of the transient learning behavior along the learning iteration number i-direction as explained in [16]. In fact, more attentions should be paid to this i-direction because the tracking task is done only on a finite time interval repeatedly. If we look at the conventional ILC updating law [1]

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t)$$

for the control of the dynamics along the ILC iteration number i-direction, it is obviously a pure integral controller. Suppose the initial control $u_0(t) = 0$, then

$$u_{i+1}(t) = \Gamma \sum_{j=0}^{i} \dot{e}_j(t)$$

which is in an integral (I) controller form in the i-direction. If we use the PI controller in the i-direction

$$u_{i+1}(t) = k_P \dot{e}_i(t) + k_I \sum_{j=0}^{i} \dot{e}_j(t),$$

the ILC updating law takes the form that

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t)$$

where $\Gamma = k_P + k_I$ and $\Gamma_1 = -k_P$. By using the difference $\dot{e}_i(t) - \dot{e}_{i-1}(t)$ as the approximation of the *derivative* along the i-direction, the PID controller in i-direction

$$u_{i+1}(t) = k_P \dot{e}_i(t) + k_I \sum_{j=0}^{i} \dot{e}_j(t) + k_D(\dot{e}_i(t) - \dot{e}_{i-1}(t))$$

will result in the following form of the ILC updating law

$$u_{i+1}(t) = u_i(t) + \Gamma \dot{e}_i(t) + \Gamma_1 \dot{e}_{i-1}(t) + \Gamma_2 \dot{e}_{i-2}(t).$$

where $\Gamma = k_P + k_I + k_D$, $\Gamma_1 = -k_P - 2k_D$ and $\Gamma_2 = k_D$. This is a high-order iterative learning controller. The above arguments indicate that the high-order ILC is capable of giving better ILC performance than the traditional first-order case where only an integral controller is actually used.

1.2.2 Problem Formulation

Consider the following discrete-time uncertain nonlinear time-varying system which performs a given task repeatedly.

$$\begin{cases} x_i(t+1) = f(x_i(t), t) + B(x_i(t), t)u_i(t) + w_i(t) \\ y_i(t) = C(t)x_i(t) + v_i(t) \end{cases} \tag{1.2}$$

where i denotes the i-th repetitive operation of the system; t is the discrete time index and $t \in [0, N]$ which means that $t \in \{0, 1, \cdots, N\}$; $x_i(t) \in R^n$, $u_i(t) \in R^m$, and $y_i(t) \in R^r$ are the state, control input, and output of the system, respectively; $C(t) \in R^{r\times n}$ is a time-varying matrix; the functions $f(\cdot,\cdot) : R^n \times [0, N] \mapsto R^n$ and $B(\cdot,\cdot) : R^n \times [0, N] \mapsto R^m$ are uniformly globally Lipschitzian in x, i.e.,$\forall t \in [0, N], \forall i, \exists$ constants k_f, k_B, such that

$$\|\Delta f_i(t)\| \le k_f \|\Delta x_i(t)\|, \quad \|\Delta B_i(t)\| \le k_B \|\Delta x_i(t)\|$$

where $\Delta f_i(t) \stackrel{\triangle}{=} f(x_i(t), t) - f(x_{i-1}(t), t)$, $\Delta B_i(t) \stackrel{\triangle}{=} B(x_i(t), t) - B(x_{i-1}(t), t)$, $\Delta x_i(t) \stackrel{\triangle}{=} x_i(t) - x_{i-1}(t)$; $w_i(t)$, $v_i(t)$ are the system's uncertainties or disturbances which are bounded with unknown bounds b_w, b_v defined as

$$b_w \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|w_i(t)\|, \quad b_v \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|v_i(t)\|, \ \forall i. \tag{1.3}$$

Denote the output tracking error $e_i(t) \stackrel{\triangle}{=} y_d(t) - y_i(t)$ where $y_d(t)$ is the given desired output trajectory, which is realizable, i.e., given a bounded $y_d(t)$, there exists a unique bounded desired input $u_d(t)$, $t \in [0, N]$ such that when $u(t) = u_d(t)$, the system has a unique bounded desired state $x_d(t)$ satisfying

$$\begin{cases} x_d(t+1) = f(x_d(t), t) + B(x_d(t), t)u_d(t) \stackrel{\triangle}{=} f_d + B_d u_d \\ y_d(t) = C(t)x_d(t) \stackrel{\triangle}{=} C(t)x_d. \end{cases} \tag{1.4}$$

Denote the bound of the desired control u_d as $b_{u_d} \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|u_d(t)\|$.

The problem is formulated as follows. Starting from an arbitrary continuous initial control input $u_0(t)$, obtain the next control input $u_1(t)$ and the subsequent series $\{u_i(t) \mid i = 2, 3, \cdots\}$ for system (1.2) by using a proper learning control updating law in such a way that when $i \to \infty, y_i(t) \to y_d(t) \pm \varepsilon^*$ in the presence of bounded uncertainty, disturbance and re-initialization error. ε^* is the tracking error bound.

To solve the above problem, we propose the feedback-assisted high-order iterative learning controller as follows. At the i-th ILC iteration, the control input $u_i(t)$ to the system (1.2) is the output of a satuator, i.e.,

$$u_i(t) = \mathrm{sat}(\bar{u}_i(t)) \tag{1.5}$$

where $\mathrm{sat}(\bar{u}_i(t)) \triangleq [\mathrm{sat}(\bar{u}_{1_i}(t)), \cdots, \mathrm{sat}(\bar{u}_{m_i}(t))]^T$ and

$$\mathrm{sat}(\bar{u}_{j_i}(t)) \triangleq \begin{cases} \bar{u}_{j_i}(t), & \text{if } \mid \bar{u}_{j_i}(t) \mid \leq \bar{u}_j^* \\ \frac{\bar{u}_{j_i}(t)}{|\bar{u}_{j_i}(t)|}\bar{u}_j^*, & \text{if } \mid \bar{u}_{j_i}(t) \mid > \bar{u}_j^* \end{cases} \tag{1.6}$$

where $j = 1, 2, \cdots, m$ and $\bar{u}_j^* > 0$ are the satuation bounds which may be selected in accordance with actuator limitations. The introduction of input satuators here is to show that the ILC is also effective in such realistic cases as argued in [17]. The satuator input is that

$$\bar{u}_i(t) = u_i^f(t) + u_i^b(t) \tag{1.7}$$

where $u_i^f(t)$ is from the feed*f*orward iterative learning controller and $u_i^b(t)$ is from the feed*b*ack stabilizing controller. The feedback stabilizing controller is assumed to be in the following general form

$$z_i(t+1) = h_a(z_i(t)) + H_b(z_i(t))e_i(t), \tag{1.8}$$
$$u_i^b(t) = h_c(z_i(t)) + H_d(z_i(t))e_i(t) \tag{1.9}$$

where $z_i(t) \in R^{n_c}$ is the state of the feedback stabilizing controller with $z_i(0) = 0,\ \forall i$. The vector-valued functions $h_a(\cdot) : R^{n_c} \mapsto R^{n_c}$ and $h_c(\cdot) : R^{n_c} \mapsto R^m$ are designed to be sector-bounded as

$$\|h_a(z_i(t))\| \leq b_{h_a}\|z_i(t)\|,\ \|h_c(z_i(t))\| \leq b_{h_c}\|z_i(t)\|.$$

The function matrices $H_b(\cdot) : R^{n_c} \mapsto R^{n_c\times r}$ and $H_d(\cdot) : R^{n_c} \mapsto R^{m\times r}$ are designed to be with uniform bounds, i.e., $\forall t \in [0, N], \forall z_i(t) \in R^{n_c}$,

$$\|H_b(z_i(t))\| \leq b_{H_b},\ \|H_d(z_i(t))\| \leq b_{H_d}.$$

The $b_{h_a}, b_{h_c}, b_{H_b}, b_{H_d}$ above are positive constants which are not necessarily known. A simple ILC updating law is used which includes tracking errors of M previous iterations, i.e.,

$$u_{i+1}^f(t) = u_i(t) + \sum_{k=1}^{M} Q_k(t)e_{i-k+1}(t+1) \tag{1.10}$$

where M is the order of the ILC updating law with $M \geq 1$; $Q_k(t) \in R^{m\times r} (k = 1, \cdots, M)$ are the learning matrices which are to be determined to ensure the ILC convergence.

Remark 1.2.1 *Because the tracking errors of previous ILC iterations are prestored in the memory and can be easily manipulated to shift the time index* *by 1, the above proposed ILC updating law (1.10) can be taken as a P-type in the discrete-time t-direction and a PID-type in the i-direction, i.e., P-PID ILC updating law, according to the arguments in the Introduction section. However, from the sampled-data point of view, in the t-axis, using the information of the moment $(t+1)$ at the instant t implies a PD-type because $e_i(t+1) \approx e_i(t) + T_s \dot{e}_i(t)$ where T_s is the sampling time. The scheme (1.10) is thus essentially a PD-PID type ILC.*

To restrict our discussion, the following assumptions are made.

A1). The initialization error is bounded as follows, $\forall t \in [0, N],\ \forall i,\ \|x_d(0) - x_i(0)\| \leq b_{x_0},\ \|y_d(0) - y_i(0)\| \leq b_C b_{x_0} + b_v$, where $b_C \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|C(t)\|$.

A2). Matrix $C(\cdot)B(\cdot,\cdot)$ has a full column rank $\forall t \in [0, N],\ x(t) \in R^n$.

A3). Operator $B(\cdot,\cdot)$ is bounded, i.e., $\exists$ a constant b_B such that for all i, $\sup_{t\in[0,N]} \|B(x_i(t), t)\| \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|B_i(t)\| \leq b_B$.

A4). The desired output $y_d(t), t \in [0, N]$ is achievable by the desired input $u_d(t)$ which is inside the satuation bounds, i.e., $u_d(t) \equiv \mathrm{sat}(u_d(t)), \forall t \in [0, N]$.

Assumption A1) restricts that the initial states or the initial outputs in each repetitive operation should be inside a given ball centered at the desired initial ones. The radius of the ball may be unknown. The number of outputs r can be greater than or equal to the number of inputs m according to A2). A3) says that the range of operator $B(\cdot,\cdot)$ is always finite. This is reasonable because the repetitive tasks are performed in a finite time interval $[0, NT_s]$. A4) requires that the desired trajectory should be planned in such a way that those large and sudden changes, which the system can not follow, should be excluded from the desired trajectories.

To analyze the robust convergence property of the proposed feedback-assisted high-order ILC algorithm, the following $\lambda-$norm is introduced for a discrete-time vector $h(t), t = 0, 1, \cdots, N$.

$$\|h(t)\|_\lambda \stackrel{\triangle}{=} \sup_{t\in[0,N]} \hat{e}^{-\lambda t}\|h(t)\| \tag{1.11}$$

where $\lambda > 0$ when $\hat{e} > 1$ or $\lambda < 0$ when $\hat{e} \in (0, 1)$. The positive constant $\hat{e}$ will be specified later. It should be pointed out that the $\lambda-$norm used in this section is equivalent to the infinity-norm [18] because $\|h(\cdot)\|_\lambda \leq \|h(\cdot)\|_\infty \leq \|h(\cdot)\|_\lambda \hat{e}^{\lambda N}$, where the infinity-norm $\|h(t)\|_\infty \stackrel{\triangle}{=} \sup_{t\in[0,N]} \|h(t)\|$.

To facilitate the later derivations, some basic relations are presented in the following. The main purpose is to explore the relationship between $(\|\delta x_i(t)\|_\lambda + \|z_i(t)\|_\lambda)$ and $\|\delta u_i^f(t)\|_\lambda$ where $\delta u_i^f(t) \stackrel{\triangle}{=} u_d(t) - u_i^f(t),\ \delta x_i(t) \stackrel{\triangle}{=} x_d(t) - x_i(t)$. Denote $\delta u_i(t) \stackrel{\triangle}{=} u_d(t) - u_i(t)$.

First, it's an easy exercise to verify the following recursive formula.

$$\bar{z}_{i+1} = a_1\bar{z}_i + a_2\bar{z}_i^* + a_3 = a_1^{i+1}\bar{z}_0 + \sum_{j=0}^{i} a_1^{i-j}(a_2\bar{z}_j^* + a_3) \tag{1.12}$$

where $\{\bar{z}_i, \bar{z}_i^* \mid i = 0, 1, \cdots\}$ are two series and related to each other by coefficients a_1, a_2, a_3.

Denote

$$\delta f_i(t) \stackrel{\triangle}{=} f_d - f(x_i(t), t), \quad \delta B_i(t) \stackrel{\triangle}{=} B_d - B_i(t).$$

Then, from (1.2) and (1.4), it can be obtained that

$$\delta x_i(t+1) = \delta f_i(t) + \delta B_i(t)u_d + B_i(t)\delta u_i(t) - w_i(t). \tag{1.13}$$

Taking the norm for (1.13) yields

$$\|\delta x_i(t+1)\| \le (k_f + b_{u_d}k_B)\|\delta x_i(t)\| + b_B\|\delta u_i(t)\| + b_w. \tag{1.14}$$

From (1.9), it can be seen that

$$\|u_i^b(t)\| \le b_{h_c}\|z_i(t)\| + b_{H_d}b_C\|\delta x_i(t)\| + b_{H_d}b_v. \tag{1.15}$$

By noticing that $u_d(t) \equiv \mathrm{sat}(u_d(t))$ and that

$$\begin{aligned} \|\delta u_i(t)\| &= \|u_d(t) - \mathrm{sat}(u_i^f(t) + u_i^b(t))\| \\ &\le \|\delta u_i^f(t) - u_i^b(t)\| \le \|\delta u_i^f(t)\| + \|u_i^b(t)\|, \end{aligned} \tag{1.16}$$

then, (1.14) becomes

$$\begin{aligned} \|\delta x_i(t+1)\| &\le (k_f + b_{u_d}k_B + b_B b_{H_d} b_C)\|\delta x_i(t)\| \\ &+ b_B b_{h_c}\|z_i(t)\| + b_B\|\delta u_i^f(t)\| + b_B b_{H_d} b_v + b_w. \end{aligned} \tag{1.17}$$

On the other hand, it can be observed from (1.8) that

$$\|z_i(t+1)\| \le b_{h_a}\|z_i(t)\| + b_{H_b}b_C\|\delta x_i(t)\| + b_{H_b}b_v, \tag{1.18}$$

thus, adding (1.18) into (1.17) yields

$$\begin{aligned} (\|\delta x_i(t+1)\| + \|z_i(t+1)\|) &\le \hat{e}(\|\delta x_i(t)\| + \|z_i(t)\|) \\ &+ b_B\|\delta u_i^f(t)\| + \hat{\varepsilon} \end{aligned} \tag{1.19}$$

where

$$\hat{e} \stackrel{\triangle}{=} \max\{k_f + b_{u_d}k_B + b_B b_{H_d} b_C + b_{H_b} b_C,\ b_{h_a} + b_B b_{h_c}\} \neq 1;$$
$$\hat{\varepsilon} \stackrel{\triangle}{=} (b_{H_b} + b_B b_{H_d})b_v + b_w.$$

Applying (1.12), we can get

$$\|\delta x_i(t+1)\| + \|z_i(t+1)\| \leq \hat{e}^{t+1} b_{x_0} + \sum_{j=0}^{t} \hat{e}^{t-j}(b_B \|\delta u_i^f(j)\| + \hat{\varepsilon}). \tag{1.20}$$

To see a simpler relationship between $\|\delta x_i(t)\|_\lambda + \|z_i(t)\|_\lambda$ and $\|\delta u_i^f(t)\|_\lambda$, by noticing the following facts that

- $\|c\|_\lambda \equiv \mid c \mid, \ \forall c \in R;$
- $\forall \mid \lambda \mid > 1, \ \sup_{t\in[0,N]} \hat{e}^{-(\lambda-1)t} = 1;$
- $\forall t_1 \in [0, N_1], t_2 \in [0, N_2]$, if $0 \leq N_1 \leq N_2 \leq N$, then $\|\delta h(t_1)\|_\lambda \leq \|\delta h(t_2)\|_\lambda$;

then, taking the $\lambda-$norm ($\mid \lambda \mid > 1$) operation of (1.20) gives

$$\|\delta x_i(t)\|_\lambda + \|z_i(t)\|_\lambda \leq b_{x_0} + b_B O(\mid \lambda \mid^{-1}) \|\delta u_i^f(t)\|_\lambda + c_0 \hat{\varepsilon} \tag{1.21}$$

where

$$O(\mid \lambda \mid^{-1}) \triangleq \frac{1 - \hat{e}^{-(\lambda-1)N}}{\hat{e}^\lambda - \hat{e}}, \quad c_0 \triangleq \sup_{t\in[0,N]} \frac{\hat{e}^{-(\lambda-1)t}(1 - \hat{e}^{-t})}{\hat{e} - 1}.$$

For brevity of our discussion, in the sequel, the following notations are used.

$$b_{Q_k} \triangleq \sup_{t\in[0,N]} \|Q_k(t)\|, \ (k = 1, 2, \cdots, M),$$

$$\rho_1 \triangleq \sup_{t\in[0,N]} \|I_m - Q_1(t)C(t+1)B_i(t)\|, \quad \forall i,$$

$$\rho_k \triangleq \sup_{t\in[0,N]} \|Q_k(t)C(t+1)B_{i-k+1}(t)\|, \quad \forall i, \ (k = 2, 3, \cdots, M).$$

1.2.3 Robust Convergence

A main result is presented in the following theorem.

Theorem 1.2.1 *For the repetitive discrete-time uncertain time-varying nonlinear system (1.2) under assumptions A1)-A4), given the realizable desired trajectory* $y_d(t)$ *over the fixed time interval* $[0, NT_s]$, *by using the ILC updating law (1.10) and the feedback controller (1.8)-(1.9), if the condition*

$$\rho \triangleq \sum_{k=1}^{M} \rho_k < 1, \tag{1.22}$$

is satisfied, then the λ*-norm of the tracking errors* $e_i(t)$, $\delta u_i(t)$, $\delta x_i(t)$ *are bounded for all* i. *For a sufficiently large* $\mid \lambda \mid$, $\forall t \in [0, N]$,

$$b_{u^f} \triangleq \lim_{i\to\infty} \|\delta u_i^f(t)\|_\lambda \leq b_{u^f}(b_{x_0}, b_w, b_v), \tag{1.23}$$

$$b_u \triangleq \lim_{i\to\infty} \|\delta u_i(t)\|_\lambda \leq b_u(b_{x_0}, b_w, b_v), \qquad (1.24)$$

$$b_x \triangleq \lim_{i\to\infty} \|\delta x_i(t)\|_\lambda \leq b_{x_0} + b_B O(|\ \lambda\ |^{-1}) b_{u^f} + c_0\hat{\varepsilon}, \qquad (1.25)$$

$$b_e \triangleq \lim_{i\to\infty} \|e_i(t)\|_\lambda \leq b_C b_x + b_v. \qquad (1.26)$$

Moreover, b_u, b_x, b_e will all converge uniformly to zero for $t = 0, 1, \cdots, N$ as $i \to \infty$ in the absence of uncertainty, disturbance and initialization error, i.e., $b_w, b_v, b_{x_0} \to 0$.

Theorem 1.2.1 shows that the uniform boundedness of the tracking error can be established in the presence of bounded uncertainty, disturbance and the re-initialization error even without the assistance of the feedback controller. It is also shown that the tracking error bound is a class-K function of the bounds of the differences of uncertainties, disturbances and the re-initialization errors between two successive ILC iterations under some additional conditions.

Before proving Theorem 1.2.1, the following lemma [19] is needed.

Lemma 1.2.1 *Suppose a real positive series $\{a_n\}_1^\infty$ satisfies $a_n \leq \rho_1 a_{n-1} + \rho_2 a_{n-2} + \cdots + \rho_N a_{n-N} + \varepsilon$, $(n = N+1, N+2, \cdots)$, where $\rho_i \geq 0$, $(i = 1, 2, \cdots, N)$, $\varepsilon \geq 0$ and $\rho = \sum_{i=1}^N \rho_i < 1$, then the following holds:*

$$\lim_{n\to\infty} a_n \leq \varepsilon/(1-\rho). \qquad (1.27)$$

Proof: Refer to [19].

Now we proceed to prove Theorem 1.2.1.

Proof of Theorem 1.2.1:
The tracking error at $(i+1)-$th repetition is

$$e_i(t) = y_d(t) - y_i(t) = C(t)\delta x_i(t) - v_i(t). \qquad (1.28)$$

Investigating the learning control deviation at the $(i+1)$-th repetition $\delta u_{i+1}^f(t)$ gives

$$\begin{aligned} \delta u_{i+1}^f(t) &= \delta u_i(t) - \sum_{k=1}^M Q_k(t) e_{i-k+1}(t+1) \\ &= \delta u_i(t) - \sum_{k=1}^M Q_k(t) C(t+1)\delta x_{i-k+1}(t+1) \\ &\quad + \sum_{k=1}^M Q_k(t) v_{i-k+1}(t+1). \end{aligned} \qquad (1.29)$$

By referring to (1.13), (1.29) can be written as

$$\begin{aligned}\delta u^f_{i+1}(t) &= \delta u_i(t) - \sum_{k=1}^{M} Q_k(t)C(t+1)\\ &\quad [\delta f_{i-k+1}(t) + \delta B_{i-k+1}(t)u_d + B_{i-k+1}(t)\delta u_{i-k+1}(t)\\ &\quad -w_{i-k+1}(t)] + \sum_{k=1}^{M} Q_k(t)v_{i-k+1}(t+1). \qquad (1.30)\end{aligned}$$

Collecting terms and then performing the norm operation for (1.30) yield

$$\begin{aligned}\|\delta u^f_{i+1}(t)\| &\le \sum_{k=1}^{M} \rho_k \|\delta u_{i-k+1}(t)\|\\ &\quad + \sum_{k=1}^{M} b_{Q_k} b_C (k_f + b_{u_d} k_B)\|\delta x_{i-k+1}(t)\|\\ &\quad + \sum_{k=1}^{M} b_{Q_k}(b_C b_w + b_v). \qquad (1.31)\end{aligned}$$

Based on (1.16) and (1.15), (1.31) becomes

$$\begin{aligned}\|\delta u^f_{i+1}(t)\| &\le \sum_{k=1}^{M} \rho_k \|\delta u^f_{i-k+1}(t)\|\\ &\quad + \sum_{k=1}^{M} \alpha_k (\|\delta x_{i-k+1}(t)\| + \|z_{i-k+1}(t)\|) + \varepsilon. \qquad (1.32)\end{aligned}$$

where $\alpha_k \triangleq \max\{b_{Q_k} b_C (k_f + b_{u_d} k_B) + b_{H_d} b_C \rho_k, b_{h_c}\rho_k\}$ and $\varepsilon \triangleq \sum_{k=1}^{M}[b_{Q_k}(b_C b_w + b_v) + b_{H_d} b_v \rho_k]$.

By utilizing the relationship in (1.21), taking the $\lambda-$norm for (1.31) gives

$$\begin{aligned}\|\delta u^f_{i+1}(t)\|_\lambda &\le \sum_{k=1}^{M} \rho_k \|\delta u^f_{i-k+1}(t)\|_\lambda\\ &\quad + \sum_{k=1}^{M} \alpha_k b_B O(|\lambda|^{-1}) \|\delta u^f_{i-k+1}(t)\|_\lambda\\ &\quad + \sum_{k=1}^{M} \alpha_k (b_{x_0} + c_0 \hat{\varepsilon}) + \varepsilon. \qquad (1.33)\end{aligned}$$

Referring to (1.22), it is clear that a sufficiently large $|\lambda|$ can be used to ensure that

$$\sum_{k=1}^{M} (\rho_k + \alpha_k b_B O(|\lambda|^{-1})) \triangleq \hat{\rho} < 1, \qquad (1.34)$$

therefore, according to Lemma 1.2.1, we have

$$b_{u^f} = \lim_{i\to\infty} \|\delta u_i^f(t)\|_\lambda = \frac{\varepsilon_0}{1-\hat{\rho}} \triangleq b_{u^f}(b_{x_0}, b_w, b_v) \tag{1.35}$$

where $\varepsilon_0 \triangleq \varepsilon + \sum_{k=1}^{M} \alpha_k (b_{x_0} + c_0 \hat{\varepsilon})$. From (1.28), it is easy to verify (1.25) and (1.26). It can be observed from (1.21) that

$$\begin{aligned} b_{xz} &\triangleq \lim_{i\to\infty} (\|\delta x_i(t)\|_\lambda + \|z_i(t)\|_\lambda) \\ &\le b_{x_0} + b_B O(\mid \lambda \mid^{-1}) b_{u^f} + c_0 \hat{\varepsilon}. \end{aligned} \tag{1.36}$$

Therefore, from (1.16) and (1.15), by referring to (1.36), we have

$$\begin{aligned} b_u &\triangleq \lim_{i\to\infty} \|\delta u_i(t)\|_\lambda \le b_{u^f} + \max\{b_{h_c}, b_{H_d} b_C\} b_{xz} + b_{H_d} b_v \\ &\triangleq b_u(b_{x_0}, b_w, b_v) \end{aligned} \tag{1.37}$$

which verifies (1.24). Moreover, it is easy to observe that b_{u^f}, b_u, b_x, and b_e will all tend to zero uniformly for $t = 0, 1, \cdots, N$ as $i \to \infty$ in the absence of uncertainty, disturbance and initialization error, i.e., when $b_w, b_v, b_{x_0} \to 0$.

Remark 1.2.2 *Consider the case that the desired trajectory varies with respect to ILC iteration number i. Suppose the desired trajectory at the i-th iteration is changed to $y_{d_i}(t)$. If*

$$\|y_{d_i}(t) - y_d(t)\| < b_{y_d}, \quad \forall t \in [0, N] \text{ and } \forall i, \tag{1.38}$$

then, all the discussions made above are all valid by replacing b_v with $b_v + b_{y_d}$.

Remark 1.2.3 *The proposed high-order ILC updating law (1.10) can be regarded as a PID-type learning controller in the i-direction where the higher order terms are for a better approximation of the* D *information in the i-direction. Better ILC performance can be expected compared to traditional I-type controller even in the absence of the feed-back controller. When a feed-back controller is designed to better perform in the t-domain, the control system performance will be improved along the i-domain. In other words, the transient performance can be improved both in the i-direction and in the t-direction by the proposed feedback-assisted high-order iterative learning controller.*

The tracking errors in the ILC updating law (1.10) are shifted by one sampling step, i.e., $e_{i-k+1}(t+1)$. In the following Corollary, we will show that the result of Theorem 1.2.1 still holds if tracking errors $e_{i-k+1}(t)$ are included in (1.10), i.e., when the ILC updating law

$$u_{i+1}^f(t) = u_i(t) + \sum_{k=1}^{M} [Q_k(t) e_{i-k+1}(t+1) + \bar{Q}_k(t) e_{i-k+1}(t)] \tag{1.39}$$

is applied, where $\bar{Q}_k(t)$ are the bounded learning matrices $\forall t \in [0, N]$.

Corollary 1.2.1 *The ILC convergency in Theorem 1.2.1 still holds when the ILC updating law (1.10) is replaced by the ILC updating law (1.39).*

Proof of Corollary 1.2.1: The proof is quite similar to the proof of Theorem 1.2.1. When (1.39) is considered, the following term

$$-\sum_{k=1}^{M} \bar{Q}_k(t)e_{i-k+1}(t) + \sum_{k=1}^{M} \bar{Q}_k(t)v_{i-k+1}$$

should be appended in the left-hand sides of (1.29) and (1.30) based on (1.28). Hence, we get the following inequality of the norm estimates similar to (1.31).

$$\begin{aligned} \|\delta u_{i+1}^f(t)\| \quad \leq \quad & \sum_{k=1}^{M} \rho_k \|\delta u_{i-k+1}(t)\| \\ & + \sum_{k=1}^{M} [b_{Q_k} b_C (k_f + b_{u_d} k_B) + b_{\bar{Q}_k} b_C] \|\delta x_{i-k+1}(t)\| \\ & + \sum_{k=1}^{M} [b_{Q_k}(b_C b_w + b_v) + b_{\bar{Q}_k} b_v] \end{aligned} \tag{1.40}$$

where $b_{\bar{Q}_k} \triangleq \sup_{t\in[0,N]} \|\bar{Q}_k(t)\|$. The remaining part of the proof is the same as the proof of Theorem 1.2.1 if we set

$$\alpha_k \triangleq \max\{b_{Q_k} b_C (k_f + b_{u_d} k_B) + b_{\bar{Q}_k} b_C + b_{H_d} b_C \rho_k, b_{h_c} \rho_k\}$$

and

$$\varepsilon \triangleq \sum_{k=1}^{M} [b_{Q_k}(b_C b_w + b_v) + b_{\bar{Q}_k} b_v + b_{H_d} b_v \rho_k],$$

which ends the proof.

Remark 1.2.4 *It can be observed that the convergence of the discrete-time ILC is only governed by the learning components $Q_k(t)e_{i-k+1}(t+1)$. However, the additional terms $\bar{Q}_k(t)e_{i-k+1}(t)$ in the ILC updating law may improve the performance of ILC convergence. If we set $\bar{Q}_k(t) = -Q_k(t)$, it is obvious that the result of Corollary 1.2.1 is a discrete analogy of the result of [17].*

In the ILC updating law (1.10), only the tracking errors of previous cycles were utilized. The control functions of previous cycles can be utilized also as in the high-order ILC schemes [15; 16]. The general form of the high-order ILC updating law is given as

$$\begin{aligned} u_{i+1}^f(t) \quad = \quad & \sum_{k=1}^{M} [P_k(t)u_{i-k+1}(t) + Q_k(t)e_{i-k+1}(t+1) \\ & + \bar{Q}_k(t)e_{i-k+1}(t)] \end{aligned} \tag{1.41}$$

where the learning matrices $P_k(t)$ $(k = 1, 2, \cdots, M)$ satisfy

$$\sum_{k=1}^{M} P_k(t) = I_m, \quad \forall t \in [0, N]. \tag{1.42}$$

We can get a similar robust convergence result to Theorem 1.2.1.

Theorem 1.2.2 *Consider the repetitive discrete-time uncertain time-varying nonlinear system (1.2) satisfying Assumptions A1)-A4). For a given realizable desired trajectory* $y_d(t)$ *over the fixed time interval* $[0, NT_s]$*, the iterative learning controller (1.41) and feedback controller (1.8)-(1.9) are applied. If*

$$\rho' \triangleq \sum_{k=1}^{M} \rho'_k < 1, \tag{1.43}$$

where $\forall i$*, and* $k = 1, 2, \cdots, M$,

$$\rho'_k \triangleq \sup_{t \in [0,N]} \|P_k(t) - Q_k(t)C(t+1)B_{i-k+1}(t)\|, \tag{1.44}$$

then the λ*-norm of the tracking errors* $e_i(t)$, $\delta u_i^f(t)$, $\delta u_i(t)$, $\delta x_i(t)$ *are all bounded for all* i*. The bounds of the tracking errors are functions of* b_{x_0}, b_w, b_v *and moreover, will all converge uniformly to zero for* $t = 0, 1, \cdots, N$ *as* $i \to \infty$ *in the absence of uncertainty, disturbance and initialization error, i.e.,* $b_w, b_v, b_{x_0} \to 0$.

Proof of Theorem 1.2.2: The main body of the proof is similar to the proof of Theorem 1.2.1 where (1.29) and (1.30) should be re-written as

$$\begin{aligned} \delta u_{i+1}^f(t) &= u_d(t) - \sum_{k=1}^{M} P_k(t) u_{i-k+1}(t) \\ &\quad - \sum_{k=1}^{M} [Q_k(t) e_{i-k+1}(t+1) + \bar{Q}_k(t) e_{i-k+1}(t)] \\ &= u_d(t) - \sum_{k=1}^{M} P_k(t) u_{i-k+1}(t) \\ &\quad - \sum_{k=1}^{M} Q_k(t) C(t+1) \delta x_{i-k+1}(t+1) \\ &\quad + \sum_{k=1}^{M} Q_k(t) v_{i-k+1}(t+1) \\ &\quad - \sum_{k=1}^{M} \bar{Q}_k(t) C(t) \delta x_{i-k+1}(t) + \sum_{k=1}^{M} \bar{Q}_k(t) v_{i-k+1}(t). \end{aligned} \tag{1.45}$$

By referring to (1.13) and (1.42), (1.45) can be written as

$$
\begin{aligned}
\delta u_{i+1}^{f}(t) \quad = \quad & \sum_{k=1}^{M} P_k(t)\delta u_{i-k+1}(t) - \sum_{k=1}^{M} Q_k(t)C(t+1) \\
& [\delta f_{i-k+1}(t) + \delta B_{i-k+1}(t)u_d + B_{i-k+1}(t)\delta u_{i-k+1}(t) \\
& -w_{i-k+1}(t)] + \sum_{k=1}^{M} Q_k(t)v_{i-k+1}(t+1) \\
& -\sum_{k=1}^{M} \bar{Q}_k(t)C(t)\delta x_{i-k+1}(t) + \sum_{k=1}^{M} \bar{Q}_k(t)v_{i-k+1}(t). \qquad (1.46)
\end{aligned}
$$

Estimating the norm yields

$$
\begin{aligned}
\|\delta u_{i+1}^{f}(t)\| \quad \leq \quad & \sum_{k=1}^{M} \rho_k^{'}\|\delta u_{i-k+1}(t)\| \\
& +\sum_{k=1}^{M}[b_{Q_k}b_C(k_f + b_{u_d}k_B) + b_{\bar{Q}_k}b_C]\|\delta x_{i-k+1}(t)\| \\
& +\sum_{k=1}^{M}[b_{Q_k}(b_C b_w + b_v) + b_{\bar{Q}_k}b_v]. \qquad (1.47)
\end{aligned}
$$

The remaining part of the proof is the same as the proof of Theorem 1.2.1 if we set

$$\alpha_k \triangleq \max\{b_{Q_k}b_C(k_f + b_{u_d}k_B) + b_{\bar{Q}_k}b_C + b_{H_d}b_C\rho_k^{'}, b_{h_c}\rho_k^{'}\}$$

and

$$\varepsilon \triangleq \sum_{k=1}^{M}[b_{Q_k}(b_C b_w + b_v) + b_{\bar{Q}_k}b_v + b_{H_d}b_v\rho_k^{'}],$$

which ends the proof.

Remark 1.2.5 *The general form of high-order ILC updating law (1.41) introduces more flexibility in determining the learning matrices. This can be taken as a form of "filtered tracking error" case as considered in [20]. However, the increased number of design parameters will obscue the expected advantages. Thus, from practical point of view, it is sufficient to use only dual PIDs both in the t-direction and in the i-direction for the proposed feedback-assisted ILC, especially taking into consideration the long-history usage of the PID controller in industries.*

1.2.4 Reduction of Tracking Error Bound

According to Theorems 1.2.1 and 1.2.2, the control applied to the system has been proved to be bounded at each ILC iteration. Let

$$b_{u^*} \triangleq \sup_{t\in[0,T]} \|u_i(t)\|, \forall i. \tag{1.48}$$

In the following we will show that under certain additional restrictions, the tracking error bound is a function of the bounds of the differences of uncertainties, disturbances and the re-initialization errors between two successive ILC iterations even without the feedback controller, i.e., $u_i(t) \equiv u_i^f(t)$.

Introduce a similar $\lambda-$norm definition as in (1.11) with $\hat{e}$ replaced by

$$\tilde{e} \triangleq k_f + b_{u^*} k_B \neq 1. \tag{1.49}$$

Denote $\Delta\bar{h}_i(t) \triangleq \bar{h}_i(t) - \bar{h}_{i-1}(t)$, $\bar{h} \in \{u, y, w, v\}$. Similar to (1.13) and (1.14), the following two inequalities can be obtained

$$\Delta x_i(t+1) = \Delta f_i(t) + \Delta B_i(t) u_i(t) + B_i(t)\Delta u_i(t) + \Delta w_i(t), \tag{1.50}$$

$$\|\Delta x_i(t+1)\| \leq \tilde{e}\|\Delta x_i(t)\| + b_B\|\Delta u_i(t)\| + b'_w \tag{1.51}$$

where $b'_w \triangleq \sup_{t\in[0,N]} \|\Delta w_i(t)\|$, $\forall i$. It is also apparent by referring to (1.21) that

$$\|\Delta x_i(t)\|_\lambda \leq b'_{x_0} + b_B \tilde{O}(|\ \lambda\ |^{-1})\|\Delta u_i(t)\|_\lambda + c'_0 b'_w \tag{1.52}$$

where $b'_{x_0} \triangleq \|\Delta x_i(0)\|$, $\forall i$, and

$$\tilde{O}(|\ \lambda\ |^{-1}) \triangleq \frac{1 - \tilde{e}^{-(\lambda-1)N}}{\tilde{e}^{\lambda} - \tilde{e}}, \quad c'_0 \triangleq \sup_{t\in[0,N]} \frac{\tilde{e}^{-(\lambda-1)t}(1-\tilde{e}^{-t})}{\tilde{e}-1}.$$

The following notations are employed.

$$\bar{\rho}_1 \triangleq \sup_{t\in[0,N]} \|I_r - C(t+1)B_i(t)Q_1(t)\|, \quad \forall i,$$

$$\bar{\rho}_k \triangleq \sup_{t\in[0,N]} \|C(t+1)B_i(t)Q_k(t)\|, \quad \forall i, (k = 2, 3, \cdots, M),$$

$$b'_v \triangleq \sup_{t\in[0,N]} \|\Delta v_i(t)\|, \ \forall i.$$

To show that the tracking error bound is a function of the bounds of the differences of uncertainties, disturbances and the re-initialization errors between two successive ILC iterations even without the feedback controller, in addition to Assumptions A1) to A3), the following Assumptions A1') and A2') are imposed.

A1'). The the differences of uncertainties, disturbances and the re-initialization errors between two successive ILC iterations are bounded with unknown bounds denoted by b'_w, b'_v and b'_{x_0} respectively.

A2'). Matrix $C(\cdot)B(\cdot,\cdot)$ has a full row rank $\forall t \in [0, N],\ x(t) \in R^n$.

According to Assumption A2'), the number of outputs r should be less than or equal to the number of inputs m. Because A2) is still applied for a bounded input sequence $\{u_i(t) \mid i = 1, 2, \cdots\}$, thus the restriction here is actually that $m = r$ and $C(\cdot)B(\cdot,\cdot)$ is full rank.

Under these restrictions, new property of the tracking error convergence can be explored and is presented in the following theorem.

Theorem 1.2.3 *Given the realizable desired trajectory $y_d(t)$ over the fixed time interval $[0, NT_s]$, for the repetitive discrete-time uncertain time-varying non-linear system (1.2) under assumptions A1)-A4), A1') and A2'), by using the high-order ILC updating law (1.10), the λ-norm of the tracking errors $e_i(t)$ is bounded if*

$$\sum_{k=1}^{M} \bar{\rho}_k < 1. \tag{1.53}$$

Moreover, the bound is a function of b'_w, b'_v and b'_{x_0}. Furthermore, the tracking error $e_i(t)$ converges uniformly to zero for $t = 1, \cdots, N$ as $i \to \infty$ when the uncertainties, disturbances and the re-initialization errors between two successive ILC iterations tend to be the same, i.e., when b'_w, b'_v and b'_{x_0} tend to 0.

Proof of Theorem 1.2.3: The tracking error at $(i+1)-$th ILC iteration can be expressed as

$$\begin{aligned} e_{i+1}(t+1) &= e_i(t+1) - \Delta y_{i+1}(t+1) \\ &= e_i(t+1) - C(t+1)\Delta x_{i+1}(t+1) - \Delta v_{i+1}(t+1) \end{aligned} \tag{1.54}$$

Substituting (1.50) and (1.10) into (1.54) gives

$$\begin{aligned} e_{i+1}(t+1) = {} & e_i(t+1) - C(t+1)[\Delta f_{i+1}(t) + \\ & \Delta B_{i+1}(t)u_{i+1}(t)] - C(t+1)B_i(t)\sum_{k=1}^{M} Q_k(t)e_{i-k+1}(t+1) + \\ & -C(t+1)\Delta w_{i+1}(t) - \Delta v_{i+1}(t+1). \end{aligned} \tag{1.55}$$

Collecting terms and then taking the norm for (1.55) yield

$$\begin{aligned} \|e_{i+1}(t+1)\| \leq {} & \sum_{k=1}^{M} \bar{\rho}_k \|e_{i-k+1}(t+1)\| + b_C\tilde{e}\|\Delta x_{i+1}(t)\| \\ & + b_C b'_w + b'_v. \end{aligned} \tag{1.56}$$

Taking the $\lambda-$norm for (1.56) and using the relationship (1.52), we simply have

$$\begin{aligned} \|e_{i+1}(t+1)\|_\lambda \quad &\le \quad \sum_{k=1}^{M} \bar{\rho}_k \|e_{i-k+1}(t+1)\|_\lambda + \varepsilon' \\ &\quad + \sum_{k=1}^{M} \tilde{O}_k(|\ \lambda\ |^{-1}) \|e_{i-k+1}(t+1)\|_\lambda \end{aligned} \tag{1.57}$$

where $\tilde{O}_k(|\ \lambda\ |^{-1}) \triangleq b_{Q_k} b_C \tilde{e} b_B \tilde{O}(|\ \lambda\ |^{-1})$ and $\varepsilon' \triangleq b_C \tilde{e}(b'_{x_0} + c'_0 b'_w) + b_C b'_w + b'_v$. Clearly, based on (1.53), a sufficiently large $|\ \lambda\ |$ can be used to ensure that

$$\sum_{k=1}^{M} [\bar{\rho}_k + \tilde{O}_k(|\ \lambda\ |^{-1})] < 1. \tag{1.58}$$

Therefore, according to Lemma 1.2.1, the λ-norm of tracking error is bounded as $i \to \infty$. Referring to the definitions of ε' and $\tilde{O}_k(|\ \lambda\ |^{-1})$, we observe that the tracking error bound is a function of b'_w, b'_v and b'_{x_0}. Furthermore, the tracking error $e_i(t)$ converges uniformly to zero for $t = 1, 2, \cdots, N$ as $i \to \infty$ when the uncertainties, disturbances and the re-initialization errors between two successive ILC iterations tend to be the same, i.e., when b'_w, b'_v and b'_{x_0} tend to 0.

Remark 1.2.6 *By referring to the Corollary 1.2.1, we know the Theorem 1.2.3 still holds if the ILC updating law (1.39) is applied.*

Remark 1.2.7 *Consider the case that the desired trajectory varies with respect to ILC iteration number i. Suppose the desired trajectory at the i-th iteration is changed to $y_{d_i}(t)$. If*

$$\|y_{d_{i+1}}(t) - y_{d_i}(t)\| < b'_{y_d}, \quad \forall t \in [0, N] \text{ and } \quad \forall i, \tag{1.59}$$

then, all the conclusions made above are valid by replacing b'_v with $b'_v + b'_{y_d}$.

Remark 1.2.8 *It is implied in Theorem 1.2.3 that the ILC method can completely reject any repetitive components of the uncertainty or disturbance. The uniform bound of the tracking error is dependent on the bounds of the differences of the uncertainties and disturbances between two successive system repetitions.*

Remark 1.2.9 *It worth to note that the system with a direct transmission term in output equation considered in [21] is a special case investigated in this chapter. The major difference is that in the learning updating law [21] $e_i(t)$ is used instead of $e_i(t+1)$ used here.*

1.2.5 Simulation Illustrations

To demonstrate the effectiveness of the proposed feedback-assisted high-order ILC algorithm for the improvement of the ILC convergence performance, a

Table 1.1 Parameters of the single-link manipulator

parameter	m_0	M_0	l	g
unit	kg	kg	m	m/sec.^{2}
value	2	4	0.5	9.8

single link direct joint driven manipulator model is used for the simulation study. The dynamic equation in the continuous-time t' domain is

$$\ddot{\theta}(t') = \frac{1}{J}[\tau(t') + \tau_n(t')] + \frac{1}{J}(\frac{1}{2}m_0 + M_0)gl\sin\theta(t') \tag{1.60}$$

where $\theta(t')$ is the angular position of the manipulator; $\tau(t')$ is the applied joint torque; $\tau_n(t')$ is the exogenous disturbance torque; m_0, l are the mass and length of the manipulator respectively, M_0 is the mass of the tip load, g is the gravitational acceleration and the J is the moment of inertia w.r.t the joint, i.e., $J = M_0 l^2 + m_0 l^2/3$. The parameters used in this simulation study are listed in Table 1.1. Let the sampling period $T_s = 0.01$ sec. One can discretize the above model by using simple Euler method as follows:

$$\begin{cases} x_1(t+1) = x_2(t) \\ x_2(t+1) = 2x_2(t) - x_1(t) + [\tau(t) + \tau_n(t) + (0.5m_0 + M_0)gl\sin(x_1(t))]T_s^2/J \end{cases} \tag{1.61}$$

where discrete time $t = 0, 1, \cdots, 100$, $x_1(t) = \theta(t), x_2(t) = \theta(t+1)$. The desired tracking trajectories of the tracking tasks over the time interval $[0, 1]$ sec. are specified as

$$\theta_d(t) = \theta_b + (\theta_b - \theta_f)(15\tau_0^4 - 6\tau_0^5 - 10\tau_0^3) \tag{1.62}$$

where $\tau_0 = tT_s/(t_f - t_0)$. In the simulation, we use $\theta_b = 0°$, $\theta_f = 90°$, $t_0 = 0$, $t_f = 1$. The initial states at each ILC repetition are all set to 0. The ILC ends when $e_{b1} \leq 1°$ where

$$e_{b1} \stackrel{\triangle}{=} \sup_{t\in[0,100]} \mid \theta_d(t) - \theta(t) \mid .$$

To simplify the presentation, the following ILC updating law is used

$$u_{i+1}^f(t) = u_i(t) + \sum_{k=1}^{M} Q_k(e_{i-k+1}(t+1) - e_{i-k+1}(t)) \tag{1.63}$$

where Q_k is the learning parameter and $e(t) = \theta_d(t) - \theta(t)$. In the ILC method, the system parameters are assumed unknown. However, the learning parameter determination should be based on the knowledge of the system. To make the results with comparability, in the following simulations, we use

$Q_1 = 50, Q_2 = 25, Q_3 = 25.$

The following five cases are presented.

Case 1. High-order Effects Without Feedback Controller
Consider the ideal situation without exogenous torque disturbance. The tracking control is achieved by the iterative learning controller only without the assistance of the feedback controller. To compare the effectiveness of the high-order ILC scheme, three subcases are considered, i.e., $M = 1, 2, 3$. The histories of the tracking error bounds are shown in Figure 1.1 where the improved ILC convergence can be observed when higher order ILC updating law is applied.
Case 2. Effects of the Feedback Controller in ILC
This case is similar to *Case 1.* We apply the fixed ILC updating law with $M = 1$. But a simple P-type feedback controller

$$u_i^b(t) = K_p e_i(t) \tag{1.64}$$

is applied to assist the ILC. Four subcases are considered for different gains K_p. When $K_p = 0$, the tracking is by ILC which is the subcase $M = 1$ in *Case 1.* The results are summarized in Figure 1.2. It is observed that an improved ILC convergence performance can be obtained when the ILC is assisted by a feedback controller. It is interesting to note in this case that the ILC performance improves as K_p increasing which implies a well designed feedback controller is quite helpful for the iterative learning control.
Case 3. High-order Effects With a Feedback Controller
Consider the ILC updating laws with different orders, i.e., $M = 1, 2, 3$. The same incorporated feedback controller is applies with $K_p = 30$. The tracking results are given in Figure 1.3, where a similar performance improvement behavior of the ILC convergence can be observed.
Case 4. Tracking the Varying Desired Trajectories
In this case, we choose $M = 1,\ K_p = 30$. The desired trajectory is supposed to vary with respect to iteration number i. The varying desired trajectories are designed with the same form (1.62) but the final angle $\theta_f(t)$ is revised to

$$\theta_{f_i}(t) = (2 - e^{-0.05i})45^\circ. \tag{1.65}$$

The varying desired trajectory is denoted by θ_{d_i}. In this case, we choose $M = 1,\ K_p = 30$. In the first ILC iteration, the desired trajectory $\theta_{d_0}(t)$ and the system output $\theta_0(t)$ are plotted in Figure 1.4. To have a clear comparison, the desired trajectory and the system output at 100-th ILC iteration, i.e., $\theta_{d_{100}}(t)$ and $\theta_{100}(t)$, are drawn in the same figure. We observe a good tracking in this case. It should be noted that in this case b_{y_d} defined in Remark 1.2.2 is fairly large. However, according to Remark 1.2.7, the tracking error bound in this case should tend to 0 because b'_{y_d} tends to 0 as i increases. This is clearly illustrated by Figure 1.4 which indicates that Remark 1.2.2 is more conservative than Remark 1.2.7.
Case 5. Tracking the Varying Desired Trajectories With Exogenous Disturbance

The same situation as *Case 4* is considered but with exogenous torque disturbance

$$\tau_n(t) = 70(2 - e^{-0.05i})\sin(\pi t/5) \ \ \text{Nm}. \tag{1.66}$$

Good tracking performance in this case can be observed. It should be noted that in this case b_w defined in Theorem 1.2.1 is fairly large. However, according to Theorem 1.2.3, the tracking error bound in this case should tend to 0 because b'_w tends to 0 as i increases. This is clearly illustrated by Figure 1.5 which indicates that the repetitive unknown exogenous disturbance can be rejected by ILC scheme.

1.2.6 Summary

A high-order iterative learning controller, assisted by a feedback controller, is proposed. It has been proven that the tracking errors are bounded in the presence of bounded uncertainty, disturbance, and re-initialization error. Moreover, it has been shown that the tracking error bounds are functions of the bounds of uncertainty, disturbance, and re-initialization error. The tracking error bounds tend to 0 in the absence of uncertainty, disturbance, and re-initialization error. Furthermore, it also has been shown that under certain conditions, the tracking error bound is a function of the bounds of the differences of uncertainties, disturbances and the re-initialization errors between two successive ILC iterations. Improved ILC performance in both time t-direction and iteration number i-direction can be achieved by the proposed feedback-assisted high-order iterative learning controller. Simulation illustrations are presented which verify the effectiveness of the proposed ILC scheme.

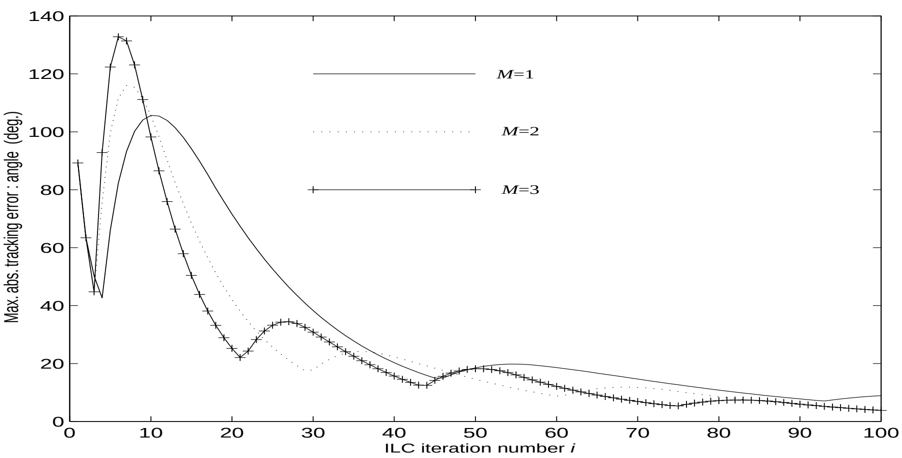

Figure 1.1 Comparison of ILC convergence histories, ideal case with different ILC orders

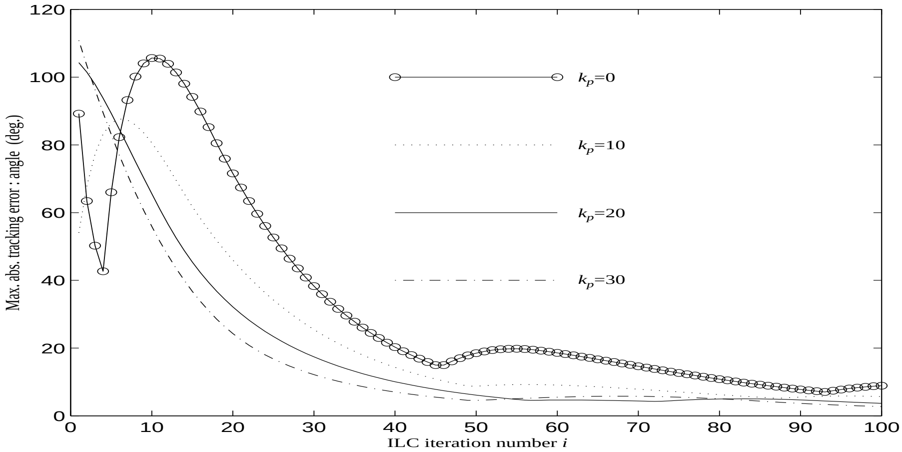

Figure 1.2 Comparison of feedback-assisted ILC convergence histories, ideal case with fixed ILC order

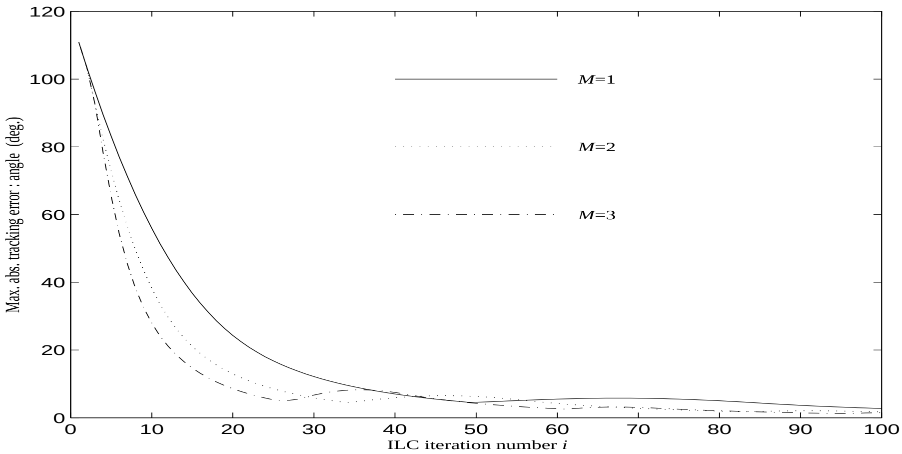

Figure 1.3 Comparison of feedback-assisted ILC convergence histories, ideal case with different ILC orders

1.3 TERMINAL HIGH-ORDER ITERATIVE LEARNING CONTROL

A special type of iterative learning control (ILC) problem is considered in this section. Due to the insufficient measurement capability in many real control problems such as Rapid Thermal Processing (RTP), it may happen that only the terminal output tracking error instead of the whole output trajectory tracking error is available. In the RTP chemical vapor deposition (CVD) of wafer fab. industry, the ultimate control objective is to control the deposition thickness (DT) at the end of the RTP cycle. The control profile for the next operation cycle has to be updated using the terminal DT tracking error alone. That is,

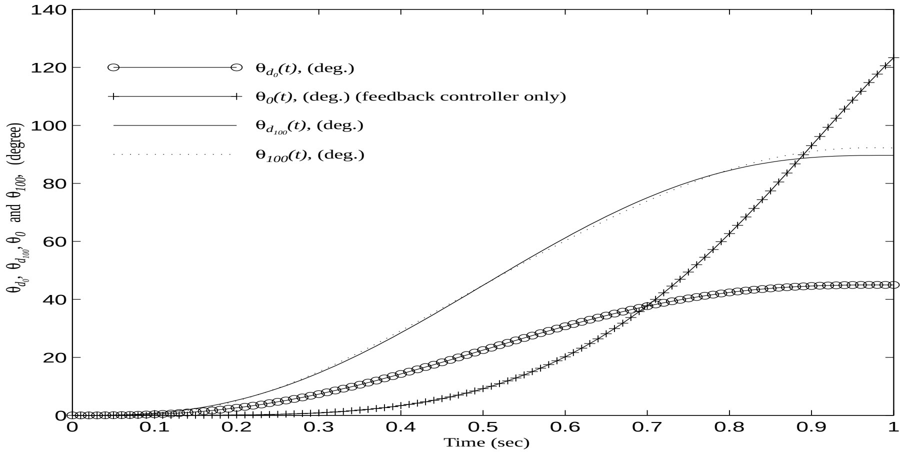

Figure 1.4 Tracking the varying desired trajectory: ideal case

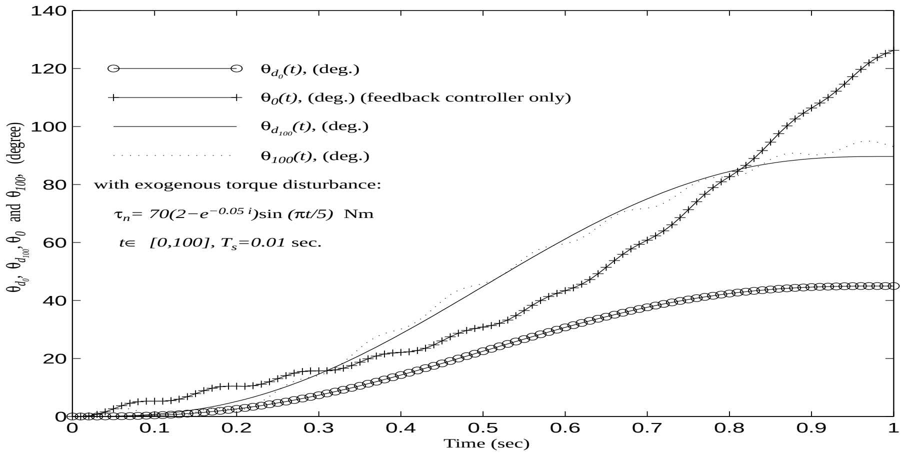

Figure 1.5 Tracking the varying desired trajectory: with exogenous disturbance

the DT is measured only at the end of the RTP cycle. A revised ILC method is proposed to address this terminal output tracking problem. By parameterizing the control profile with a piecewise continuous functional basis, the parameters are updated by a high-order updating scheme. A convergence condition is obtained for a class of uncertain discrete-time time-varying linear systems including the RTPCVD system as the subset. Simulation results for an RTPCVD thickness control problem are presented to demonstrate the effectiveness of the proposed iterative learning scheme.

1.3.1 Background and Motivation

Rapid thermal processing (RTP) systems are single-wafer, cold-wall chambers that utilize one or more radiant heat sources to rapidly heat up the semiconductor substrate at high temperatures for short times [22]. In semiconductor wafer fab. industry, RTP has been used as a *versatile* single-wafer processing technique for various thermal processing applications. Single-wafer processing will be preferred over conventional batch equipment for many applications as the wafer size increases beyond 150-200 mm. Factors in favor of single-wafer processing include compatibility with multi-chamber cluster equipment for vacuum-integrated processing, improved fabrication cycle time, and enhanced fabrication process repeatability due to improved process control. One of the typical RTP systems is for the chemical vapor deposition (RTPCVD). The terminal deposition thickness (DT) for each run of RTPCVD is to be controlled within a given tolerance by adjusting the heating lamp power profile [23].

It is critical to maintain a *uniform temperature profile* across the wafer during steady-state and transient operation to avoid the generation of slip dislocations and to ensure process uniformity. The process control can be divided into a three tier hierarchical model consisting of

- supervisory control,
- run-to-run (RtR) control, and
- real-time control.

Supervisory control, at the highest level, influences changes on a lot to lot basis. Run-to-run control updates occur after each wafer is processed. In supervisory and RtR control, only off-line sensors are required for the desired measurements which are often unavailable *in-situ*. Real-time control can often be subdivided into wafer state and process state control. Wafer state refers to physical quantities associated with the processed silicon wafer (e.g. spatial temperature distribution, film thickness, etc.). Process state, on the other hand, refers to physical quantities which are not wafer parameters (e.g. reactor wall temperatures, partial pressures of gases, etc.). Badgwell *et al.* provide a comprehensive survey of modelling and control issues in the semiconductor industry [24]. From application examples, it is apparent that the entire hierarchy may or may not be required for a particular semiconductor application. In this section, we concentrate on the control in the RtR level. Some works exist [25; 26] for a better run to run control using the idea of numerical optimization. In this research, a new scheme - *"terminal iterative learning control"* is proposed for RTPCVD application.

As the RTPCVD system executes a given task repeatedly, this repeatability can be utilized to improve the system control performance by the *iterative learning control* (ILC) method [1; 27]. The conventional ILC task is to follow a desired output trajectory in a given time interval through learning *iteratively*. In RTPCVD, however, the problem cannot be formulated in such conventional way because the exact measurement of wafer temperature is almost impossible [28]. In a more general framework, this class of control problems can be classified as the PTP (point-to-point) control problem. The main features of a PTP problem are: 1) the only available measurement is the terminal state or terminal output; 2) the ultimate control objective is also the terminal state or terminal output instead of the *whole trajectory* of the system. The precise PTP control finds increasing applications in flexible manipulators [29], mobile robots [30] etc. Learning method has been applied to PTP control problems successfully in [30; 31]. The key technique is to represent the control function as a linear combination of a pre-determined piecewise continuous functional basis (orthogonal polynomials, splines, etc.) and then to update the coefficient vector iteratively based on the terminal output error measurement at the end of each run (system repetition).

Motivated by RTPCVD control problem, in this section we developed a terminal ILC scheme to deal with a class of uncertain discrete-time linear time-varying systems in general and the RTPCVD problem in particular. A sufficient convergence condition of the terminal ILC is obtained. Terminal tracking performance under disturbance is also investigated. Simulation studies with an RTPCVD model demonstrate the effectiveness of the proposed *terminal iterative learning control* scheme. Moreover, it is also shown that a high-order learning updating scheme can provide a better convergence performance.

1.3.2 RTPCVD Model and The Control Problem

Consider a simplified RTPCVD model of poly-Si which includes the temperature dependence of deposition rate [32; 26], as follows:

$$\left\{ \begin{array}{lcl} \frac{\mathrm{d}T_w}{\mathrm{d}t} & = & [\sigma A_w E_w (T_{amb}^4 - T_w^4) + f E_w Q P]/M_w \\ \frac{\mathrm{d}S}{\mathrm{d}t} & = & k_0 \exp(-\frac{\gamma}{R T_w}) \end{array} \right. \tag{1.67}$$

where the meanings of variables and the relevant parameters are given in Table 1.2. $T_w(0)$ is known and $S(0) = 0\mu\mathrm{m}$.

The control objective is to find a lamp power profile $P(t)$ such that the controlled wafer temperature $T_w(t)$ follows a pre-planned trajectory $T_w^d(t)$ as closely as possible in a given time interval $[0, NT_s]$, where T_s is the sampling period and N is the number of samples. $T_w^d(t)$ is pre-designed [26] to satisfy the RTP requirements and especially to guarantee the final deposition thickness $S(T) \quad (T = NT_s)$. However, conventional feedback based control schemes are hardly applicable to this control task because the *in-situ* measurement of wafer temperature $T_w(t)$ is an even tougher problem. A practical way is to use the measurement of the terminal DT at the end of each run. The control task then becomes: given the RTP cycle period T and a desired DT $S_d(T)$, with the repetitive RTP runs, iteratively update the control $P(t)$ based on the terminal DT tracking error $e(T) \triangleq S_d(T) - S(T)$.

The basic idea is to parameterize the control function $P(t)$ using some known basis functions. The parameters are to be determined through an iterative learning law. The learning gain should be properly chosen to guarantee the convergence of learning process. To simplify the analysis and design, one may linearize (1.67) around an equilibrium point $(T_w^*(t), S^*(t), P^*(t))$. In practice, one may use an approximate equilibrium point as shown in the simulation study in Sec. 1.3.4.2. A small variation around the equilibrium point is denoted by $(\Delta T_w(t), \Delta S(t), \Delta P(t))$. The linearized system is

$$\left\{ \begin{array}{lcl} \frac{\mathrm{d}\Delta T_w}{\mathrm{d}t} & = & [-4\sigma A_w E_w (T_w^*)^3 \Delta T_w + f E_w Q \Delta P]/M_w + w_1(t) \\ \frac{\mathrm{d}\Delta S}{\mathrm{d}t} & = & k_0 \frac{\gamma}{R} \exp(-\frac{\gamma}{R T_w^*}) \Delta T_w / (T_w^*)^2 + w_2(t) \end{array} \right. \tag{1.68}$$

where $w_1(t), w_2(t)$ are high order terms which can be taken as the modeling uncertainties. As the process is computer-controlled, a discretized model is preferred for the analysis. Define the states $x_1 = \Delta T_w, x_2 = \Delta S$ and control $u = \Delta P$. Discretizing (1.68) with a zero order hold (ZOH) gives

$$\left\{ \begin{array}{lcl} x(t+1) & = & A(t)x(t) + Bu(t) + w(t) \\ y(t) & = & x_2(t) + v(t) \end{array} \right. \tag{1.69}$$

where $t = 0, 1, \cdots, N$; $x = [x_1, x_2]^T$; $A(t)$ and B are matrices with appropriate dimensions in terms of (1.68); $w(t)$ is the modeling uncertainty related to $w_1(t)$ and $w_2(t)$; $v(t)$ is the measurement noise (only $v(N)$ is concerned). In the subsequent subsection, a terminal iterative learning control method is developed for this class of uncertain linear time-varying discrete-time systems.

1.3.3 Terminal Output Tracking by Iterative Learning

Consider in general the uncertain time-varying linear discrete-time system

$$\left\{ \begin{array}{lcl} x_i(t+1) & = & A(t)x_i(t) + B(t)u_i(t) + w_i(t) \\ y_i(t) & = & C(t)x_i(t) + v_i(t) \end{array} \right. \tag{1.70}$$

where $t = 0, 1, \cdots, N$; the subscript i indicates the system repetition number; state $x_i(t) \in R^n$; control function $u_i(t) \in R^m$; output $y_i(t) \in R^r$; $w_i(t), v_i(t)$ are model uncertainty and measurement disturbance respectively. During the repetitive operations, only $y_i(N)$ is measurable at the end of every run i. The **control task** is to find and improve the control function $u_i(t)$ in an iterative manner such that $y_i(N)$ approaches to a given terminal output y^d as i increases.

To restrict our discussion, the following assumptions are imposed:

- B1) System (1.70) is completely controllable.
- B2) The initial state $x_i(0)$ at every iteration i can be repositioned nearby the same starting point with some misalignment

$$\|\Delta x_{i+1}(0)\| \leq \varepsilon_1$$

where ε_1 is a small positive constant.

- B3) The disturbance and modeling error are bounded and

$$\|\Delta w_{i+1}(N)\| \le \varepsilon_2, \quad \|\Delta v_{i+1}(N)\| \le \varepsilon_3.$$

Here $\Delta w_{i+1}(t) = w_{i+1}(t) - w_i(t)$ and $\Delta v_{i+1}(t) = v_{i+1}(t) - v_i(t)$; ε_2 and ε_3 are small positive constants.

Assumption B1) assures the existence of control for the given y^d. The initial states are not necessary to be strictly repetitive according to B2). B3) is a relaxed assumption on the uncertainty w_i and disturbance v_i. It is well known that any ILC scheme is able to remove repeatable uncertainty or disturbance. Thus we only need to know the "variations" of the disturbances in any two consecutive cycles. Assumption B3) describes the bounds of such variations.

As the tracking control task is only for the terminal output, conventional ILC updating law [1] cannot be applied. Following the idea of **dynamic fitting** [33], instead of solving a functional minimization problem, we parameterize the control $u_i(t)$ as

$$u_i(t) = \Phi(t)\Xi_i \tag{1.71}$$

where $\Xi_i = [\xi_{1_i}, \cdots, \xi_{p_i}]^T \in R^{p\times 1}$ is the parameter vector; $\Phi(t) =[\phi_1(t), \cdots, \phi_m(t)]^T \in R^{m\times p}$ is a properly chosen basis function matrix. $\phi_j(t) = [\varphi_{j1}(t), \cdots, \varphi_{jp}(t)], j = 1, \cdots, m$. The task is hence converted into finding an iterative scheme based on the observation of the terminal output tracking error $e_i(N) \triangleq y^d - y_i(N)$ such that the parameter Ξ_i is updated as i increases and meanwhile $e_i(N)$ converges to a prescribed ball centered at the origin.

By solving (1.70), one obtains the terminal state

$$x_i(N) = Gx_i(0) + H\Xi_i + w_i(N) \tag{1.72}$$

where

$$G = \prod_{t=0}^{N-1} A(t); \tag{1.73}$$

$$H = \sum_{k=1}^{N}\prod_{j=1}^{k-1} A(N-j)B(N-k)\Phi(N-k); \tag{1.74}$$

$$w_i(N) = \sum_{k=1}^{N}\prod_{j=1}^{k-1} A(N-j)w_i(N-k) \tag{1.75}$$

with dimensions $G \in R^{n\times n}$, $H \in R^{n\times p}$ and $w_i(N) \in R^{n\times 1}$. Thus the terminal output becomes

$$y_i(N) = CGx_i(0) + CH\Xi_i + Cw_i(N) + v_i(N). \tag{1.76}$$

Investigating the terminal tracking error of the output $y_i(N)$, one can easily get the relation between two consecutive operation cycles.

$$\begin{aligned} e_{i+1}(N) &= y^d - y_{i+1}(N) = e_i(N) - (y_{i+1}(N) - y_i(N)) \\ &= e_i(N) - CG\Delta x_{i+1}(0) - CH\Delta\Xi_{i+1} \\ &\quad -C\Delta w_{i+1}(N) - \Delta v_{i+1}(N) \end{aligned} \tag{1.77}$$

where $\Delta h_{i+1} \stackrel{\triangle}{=} h_{i+1} - h_i, \;\; h_i \in \{x_i(0), \Xi_i\}$.

The high-order learning updating law is proposed as follows:

$$\Xi_{i+1} = \Xi_i + \Delta\Xi_{i+1} = \Xi_i + \sum_{k=1}^{M} L_k e_{i-k+1}(N) \tag{1.78}$$

where M is the order of ILC and $L_k (k = 1, 2, \cdots, M)$ are learning gain matrices which are to be specified in applications. Substituting (1.78) into (1.77) yields

$$\begin{aligned} e_{i+1}(N) &= (I_r - CHL_1)e_i(N) - CG\Delta x_{i+1}(0) \\ &-CH\sum_{k=2}^{M} L_k e_{i-k+1}(N) - C\Delta w_{i+1}(N) - \Delta v_{i+1}(N) \end{aligned} \tag{1.79}$$

where I_r is an $r \times r$ unit matrix.

To analyze the convergence of the proposed terminal iterative learning control, an additional assumption is required.

- B4) CH has a full row rank.

The key issue in B4) is related to the existence of the control for PTP control and has been well discussed in [34]. As indicated in [34], a proper choice of $\Phi(t)$ is always possible if the system (1.70) is controllable.

Remark 1.3.1 *Consider the case when system (1.70) is LTI and the relative degree is d_r. We know that $CA^jB = 0$, $j = 0, 1, \cdots, d_r - 1$. Referring to (1.74),*

$$CH = \sum_{k=1}^{N} CA^{k-1}B\Phi(N-k).$$

Hence, for a properly chosen Φ, CH can be made full rank if $d_r < N$. Usually, the number of sampling points is much larger than the relative degree. Thus, B4) can be satisfied easily.

Remark 1.3.2 *Regarding the Φ selection, in general, only assumption B4) needs to be satisfied. In practice, to choose a suitable $\Phi(t)$, we need to have some knowledge about the system to be controlled. Φ can be chosen in terms of the particulars of the plant so as to further improve the control performance. A detailed example will be presented in Sec. 1.3.4.*

We summarize the above discussion in the following theorem.

Theorem 1.3.1 *Consider the system (1.70) under assumptions B1)-B4) with a given achievable terminal output y^d. By applying the control functional parameterization (1.71) and the iterative learning updating law (1.78), through the repetitive operations, the terminal tracking error $e_i(N)$ will converge to a bound if*

$$\rho \stackrel{\triangle}{=} \sum_{k=1}^{M} \rho_k < 1 \tag{1.80}$$

where $\rho_1 = \max_{t\in\{0,N\}} \|I_r - CHL_1\|$, $\rho_k = \max_{t\in\{0,N\}} \|CHL_k\|$, $k = 2, 3, \cdots, M$. *The convergence bound is*

$$\|e_i(N)\| \leq \varepsilon^* \qquad (1.81)$$

where

$$\varepsilon^* \leq (\|CG\|\varepsilon_1 + \|C\|\varepsilon_2 + \varepsilon_3)/(1-\rho). \qquad (1.82)$$

Clearly, ε^* *is a class-K function of* $\varepsilon_1, \varepsilon_2$ *and* ε_3 *which implies that* $\varepsilon^* \to 0$ *as* $\varepsilon_1, \varepsilon_2$ *and* ε_3 *approach to 0.*

Proof:
Taking norm operation of (1.79) gives

$$\begin{aligned} \|e_{i+1}(N)\| \leq & \sum_{k=1}^{M} \rho_k \|e_{i-k+1}(N)\| + \|CG\|\varepsilon_1 \\ & + \|C\|\varepsilon_2 + \varepsilon_3. \end{aligned} \qquad (1.83)$$

Comparing with the condition in Lemma 1.2.1, we have the analogy if defining $a_i = \|e_i(N)\|$ and $\varepsilon = \|CG\|\varepsilon_1 + \|C\|\varepsilon_2 + \varepsilon_3$. By virtue of the condition (1.80),

$$\lim_{i\to\infty} e_i(N) \leq \varepsilon/(1-\rho) \stackrel{\triangle}{=} \varepsilon^*. \qquad (1.84)$$

■

Remark 1.3.3 *If the uncertainty* $w(t)$ *is repeatable from run to run, their effects on the ILC can be completely rejected according to Theorem 1.3.1. The high order terms* $w_1(t)$ *and* $w_2(t)$ *in the linearized model (1.68) can be regarded as repetitive ones when a good initial control is used.*

Remark 1.3.4 *The boundedness of trajectories during each run is guaranteed theoretically because only a finite time interval is concerned in every run. However, in practice, it may happen that the tracking errors in between sampling points are large while maintaining a good pointwise tracking. In such cases, properly choosing* $\Xi^{(0)}$ *and* $\Phi(t)$ *is essential. Fully utilizing the known information about the system may be helpful as shown in the following illustrate example.*

1.3.4 Illustrative Example

1.3.4.1 RTPCVD Model. In the following, the terminal iterative learning control scheme developed above is applied to RTPCVD thickness control problem in the wafer fab. process. To simulate the actual situation, an RTPCVD model with the quartz window effect is considered:

$$\begin{cases} \frac{\mathrm{d}T_w}{\mathrm{d}t} = [\sigma A_w E_w (T_q^4 - T_w^4) + f E_w QP]/M_w \\ \frac{\mathrm{d}T_q}{\mathrm{d}t} = [QP + hA_q(T_{amb} - T_q)]/M_w \\ \frac{\mathrm{d}S}{\mathrm{d}t} = k_0 exp(-\frac{\gamma}{RT_w}) \end{cases} \qquad (1.85)$$

Table 1.2 Parameters for an RTPCVD system

symbol	value	meaning	unit
A_q		quartz window area	cm^2
A_w	400	wafer area	cm^2
E_w	0.8	wafer emissivity	unitless
f	0.5	lamp power absorbed by wafer	unitless
h		heat transfer coefficient for forced convection	$cal/cm^2/s/\ ^\circ K$
P	$\in [0,1]$	lamp power control factor	unitless
Q	1076	lamp power constant	cal/s
R	1.9872	gas constant	cal/(gmol$\cdot$ °K)
S		polysilicon deposition thickness	μm
T_w		wafer temperature	°K
T_q		quartz window temperature	°K
T_{amb}		ambient temperature	°K
M_w	1	wafer thermal mass	cal/ °K
M_q	100	quartz window thermal mass	cal/ °K
k_0	591000	pre-exponential constant of polysilicon deposition	μm/s
γ	39200	activation energy of polysilicon deposition	cal/gmol
σ	1.356×10^{-12}	Boltzmann constant	$cal/(s \cdot cm^{2} \cdot\ ^\circ K^{2})$
		$hA_q \stackrel{\triangle}{=} 1.84$ cal/s/ °K	

where the related parameters are listed in Table 1.2. In ILC design, however, we assume that only the simplified model (1.67) is available. The initial states are $T_w(0) = T_q(0) = 300$ °K , $S(0) = 0\mu$m and $N = 220$s. The desired final DT (deposition thickness) is $S(T) = 0.5\mu$m. The objective is to find the lamp power profile $P(t)$. A well pre-designed wafer temperature profile [23] can be used. Such a wafer temperature profile will give a final DT of 0.5 μm. However, online measurements of $T_w(t)$ are not available for the closed loop control. To overcome the difficulty in $T_w(t)$ measurement we apply the proposed terminal ILC method which only requires the terminal DT measurement.

1.3.4.2 Terminal ILC. To make the problem tractable, control function $P(t)$ is parameterized by

$$P(t) = \Phi(t)\Xi = \sum_{j=1}^{p} \phi_j(t)\xi_j \tag{1.86}$$

where $\phi_j(t)(j = 1, \cdots, p)$ are known basis functions and $\xi_j(j = 1, \cdots, p)$ are control parameters to be determined through iterative learning. The first-order updating law for Ξ is that

$$\Xi_i = \Xi_{i-1} + L_1 e_{i-1}(N) \tag{1.87}$$

where i is the iteration number and L_1 is the learning gain matrix to be designed. The design is based on the learning convergence condition discussed in Theorem 1.3.1 and the *a priori* knowledge about the system as shown in the following.

It should be pointed out that a good choice of $\Phi(t)$ and Ξ_0 is important to the above scheme. Although the actual RTPCVD process is complex, the simplified model (1.67) is still useful in choosing a good $\Phi(t)$, Ξ_0 and L. As the $S(t)$ is a monotonically increasing function, one may approximate it as a quadratic polynomial or an exponential function with two unknowns which can be determined by two known conditions $S(T)$ and $T_w(0)$. Hence, with this known $S(t)$, one may get $T_w(t)$ and $P(t)$ which in turn can be used to determine $\Phi(t)$ and Ξ_0. For example, setting

$$S(t) = \alpha(e^{\beta t} - 1), \tag{1.88}$$

one can calculate α, β by first equating

$$\begin{cases} \alpha(e^{\beta T} - 1) = S_d(T), \\ \mathrm{d}S(t)/\mathrm{d}t \mid_{t=0} = \alpha\beta = k_0 exp(-\gamma/[RT_w(0)]), \end{cases} \tag{1.89}$$

and solving (1.89) through numerical iterations. Immediately, one can get from the second equation of (1.67) that

$$T_w(t) = 1/(\alpha' + \beta' t) \tag{1.90}$$

where $\alpha' = -R[\ln(\alpha\beta/k_0)]/\gamma$ and $\beta' = -R\beta/\gamma$. By substituting $T_w(t)$ into the first equation of (1.67), the control can be written in the following form:

$$P(t) = c_0 + c_1/(\alpha' + \beta' t)^2 + c_2/(\alpha' + \beta' t)^4 \tag{1.91}$$

where c_0, c_1, c_2 are known constants based on (1.67) and Table 1.2. In the following, the triple $(S(t), T(t), P(t))$ in (1.88), (1.90) and (1.91) respectively, are chosen as the approximated equilibrium $(T_w^*(t), S^*(t), P^*(t))$.

The above simple derivation shows that a proper choice of $\Phi(t)$, Ξ_0 can be made based on the simplified model.

From the above discussions, the lamp power profile $P(t)$ is parameterized according to (1.91) where $c_0 = -0.0082$; $c_1 = 2.8 \times 10^{-8}$; $c_2 = 1.01 \times 10^{-12}$; $\alpha' = 0.0033$; $\beta' = -1.202 \times 10^{-5}$ based on Table 1.2. Now we can set $\Phi(t) = [1, 1/(\alpha' + \beta' t)^2, 1/(\alpha' + \beta' t)^4]$ as the basis function and set $[c_0, c_1, c_2]'$ as the initial Ξ_0. Taking $P^{(0)}(t) = \Phi(t)\Xi_0$ as a nominal control and then linearizing the plant around the approximated equilibrium $(T_w^*(t), S^*(t), P^*(t))$, we can get a discrete-time model (1.69) similar to (1.70) which enables us to choose the learning gains L_k by applying Theorem 1.3.1. For convenience, we use $L_1 = K\Xi_0$ where K is a constant. Based on the convergence analysis in Sec. 1.3.3, we know that $K > 0$.

In practice, one may use some tuning method to schedule different K's in different runs which is similar to the PID tuning in the iteration number i-axis. As the first-order ILC updating law is in fact a pure integral controller along the i-axis, higher order ILC scheme may be used which results in a PI or PID controller in the i-axis. To illustrate this, a second order scheme is used and L_2 is simply set to be $10\% L_1$. Improved results can be clearly observed in Fig. 1.6 for both cases: $K = 20$ and $K = 5$. It is interesting to note that, when $K = 20$, the oscillation has been improved a lot by using a high-order updating law.

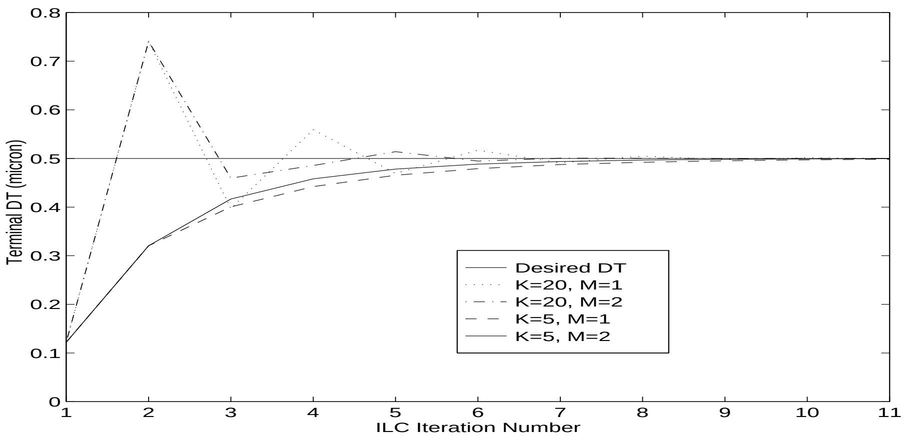

Figure 1.6 Convergence comparison of terminal DT tracking by high-order iterative learning

Larger K may result in unacceptable oscillations while smaller K slows down the convergence as shown in Fig. 1.6. An "optimal" K may exist. For example, when K is set to 10, the terminal iterative learning control gives a perfect tracking after 2 to 3 iterations as shown in Fig. 1.7. However, the "optimal" K is not achievable in practice because the full accurate knowledge about the model is not available. On the other hand, by using the idea of iterative learning, a perfect DT control can still be achieved after several repetitions even when a rough K is used. Furthermore, in Figs.1.6-1.7, the uncertainties have actually taken into account in the simulation study because a simplified model is used to design the learning gain where the high-order terms are regarded as the modelling uncertainty. It is shown from Figs.1.6-1.7 that the proposed terminal iterative learning control scheme is robust to this uncertainty. An additional case is considered where the perturbation $v_i(N)$ is a uniformly distributed random noise over [0,1]. In this case, the situation is the same as that of Fig. 1.6. The simulation results are shown in Fig. 1.8 where it is clearly observed that the tracking error bound is proportional to 0.025. This is in accordance with the result of Theorem 1.3.1. As the RTPCVD system is repetitively operated, the terminal ILC scheme demonstrated above is practically effective.

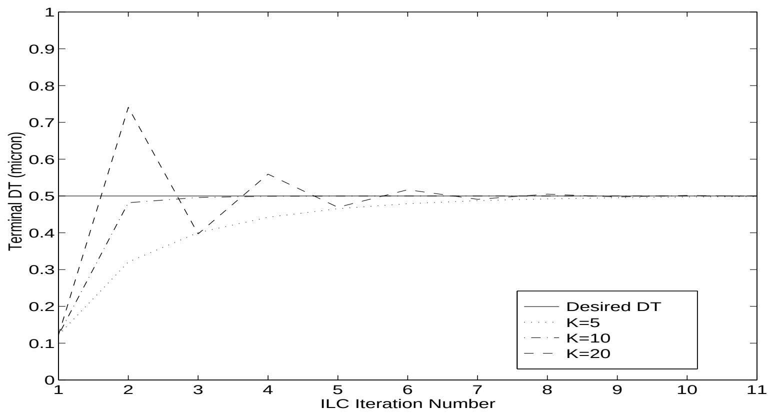

Figure 1.7 Convergence comparison of terminal DT tracking by different values of K.

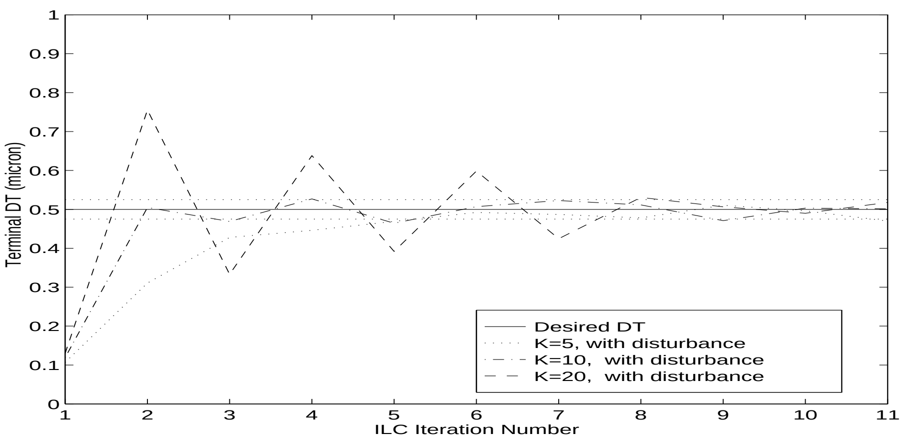

Figure 1.8 Convergence comparison of terminal DT tracking by different values of K with DT measurement errors.

1.3.5 Summary

The terminal deposition thickness control for rapid thermal process chemical vapor deposition is considered. A terminal iterative learning control scheme with a high-order updating law is proposed. The lamp power control profile is parameterized and the parameters are to be updated by using the terminal DT measurement only at the end of each run. A convergence condition is obtained for uncertain discrete-time time-varying linear systems. Simulation studies on a simplified RTPCVD model show that the desired DT can be achieved by the proposed terminal output learning scheme in a few runs. It is also illustrated that a high order scheme may provide improved convergence performance.

1.4 CONCLUSIONS

We have presented in this chapter two effective iterative learning control schemes suitable for uncertain discrete-time systems. Firstly, a high-order ILC updating law is proposed to include tracking errors in more than one previous iterations. an explicit feedback-feedforward configuration for a class of uncertain discrete-time nonlinear systems is considered where the feedback controller is in general form. It is shown that both the feedback controller and the high-order scheme are effective in speeding up learning convergence. Secondly, due to the insufficient measurement capability in many real control problems such as Rapid Thermal Processing (RTP), it may happen that only the terminal output tracking error instead of the whole output trajectory tracking error is available. The control profile for the next operation cycle has to be updated using the terminal tracking error alone. By parameterizing the control profile with a piecewise continuous functional basis, the control parameters are updated by a high-order updating scheme. A convergence condition is obtained for a class of uncertain discrete-time time-varying linear systems including the RTPCVD system as the subset.

It has been clearly shown in this chapter that iterative learning control as an engineering knowledge learning technique, does provide a systematic and an effective way of handling discrete-time system control problems when only partial process modelling or measurement knowledge is available.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] E. Rogers and D. H. Owens, *Stability Analysis for Linear Repetitive Processes*, Springer-Verlag, Berlin, 1992.

[3] Zeungnam Bien and Jian-Xin Xu, *Iterative Learning Control - Analysis, Design, Integration and Applications*, Kluwer Academic Publishers, 1998.

[4] Yangquan Chen, "Dedicated web server for Iterative Learning Control research," `http://ilc.ee.nus.edu.sg/`, Oct. 1998.

[5] K. L. Moore, "Iterative learning control - an expository overview," *Applied & Computational Controls, Signal Processing, and Circuits*, Available at `http://132.205.2.222:888/~yqchen/ILC/Ilcrep.zip.gz`, pp. 1–42, 1998.

[6] Yangquan Chen, *High-order iterative learning control: convergence, robustness and applications*, Ph.D. thesis, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Dec. 1997.

[7] Z. Geng, R. L. Carroll, and J. Xie, "Two-dimensional model algorithm analysis for a class of iterative learning control systems," *International Journal of Control*, vol. 52, pp. 833–862, 1990.

[8] J. E. Kurek and M. B. Zaremba, "Iterative learning control systhesis based on 2-D system theory," *IEEE Trans. of Automatic Control*, vol. 38, no. 1, pp. 121–125, 1993.

[9] S. S. Saab, "A discrete-time learning control algorithm for a class of linear time-invariant systems," *IEEE Trans. of Automatic Control*, vol. 40, no. 6, pp. 1138–1141, 1995.

[10] D.-H. Hwang, Z. Bien, and S.-R. Oh, "Iterative learning control method for discrete-time dynamic systems," *IEE Proceedings, Part-D, Control Theory and Applications*, vol. 138, no. 2, pp. 139–144, 1991.

[11] T. Ishihara, K. Abe, and T. Takeda, "A discrete-time design of robust iterative learning controller," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 22, no. 1, pp. 74–84, 1992.

[12] T. Suzuki, M. Yasue, S. Okuma, and Y. Uchikawa, "Discrete-time learning control for robotic manipulators," *Advanced Robotics*, vol. 9, no. 1, pp. 1–14, Jan. 1995.

[13] S. K. Tso and L. Y. X. Ma, "Discrete learning control for robots: strategy, convergence and robustness," *International Journal of Control*, vol. 57, no. 2, pp. 273–291, 1993.

[14] S. Arimoto, "Robustness of learning control for robot manipulators," in *Proc. of the 1990 IEEE Int. Conf. on Robotics and Automation*, Cincinnati, Ohio, USA, 1990, pp. 1528–1533.

[15] Z. Bien and K. M. Huh, "High-order iterative learning control algorithm," *IEE Proceedings, Part-D, Control Theory and Applications*, vol. 136, no. 3, pp. 105–112, 1989.

[16] Y. Chen, Z. Gong, and C. Wen, "Analysis of a high order iterative learning control algorithm for uncertain nonlinear systems with state delays," *Automatica*, vol. 34, no. 3, pp.345-353. March 1998.

[17] T.-J. Jang, C.-H. Choi, and H.-S. Ahn, "Iterative learning control in feedback systems," *Automatica*, vol. 31, no. 2, pp. 243–245, 1995.

[18] S. S. Saab, "Discrete-time learning control algorithm for a class of nonlinear systems," in *Proc. of American Control Conference*, Seattle, Washington, USA, June 1995, pp. 2739–2743.

[19] Y. Chen, M. Sun, B. Huang, and H. Dou, "Robust higher order repetitive learning control algorithm for tracking control of delayed repetitive systems," in *Proc. of the 31st IEEE Conf. on Decision and Control*, Tucson, Arizona, USA, Dec. 1992, pp. 2504–2510.

[20] K. H. Lee and Z. Bien, "Application of iterative learning controller with feedback loop to the focusing control system of the MODD," in *Proc. of the Asian Control Conference*, Tokyo, Japan, Jul. 1994, pp. 243–246.

[21] J.-X. Xu, "Analysis of iterative learning control for a class of nonlinear discrete-time systems," *Automatica*, Vol.33, no.10, pp.1905-1907.

[22] F. Roozeboom and N. Parekh, "Rapid thermal processing systems: A review with emphasis on temperature control," *J. Vac. Sci. Technol. B*, vol. 8, no. 6, pp. 1249–1259, November 1990.

[23] Y. Chen, J.-X. Xu, T.H. Lee, and S. Yamamoto, "An iterative learning control in rapid thermal processing," in *Proc. the IASTED Int. Conf. on Modeling, Simulation and Optimization (MSO'97)*, Singapore, Aug. 1997, pp. 189–92.

[24] T.A. Badgwell, T. Breedijk, S.G. Bushman, S.W. Butler, S. Chatterjee, T.F. Edgar, A.J. Toprac, and I. Trachtenberg, "Modeling and control of microelectronics materials processing," *Computers in Chemical Engineering*, vol. 19, pp. 1–40, 1995.

[25] P.K. Mozumder, S. Saxena, and David J. Collins, "A monitor wafer based controller for semiconductor processes," *IEEE Trans. on Semiconductor Manufacturing*, vol. 19, pp. 400–411, 1994.

[26] Evanghelos Zafiriou, R.A. Adomaitis, and G. Gattu, "An approach to run-to-run control for rapid thermal processing," in *Proc. of American Control Conf.*, Seattle, WA, USA, 1995, pp. 1286–1288.

[27] K. L. Moore, *Iterative learning control for deterministic systems*, Advances in Industrial Control. Springer-Verlag, 1993.

[28] D. Peyton, H. Kinoshita, G.Q. Lo, and D.L. Kwong, "Systems oriented survey of noncontact temperature measurement techniques for rapid thermal processing," in *Rapid Thermal and Related Processing Techniques, Vol.1393*, October 1990, pp. 259–308.

[29] S. P. Bhat and D. K. Miu, "Point-to-point positioning of flexible structures using a time domain LQ smoothness constraint," *ASME: J. of Dynamic Systems, Measurement, and Control*, vol. 114, pp. 416–421, 1992.

[30] G. Oriolo, S. Panzieri, and G. Ulivi, "Cyclic learning control of chain-form systems with applications to car-like robots," in *Proc. of 13-th Triennial World Congress of IFAC*, San Francisco, USA, 1996, pp. 187–192.

[31] P. Lucibello, "State steering by learning for a class of nonlinear control systems," *Automatica*, vol. 30, no. 9, pp. 1463–1468, Sept. 1994.

[32] R. S. Gyurcsik, T. J. Riley, and F. Y. Sorrell, "A model for rapid thermal processing: achieving uniformity through lamp control," *IEEE Trans. on Semiconductor Manufacturing*, vol. 6, no. 4, pp. 9–13, 1991.

[33] Y. Chen, C. Wen, Z. Gong, and M. Sun, "Drag coefficient curve identification of projectiles from flight tests via optimal dynamic fitting," *IFAC J. of Control Engineering Practice*, vol. 5, no. 5, pp. 627–636, May 1997.

[34] S. P. Bhat and D. K. Miu, "Solutions to point-to-point control problem using laplace transform technique," *ASME: J. of Dynamic Systems, Measurement, and Control*, vol. 113, pp. 425–431, 1991.

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