Iterative Learning Control Via Weighted Local-Symmetrical-Integration

Yangquan Chen+, Huifang Dou* and Kok Kiong Tan*
+ Center for Self-Organizing and Intelligent Systems (CSOIS)
Dept. of Electrical and Computer Engineering
Utah State University, UT 84322-4160, USA
*Mechatronics and Automation Laboratory
Dept. of Electrical Engineering, National University of Singapore,
10 Kent Ridge Crescent, Singapore 119260.

Abstract—

A new iterative learning control (ILC) updating law is proposed for tracking control of continuous linear system over a finite time interval. The ILC is applied as a feedforward controller to the existing feedback controller. By using the weighted local symmetrical integral (WLSI) of feedback control signal of previous iteration, the ILC updating law takes a simple form with only two design parameters: the learning gain and the range of local integration. Convergence analysis is presented together with a design procedure. A set of experimental results are presented to illustrate the effectiveness of the proposed WLSI-ILC scheme.

Keywords— **Iterative learning control; weighted local symmetrical integration; convergence analysis; controller design and tuning.**

I. Introduction

Since early eighties, increasing attention to 'Iterative Learning Control' (ILC) [1], [2] has been drawn from the control community. Most of the existing work focused on the *analysis* issue of ILC schemes. However, the obtained convergence condition is not enough for actual ILC applications. Therefore, in recent years, increasing efforts have been made on the *design* issue of ILC [3], [4]. A recent survey on ILC *design* issue [5] documented various practically tested design schemes. To draw the attention from the industry, it is desirable to design the ILC in a way similar to the successful use of PID (proportional-integral-derivative) controllers in industries.

This paper attempts to provide a PID-autotuning type ILC design procedure with *less* modeling efforts. A new iterative learning control (ILC) updating law is proposed for tracking control of continuous linear system over a finite time interval. By using the weighted local symmetrical integral (WLSI) of feedback control signal of the previous iteration, the ILC updating law takes a simple form with only two design parameters: the learning gain and the range of local integration. Convergence analysis is presented together with a design procedure.

Corresponding author: Dr YangQuan Chen. Tel. +1-435-797-0148. Fax: +1-435-797-2003. E-mail: yqchen@ieee.org. URL: http://www.crosswinds.net/~yqchen

II. WLSI-type ILC

The feedforward-feedback configuration of ILC algorithms has already been a standard consideration in either '*analysis*' or '*design*' [5] work on ILC. ILC, originally proposed as an open-loop control [1], has been considered as a feedforward control in addition to an existing feedback controller. A block-diagram is shown in Fig. 1 where **FBC** stands for "feedback controller" and y_d is the given desired output trajectory to be tracked. After the i-th iteration (repetitive operation), the feedforward control signal u_{ff}^i and the feedback control signal u_{fb}^i are to be stored in the memory bank for constructing the feedforward control signal at the next iteration, i.e., u_{ff}^{i+1}. The stored feedback control signal u_{fb}^i are to be multiplied with a weighting function $h(\tau)$, locally symmetrically integrated (WLSI) and then multiplied by a learning gain γ.

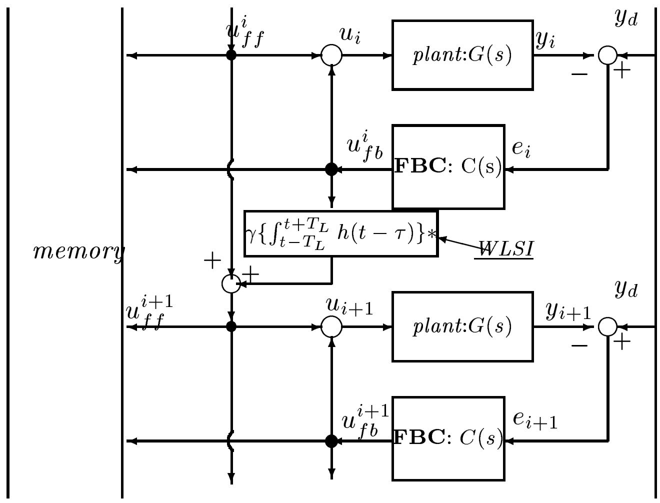

Fig. 1. Block Diagram of Weighted Local Symmetrical Double-Integral-type (WLSI) Iterative Learning Control

According to Fig. 1, the ILC updating law is hence written by

$$u_{ff}^{i+1}(t) = u_{ff}^i(t) + \gamma \int_{t-T_L}^{t+T_L} h(t-\tau) u_{fb}^i(\tau) \mathrm{d}\tau \qquad (1)$$

where γ is the learning gain, $h(\cdot)$ is the local weighting function and T_L is the width of local integration. $h(\cdot)$ should be chosen according to two properties, i.e., a) *locality and symmetry:*

$$\left\{ \begin{array}{lcl} h(t) & = & h(-t), \quad \forall t; \\ h(t) & = & 0, \quad t \notin [-T_L, T_L]; \end{array} \right. \tag{2}$$

and b) *normalization:* $\int_{-T_L}^{T_L} h(\tau)\mathrm{d}\tau = 1$. The overall control is simply that

$$u_{i+1}(k) = u_{ff}^{i+1}(k) + u_{fb}^{i+1}(k) \tag{3}$$

as shown in Fig. 1 where two parameters - γ the learning gain and T_L the width of the WLSI, are to be designed and specified. Here, two examples of the local weighting function are presented for illustration shown in Fig. 2. In Fig. 2,

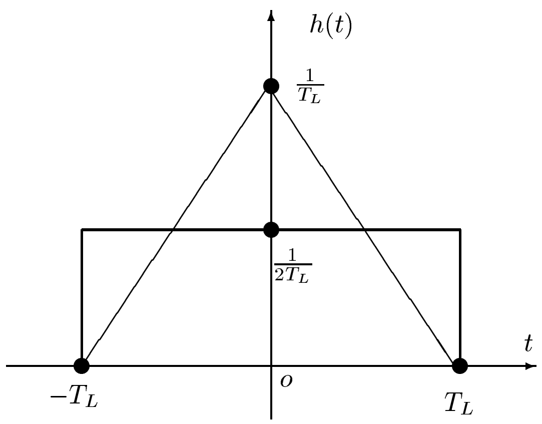

Fig. 2. Two simple local weighting functions (rectangular: $h_1(t)$; triangular: $h_2(t)$)

the rectangular one is denoted by $h_1(t)$ and the triangular $h_2(t)$. They clearly satisfy the above two properties set for general form of $h(t)$.

In practice, the WLSI-ILC is to be implemented digitally. Consider that the sampling period is T_s. Let $T_L/T_s = M$. Using the simple rectangular formula of the numerical quadrature, for $h(t) = h_1(t)$ shown in Fig. 2, one obtains

$$u_{ff}^{i+1}(k) = u_{ff}^{i}(k) + \frac{\gamma}{2M+1}\sum_{j=-M}^{M} u_{fb}^{i}(k+j) \tag{4}$$

which is the case considered in [6]. For $h(t) = h_2(t)$, similarly simple updating formula as (4) can be obtained.

Here, our control task is to track the given desired output trajectory $y_d(t)$ over a fixed time interval $[0, T]$ as closely as possible. With an existing feedback controller $C(s)$, the main objective of this paper is to use a learning feed forward controller (LFFC) given by ILC updating law (1) to achieve a better tracking performance.

III. Convergence Analysis

The convergence of the proposed learning controller is in the sense that u_{ff}^i approaches to a fixed point signal as i increases and meanwhile, $y_i(t) \to y_d(t)$. This is summarized in the following theorem.

Theorem III.1: A linear system shown in Fig. 1 is controlled by a suitable feedback controller which performs a given task repeatedly. A weighted local-symmetrical-integral-type ILC scheme (1) is applied as a learning feedforward controller (LFFC). There exists a real constant γ and a positive $T_L (0 < T_L < T)$ such that the learning process is convergent and furthermore,

$$\lim_{i\to\infty} U_{ff}^i(s) \to Y_d(s)/G(s). \tag{5}$$

where $U_{ff}^i(s) = \mathcal{L}[u_{ff}^i(t)]$ and $Y_d(s) = \mathcal{L}[y_d(t)]$. The convergence rate is given by

$$\rho(\omega, \gamma, T_L) \stackrel{\triangle}{=} \mid 1 - \gamma H(\omega, T_L) G_c(j\omega) \mid < 1, \tag{6}$$

where $G_c(s)$ is the closed loop transfer function and $G_c(s) = C(s)G(s)/(1 + C(s)G(s))$.

Theorem III.1 implies that the iterative learning controller is essentially applied to inverse the plant to be controlled in an iterative manner. Since linear system is considered in this paper, in the sequel, frequency domain notion is used. Using Laplace transformation and its convolution theorem, the updating law (1) becomes

$$U_{ff}^{i+1}(s) = U_{ff}^{i}(s) + \gamma H(\omega, T_L) U_{fb}^{i}(s) \tag{7}$$

where $U_{fb}^i(s) = \mathcal{L}[u_{fb}^i(t)]$, $s = j\omega$ and

$$H(\omega, T_L) = \int_{-T_L}^{T_L} h(t)e^{-st}\mathrm{d}t. \tag{8}$$

Due to its symmetry, $H(\omega, T_L)$ is with zero phase.

Now we proceed to present a proof of Theorem III.1.

Proof: From Fig. 1, the feedback signal can be written as

$$U_{fb}(s) = -G_c(s)U_{ff}(s) + G_c(s)Y_d(s)/G(s). \tag{9}$$

Learning updating law (7) becomes,

$$\begin{array}{rcl} U_{ff}^{i+1}(s) & = & [1 - \gamma H(\omega, T_L)G_c(s)]U_{ff}^{i}(s) \\ & & + \gamma H(\omega, T_L)G_c(s)Y_d(s)/G(s). \end{array} \tag{10}$$

Iterating (10), one obtains

$$\begin{array}{rcl} U_{ff}^{i+1}(s) & = & [1 - \gamma H(\omega, T_L)G_c(s)]^i U_{ff}^{0}(s) \\ & & + \{1 - [1 - \gamma H(\omega, T_L)G_c(s)]^{i+1}\}Y_d(s)/G(s). \end{array} \tag{11}$$

Since $H(\omega, T_L)$ and $G_c(s)$ are essentially with a low pass filter characteristics, it is clearly possible to choose a suitable γ such that (6) is true. In addition, T_L can be used to shape $\rho(\omega, \gamma, T_L)$ which is the convergence rate in (11). Therefore, there exists a design of γ and T_L such that (6) holds and from (11) $\lim_{i\to\infty} U_{ff}^i(s) \to Y_d(s)/G(s)$ and moreover, $y_i(t) \to y_d(t)$ for all $t \in [0, T]$ as $i \to \infty$. ■

Remark III.1: The convergence condition (6) is similar to the one obtained in [7] or [8] which is in the form of

$$\mid 1 - L(j\omega)G(j\omega) \mid < 1, \tag{12}$$

where $G(j\omega)$ is the open-loop transfer function of the plant to be controlled and $L(j\omega)$ is the learning operator such that $U_{k+1}(j\omega) = U_k(j\omega) + L(j\omega)E_k(j\omega)$ where $E_k(j\omega)$ is the tracking error at the k-th iteration. However, it is harder to find a realizable $L(j\omega)$ for all ω that satisfies (12). Note that in (6), the closed-loop transfer function $G_c(j\omega)$, instead of $G(j\omega)$, is involved which is normally close to 1 at low frequencies and damps rapidly as the frequency increases. Therefore, it is relatively easier to find a suitable $H(\cdot,\cdot)$ and a λ to satisfy (6). The proposed WLSI-ILC is hence less model dependant as shown in our experimental studies.

IV. Design Issues

A. Prior Knowledge

In [1], the range of the first Markov parameter should be known *a priori*. This is an unusual and nonconventional requirement. Moreover, the derivatives of output tracking error are prone to noise amplification. As argued in [5], when faced to an actual system, one cannot assume zero knowledge available. In most engineering practice, it is quite common that part of the Nyquist curve information about the system is available. In this paper, the knowledge we used includes

- The frequency of the desired trajectory, which is less than a known frequency denoted by ω_d and $\omega_d < \omega_c$. ω_c is the systems cut-off frequency;
- An estimate of $G_c(j\omega_c)$ or $G_c(j\omega_d)$.

Clearly, the above knowledge is minimal for controller design. In addition, the local weighting function $h(t)$ for a given T_L should be decided beforehand. Note that here the full information of the Nyquist curve is not necessarily known.

B. Design Method for T_L

A suitably chosen T_L is very important. Small T_L will bring in more high frequency signal components stored in the memory bank. These high frequency signal components may be accumulated due to different phase relationship from iteration to iteration. This is the major reason of the divergence for some ILC schemes which may be convergent at the initial iterations but as ILC runs, divergence can be observed in practical applications [5]. Meanwhile, a too large T_L deteriorates the signal's low frequency components when smoothing out the high frequency components.

A simple consideration is that, the signal's energy can not be attenuated by half via $H(\omega, T_L)$. One obtains

$$H(\omega_d, T_L) = \frac{\sqrt{2}}{2}. \tag{13}$$

For $h_1(t)$ and $h_2(t)$ shown in Fig. 2, by referring to (8), one gets

$$\begin{aligned} H_1(\omega, T_L) &= \frac{\sin(\omega T_L)}{\omega T_L}; & (14)\\ H_2(\omega, T_L) &= \frac{2(1-\cos(\omega T_L))}{\omega^2 T_L^2} = \{\frac{\sin(\omega T_L/2)}{\omega T_L/2}\}^2 \\ &= H_1^2(\omega, T_L/2). & (15)\end{aligned}$$

Using $H_1(\omega, T_L)$ and applying (13) will give an estimate of T_L by

$$\omega_d T_L \approx 1.392 \tag{16}$$

using `solve('sin(x)/x =sqrt(2)/2')` of MATLAB Symbolic Math Toolbox. Therefore,

$$T_L \geq \frac{1.392}{\omega_d}. \tag{17}$$

Similarly, when using $H_2(\omega, T_L)$,

$$\omega_d T_L \approx 2.004, \quad \text{and} \quad T_L \geq \frac{2.004}{\omega_d} \tag{18}$$

using `solve('2*(1-cos(x))/x^2 = sqrt(2)/2')`.

Remark IV.1: The learning scheme analyzed in [9] is actually the case when $T_L \to 0$ which caters for tracking desired trajectory with ultra high signal frequency, according to the discussion of this subsection. This is too stringent to be practically useful. Therefore, in practical use of learning control scheme like the one proposed in [9], a local symmetrical integration is required with a suitable T_L.

C. Design Method for γ

It is an open problem to get a reasonable estimate of the upper limit of γ with **less** modeling effort. However, during practice, one can always start with a smaller, conservative γ via which the learning process converges. The above discussions indicate that it is an easy task to make ILC work.

Full knowledge of $G_c(j\omega)$, $\omega \leq \omega_c$ may be sometimes impractical. Therefore, it is assumed that at least the value of $G_c(j\omega_c)$ is available. In what follows, it will be shown that $G_c(j\omega_c)$ can be used to design a reasonable γ. Let $G(j\omega) = A(\omega)e^{\theta(\omega)}$. Denote $\bar{\rho} = \rho^2(\omega, \gamma, T_L)$. Then, from (6),

$$\bar{\rho} = 1 - 2\gamma H(\omega, T_L)A(\omega)\cos\theta(\omega) + \gamma^2 H^2(\omega, T_L)A^2(\omega). \tag{19}$$

Clearly, γ should be chosen to minimize $\bar{\rho}$. From (19), the best γ should be

$$\gamma = \frac{\cos\theta(\omega)}{H(\omega, T_L)A(\omega)} \tag{20}$$

by setting $\frac{d\bar{\rho}}{d\gamma} = 0$. At frequency ω_c, $\gamma = \frac{\cos\theta(\omega_c)}{H(\omega_c, T_L)A(\omega_c)}$. It should be noted that γ may be negative at certain frequency range. For most applications, ω_d is quite small and in this case γ can be given approximately by $\gamma \leq \sqrt{2}$. When only $G_c(j\omega_d)$ is known, γ can be designed similarly according to (20). If the knowledge of $G_c(j\omega)$ within a frequency range $[\omega_L, \omega_H]$ is known. A plot of $\gamma(\omega)$ is available from (20).This plot is useful in selecting a suitable γ when different frequencies of interest are to be considered in $[\omega_L, \omega_H]$.

V. An Experimental Illustration

Our extensive studies on a PMLM (permanent magnet linear motor) platform were focused on ultra precision motion control with *less* or *modest* modeling effort using a feedback-feedforward structure as shown in Fig. 1. First, a PID feedback controller is designed using a relay automatic tuning method. An artificial delay is introduced to achieve self-induced controlled oscillations from which the PID controller can be automatically tuned. An iterative learning controller (ILC) is applied as a feedforward controller to the existing relay-tuned PID feedback controller to enhance the trajectories tracking performance by utilizing the experience gained from the repeated execution of the same operations. Detailed design steps and achieved results can be found in [10].

Here, a set of representative experiment results are presented to verify the theoretical results of this paper. For detailed experiment setup, again, refer to [10]. First, a cautious γ is chosen to be 0.1. As discussed in the above, it is always possible to make the learning control work. We use a rectangular integration formula for the WLSI and the weights are set to be equal for simple implementation. This is the case for rectangular $H(\cdot,\cdot)$ shown in Fig. 2. M is proportional to the length of the WLSI interval T_L. Different M's were used to test the ILC convergence. As shown in Fig. 3, when $M = 0$, it is not convergent for the reasons in Remark IV.1. However, the ILC convergence at several initial iterations can be observed. The similar situation happened when $M = 1, 2, 3$. When $M > 4$, the ILC exhibits good convergence property. When M increases to 20, no further significant improvement can be obtained. This verifies the ILC design formula presented in the above for T_L.

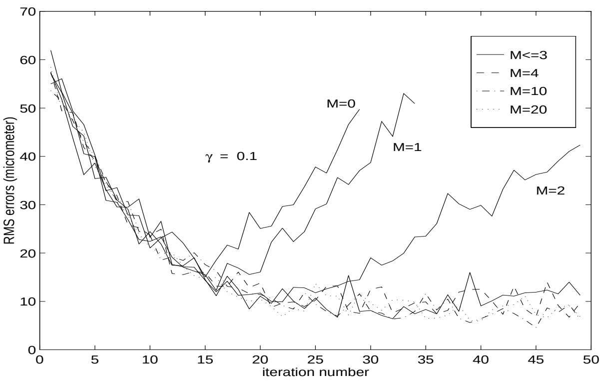

Fig. 3. Learning convergence comparison, fixed $\gamma = 0.1$ with different M (Tracking error RMS vs. ILC iteration number).

As verified in Fig. 3, a suitable M ($M = 10$) can be chosen and fixed. Now, different learning gains were used to test the ILC convergence. As shown in Fig. 4, when γ increases, ILC converges faster. However, sustainable ILC convergence (or long term stability of ILC [5]) will not be guaranteed as γ increases from 0.3 and beyond.

Similar results were obtained if a triangular $H(\cdot,\cdot)$, as shown in Fig. 2, is used. It should be noted again that the WLSI-ILC scheme proposed in this paper is *less* model dependant.

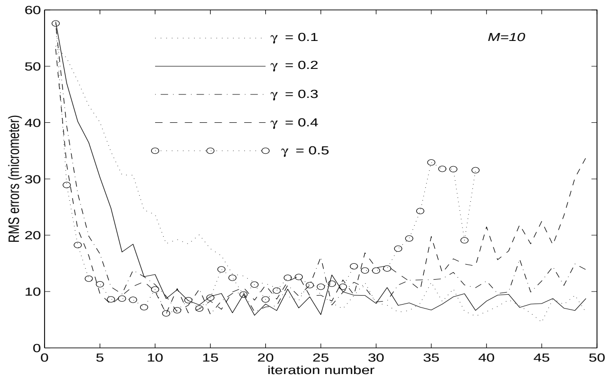

Fig. 4. Learning convergence comparison, fixed $M = 10$ with different γ (Tracking error RMS vs. ILC iteration number).

VI. Conclusions

A new iterative learning control (ILC) updating law is proposed for tracking control of continuous linear system over a finite time interval. The ILC is applied as a feedforward controller to the existing feedback controller. By using the weighted local symmetrical integral (WLSI) of feedback control signal of previous iteration, the ILC updating law takes a simple form with only two design parameters: the learning gain and the range of local integration. Convergence analysis is presented together with a design procedure. A set of experimental results are briefly presented to illustrate the effectiveness of the proposed WLSI-ILC scheme.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] K. L. Moore, "Iterative learning control - an expository overview," *Applied & Computational Controls, Signal Processing, and Circuits*, vol. 1, no. 1, pp. 151-214, 1999.

[3] Zeungnam Bien and Jian-Xin Xu, *Iterative Learning Control - Analysis, Design, Integration and Applications*, Kluwer Academic Publishers, 1998.

[4] Yangquan Chen, "Dedicated web server for Iterative Learning Control research," `http://cicserver.ee.nus.edu.sg/~ilc/`, Oct. 1998.

[5] Richard W. Longman, "Designing Iterative Learning and Repetitive Controllers". In Z. Bien and Xu J.-X. eds, "*Iterative Learning Control - Analysis, Design, Integration and Application*", Kluwer Academic Publishers, pp. 107–145, 1998.

[6] E. Burdet, L. Rey, and A. Codourey, "A trivial method of learning control," in *Preprints of the 5th IFAC Symposium on Robot Control, volume 2*, Nante, France, 1997, IFAC.

[7] A. De Luca, G. Paesano, and G. Ulivi, "A frequency-domain approach to learning control: implementation for a robot manipulator," *IEEE Trans. on Industrial Electronics*, vol. 39, no. 1, pp. 1–10, 1992.

[8] T. Kavli, "Frequency-domain synthesis of trajectory learning controllers for robot manipulators," *J. of Robotic Systems*, vol. 103, pp. 663–680, 1991.

[9] T.-Y. Kuc, J. S. Lee, and K. Nam, "An iterative learning control theory for a class of nonlinear dynamic systems," *Automatica*, vol. 28, no. 6, pp. 1215–1221, 1992.

[10] K. K. Tan, H. F. Dou, Y. Chen, and T. H. Lee, "High precision linear motor control via relay-tuned iterative learning based on zero-phase filtering," *IEEE Trans. on Control Systems Technology*, vol. 9, no. 1, pp. 1–8, 2001.