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Feedback Controller Design to Ensure Monotonic Convergence in Discrete-Time, P-Type Iterative Learning Control

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Abstract

In a previous paper we considered the equivalence between the necessary and sufficient condition for convergence and the sufficient condition for monotonic convergence in discrete-time, P-type iterative learning control. Specifically, requirements on the discrete-time, linear, time-invariant plant were given so that convergence of the learning algorithm implies monotonic convergence. In this note we consider the design of current cycle feedback controllers for the plant so that these requirements are met, thereby ensuring that any P-type learning control algorithm that converges will also converge monotonically.

1 Introduction

Consider a discrete-time, linear, time-invariant system of relative degree one:

$$Y_k(z) = H(z)U_k(z) = (h_1 z^{-1} + h_2 z^{-2} + \cdots)U_k(z), \quad (1)$$

where z^{-1} is the standard delay operator in time, $Y_k(z)$ and $U_k(z)$ are the z-transforms of the system's output and input sequences, $y_k(t)$ and $u_k(t)$, respectively, "t" is the time index and satisfies $t \in [0, N]$, and "k"denotes the repetition index. The variables h_i are the standard Markov parameters of the system $H(z)$. We assume $y_k(0) = y_d(0) = y_0$ for all k, where $y_d(t)$ is the desired output. As is common, we also assume the plant to be controlled is stable. If not, a suitable current cycle feedback controller should first be designed and the ILC algorithm would be applied to the closed-loop system. Now we introduce what we call the Arimoto P-type ILC algorithm [1]:

$$u_{k+1}(t) = u_k(t) + \gamma e_k(t+1), \quad (2)$$

where $e_k(t) = y_d(t) - y_k(t)$ is the error and γ is the learning gain (we call this P-type because the derivative of the error $e_k(t)$ does not explicitly appear).

In this paper we consider some aspects of the monotonic convergence of the ILC system formed by the combination of Equations 1 and 2 (hereafter referred to as the "ILC scheme"). Specifically, we show how to design a conventional feedback controller for the system, if possible, so that any convergent P-type ILC law is guaranteed to be monotonically convergent. The paper begins with a discussion about the convergence of the ILC scheme. We then present two feedback controller design approaches for ensuring monotonic convergence. The first results in a finite impulse response (FIR) controller and the second gives an infinite impulse response (IIR) controller. Both produce deadbeat responses in the time domain. We include examples to illustrate the ideas.

2 ILC Convergence

It is well-known that the combination of Equation 1 with Equation 2 converges in a given norm topology if the induced operator norm satisfies

$$\|I - \gamma H\|_i < 1.$$

Of course, this sufficient condition ensures monotonic convergence. However, it is also well-known that

$$|1 - \gamma h_1| < 1$$

provides a necessary and sufficient condition for convergence [2].

Unfortunately, this second condition does not guarantee monotonic convergence [3, 4]. Beginning in [5] and finishing in [6], we proved the following equivalence between these two conditions in the infinity-norm topology:

Theorem 1 For the ILC scheme, if γ is chosen so that $|1 - \gamma h_1| < 1$, then $\|I - \gamma H\|_1 < 1$ if

$$|h_1| > \sum_{j=2}^{N} |h_j|. \quad (3)$$

This result was illustrated through examples in [5]. For instance, the system

$$Y_k(z) = \frac{z - 0.9}{z^2 + 0.2z - 0.125} U_k(z), \tag{4}$$

operating on the interval $[1, 33]$, with the ILC update law

$$u_{k+1}(t) = u_k(t) + 0.9 e_k(t+1)$$

and the desired output signal shown in Figure 1, results in the plot of $\|e_k(t)\|_2$ versus iteration number k shown in Figure 2. For this system, $h_1 = 1$ and $\gamma = 0.9$, so the necessary and sufficient condition for convergence is satisfied. But,

$$|h_1| = 1 < \sum_{j=2}^{N} |h_j| = 1.4127.$$

Hence, we expect that although the ILC scheme will converge, it will not necessarily do so monotonically. Indeed, as seen in Figure 2, the norm of the error first converges, but then grows before finally converging to zero.

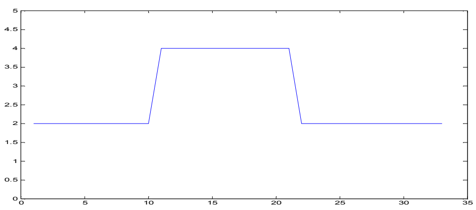

Figure 1: Desired output signal.

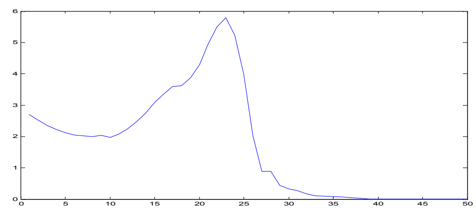

Figure 2: $\|e_k(t)\|_2$ (y-axis) versus iteration number k (x-axis) for uncontrolled system.

3 Monotonic Convergence via Feedback Controller Design

A given plant may not satisfy the sufficient condition needed to ensure that ILC convergence implies monotonic convergence. However, just as one may often design a so-called "current cycle" feedback controller for an unstable system before using ILC, one can also design a current cycle feedback controller to condition the plant so that Theorem 1 is satisfied for the closed-loop system. One can then apply the ILC scheme to the closed-loop system and assured of monotonic convergence.

We will present two different methodologies for designing a current cycle feedback controller to condition a system to ensure that Theorem 1 is satisfied. The first will specify the coefficients of a FIR controller. The second will design an IIR controller. In addition to these two approaches, however, there is an additional consideration that must be addressed. This concerns where the ILC signal is injected into the control loop. There are two possibilities, as shown in Figure 3. In Figure 3(a) we see the ILC signal added to the feedback signal. The reference signal into the closed-loop system is $y_d(t)$. Notice that in this case the effective plant seen by the ILC algorithm is

$$H_{cl}^A = \frac{H(z)}{1 + C(z)H(z)}.$$

We will call this Case A. Figure 3(b) shows the other possibility. In this case the ILC signal updates the reference signal used to drive the closed-loop system and the effective plant seen by the ILC algorithm is given by

$$H_{cl}^B = \frac{C(z)H(z)}{1 + C(z)H(z)}.$$

We will call this Case B. The difference between these two cases is that in Case B the zeros of both the controller and the plant influence the zero dynamics of the system acted on by the ILC algorithm, whereas in Case A only the zeros of the plant play a role.

4 FIR Controller Design

Considering Figure 3, let

$$\begin{aligned} H(z) &= h_1 z^{-1} + h_2 z^{-2} + \cdots, \\ C(z) &= c_0 + c_1 z^{-1} + c_2 z^{-2} + \cdots, \end{aligned}$$

and define $M(z) = C(z)H(z)$, so that for

$$M(z) = m_1 z^{-1} + m_2 z^{-2} + \cdots.$$

Then we can write

$$m_i = \sum_{j=1}^{i} h_j c_{i-j}.$$

Notice that $C(z)$ has relative degree zero. Also, because we are operating on a finite interval, we can assume $C(z)$ to have a finite number of terms. Thus it is FIR. Using the expression for $M(z)$ we can compute the Markov parameters for the two cases:

Case A: We have

$$\begin{aligned} H_{cl}^A &= \frac{H(z)}{1 + C(z)H(z)} = \frac{H(z)}{1 + M(z)}, \\ &= h_1^{cl-A} z^{-1} + h_2^{cl-A} z^{-2} + \cdots, \end{aligned}$$

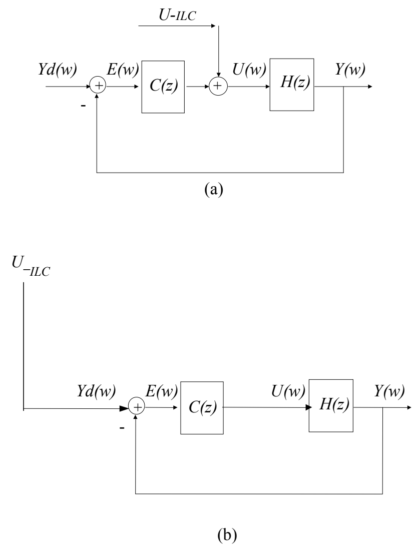

Figure 3: (a) Case A, ILC signal injected into the loop; (b) Case B, ILC signal modifies closed-loop reference signal.

where

$$\begin{aligned} h_1^{cl-A} &= h_1, \\ h_2^{cl-A} &= h_2 - h_1 m_1, \\ h_3^{cl-A} &= h_3 - h_1 m_2, \\ &\vdots \\ h_i^{cl-A} &= h_i - h_1 m_{i-1}. \end{aligned}$$

From this we can Equation 3 from Theorem 1 as

$$|h_1^{cl-A}| > \sum_{j=2}^{N} |h_j^{cl-A}, |$$

or

$$|h_1| \quad > \quad \sum_{i=2}^{N} |h_i - h_1 m_{i-1}|,$$

which can be written

$$|h_1| \quad > \quad \sum_{i=2}^{N} |h_i - h_1 \sum_{j=1}^{i-1} h_j c_{i-1-j}|. \tag{5}$$

Case B: Here we have

$$\begin{aligned} H_{cl}^{B} &= \frac{C(z)H(z)}{1+C(z)H(z)} = \frac{M(z)}{1+M(z)}, \\ &= h_1^{cl-B} z^{-1} + h_2^{cl-B} z^{-2} + \cdots, \end{aligned}$$

where

$$\begin{aligned} h_1^{cl-B} &= m_1, \\ h_2^{cl-B} &= m_2 - h_1 m_1, \\ h_3^{cl-B} &= m_3 - h_1 m_2, \\ &\vdots \\ h_i^{cl-A} &= m_i - h_1 m_{i-1}. \end{aligned}$$

From this we can write Equation 3 of Theorem 1 as

$$|h_1^{cl-B}| > \sum_{j=2}^{N} |h_j^{cl-B}, |$$

or

$$|m_1| \quad > \quad \sum_{i=2}^{N} |m_i - h_1 m_{i-1}|,$$

which can be written

$$|c_0 h_1| \quad > \quad \sum_{i=2}^{N} |\sum_{j=1}^{i} h_j c_{i-j} - h_1 \sum_{j=1}^{i-1} h_j c_{i-1-j}|. \tag{6}$$

Equations 5 and 6 define constraints that must be satisfied between the Markov parameters of the plant and those of the controller in order to satisfy the sufficient condition of Theorem 1. In general it is always possible to identify a controller that will satisfy these conditions. For, example, consider Case A. If we pick the controller Markov parameters so that

$$|h_i - h_1 \sum_{j=1}^{i-1} h_j c_{i-1-j}| = 0,$$

then Equation 5 will be satisfied. This means we need

$$\begin{aligned} 0 &= |h_2 - h_1 c_0|, \\ 0 &= |h_3 - h_1(h_1 c_1 + h_2 c_0)|, \\ 0 &= |h_4 - h_1(h_1 c_2 + h_2 c_1 + h_3 c_0)|, \\ &\vdots \end{aligned}$$

Clearly such a design will always be achievable, as these equations can be solved recursively, with one new controller parameter entering at each recursion.

As an example, consider the plant given by Equation 4, which has the Markov parameters shown in the Figure 4. Using the recursion suggested above, we can compute an FIR controller whose impulse response (Markov parameters) is given in Figure 5. Notice that in this case all the Markov parameters of the closed-loop system, as seen from the ILC algorithm, except $h = 1$, are zero. Thus the response is deadbeat with a one step time-delay. Such a system will clearly satisfy the requirement of Theorem 1 and so for this plant with this controller, any convergent P-type ILC algorithm will converge monotonically.

There are, however, some problems associated with the approach given in this section. First, we do not expect

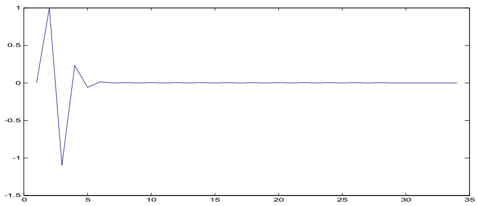

Figure 4: Plant Markov parameters.

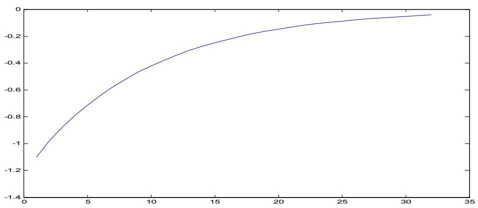

Figure 5: FIR filter coefficients for controller.

the procedure to be very robust. This could be avoided as follows. Again for Case A, we could require

$$|h_i - h_1 \sum_{j=1}^{i-1} h_j c_{i-1-j}| < \frac{|h_1|}{N-1},$$

rather than setting these terms to zero. Then Equation 5 will be satisfied. This means we need

$$\begin{aligned} \frac{|h_1|}{N-1} &> |h_2 - h_1 c_0| \\ \frac{|h_1|}{N-1} &> |h_3 - h_1(h_1 c_1 + h_2 c_0)| \\ \frac{|h_1|}{N-1} &> |h_4 - h_1(h_1 c_2 + h_2 c_1 + h_3 c_0)| \\ &\vdots \end{aligned}$$

Such a design will be more robust than the exact inversion illustrated in our example. However, another other difficulty is that the controller is specified in a high-order FIR format. This is inconvenient at best and problematic at worst, as it becomes difficult to ensure stability and in applications the size of N may be quite large. For these reasons we consider an IIR solution in the next section. We conclude this section by noting that similar analysis can be done for Case B.

5 IIR Controller Design

Consider again Figure 3and let

$$\begin{aligned} H(z) &= \frac{n_h(z)}{d_h(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}, \\ C(z) &= \frac{n_c(z)}{d_c(z)} = \frac{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \cdots + \beta_n z^{-q}}{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_n z^{-q}}. \end{aligned}$$

We again have $C(z)$ with relative degree zero, but now it is IIR. Consider the two cases:

<u>Case A</u>: In this case we can write the closed-loop system as:

$$\begin{aligned} H_{cl}^A &= \frac{\Gamma^A(z)}{\Delta(z)} = \frac{n_h(z)}{n_h(z)n_c(z) + d_h(z)d_c(z)}, \\ &= \frac{\gamma_1^A z^{-1} + \cdots + \gamma_{(q+n)}^A z^{-(q+n)}}{\delta_0 + \delta_1 z^{-1} + \cdots + \delta_{(q+n)} z^{-(q+n)}}, \\ &= h_1^{cl-A} z^{-1} + h_2^{cl-A} z^{-2} + \cdots. \end{aligned}$$

<u>Case B</u>: In this case we can write the closed-loop system as:

$$\begin{aligned} H_{cl}^B &= \frac{\Gamma^B(z)}{\Delta(z)} = \frac{n_h(z)n_c(z)}{n_h(z)n_c(z) + d_h(z)d_c(z)}, \\ &= \frac{\gamma_1^B z^{-1} + \cdots + \gamma_{(q+n)}^B z^{-(q+n)}}{\delta_0 + \delta_1 z^{-1} + \cdots + \delta_{(q+n)} z^{-(q+n)}}, \\ &= h_1^{cl-B} z^{-1} + h_2^{cl-B} z^{-2} + \cdots. \end{aligned}$$

Now, define the following vectors:

$$\begin{aligned} \mathbf{a} &= (a_0, a_1, a_2, \cdots, a_n)^T, \\ \mathbf{b} &= (0, b_1, b_2, \cdots, b_n)^T, \\ \boldsymbol{\alpha} &= (\alpha_0, \alpha_1, \alpha_2, \cdots, \alpha_q)^T, \\ \boldsymbol{\beta} &= (\beta_0, \beta_1, \beta_2, \cdots, \beta_q)^T, \\ \boldsymbol{\gamma}^A &= (0, \gamma_1^A, \gamma_2^A, \cdots, \gamma_{(q+n)}^A)^T, \\ \boldsymbol{\gamma}^B &= (0, \gamma_1^B, \gamma_2^B, \cdots, \gamma_{(q+n)}^B)^T, \\ \boldsymbol{\delta} &= (\delta_0 \delta_1, \delta_2, \cdots, \delta_{(q+n)})^T. \end{aligned} \tag{7}$$

Let the appropriately-dimensioned matrices A and B be given, respectively, as

$$\begin{bmatrix} \mathbf{a} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{a} & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{a} & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \mathbf{a} \end{bmatrix}, \begin{bmatrix} \mathbf{b} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{b} & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{b} & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \mathbf{b} \end{bmatrix},$$

and let

$$H^{cl-A} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ h_1^{cl-A} & 0 & \cdots & \vdots \\ h_2^{cl-A} & h_1^{cl-A} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ h_{q+n+1}^{cl-B} & h_{q+n}^{cl-A} & \cdots & h_1^{cl-A} \\ \vdots & \vdots & \ddots & \vdots \\ h_N^{cl-A} & h_{N-1}^{cl-A} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}.$$

A similar expression can be given for H^{cl-B}. Also note the following relationships:

1. $\boldsymbol{\gamma}^A = \mathbf{b}$.
2. $\boldsymbol{\gamma}^B = B\boldsymbol{\beta}$.
3. $\boldsymbol{\delta} = [B|A]\begin{pmatrix}\boldsymbol{\beta}\\ \boldsymbol{\alpha}\end{pmatrix}$.
4. The matrices H^{cl-A} and H^{cl-B} are "tall" matrices with full column rank equal $(q+n+1)$.
5. By definition for Case A we have

$$H_{cl}^A = \frac{\Gamma^A(z)}{\Delta(z)},$$

so we can multiply through by $\Delta(z)$ to get $\Gamma^A(z) = H_{cl}^A\Delta(z)$ and then we can expand and equate powers of z^{-1} to get

$$\begin{pmatrix}\boldsymbol{\gamma}^A\\ 0\\ \vdots\end{pmatrix} = H^{cl-A}\boldsymbol{\delta},$$

or

$$\begin{pmatrix}\mathbf{b}\\ 0\\ \vdots\end{pmatrix} = H^{cl-A}[B|A]\begin{pmatrix}\boldsymbol{\beta}\\ \boldsymbol{\alpha}\end{pmatrix}. \tag{8}$$

Similarly, for Case B we can write

$$\begin{pmatrix}\mathbf{B} & 0\\ 0 & 0\\ \vdots & \vdots\end{pmatrix}\begin{pmatrix}\boldsymbol{\beta}\\ \boldsymbol{\alpha}\end{pmatrix} = H^{cl-A}[B|A]\begin{pmatrix}\boldsymbol{\beta}\\ \boldsymbol{\alpha}\end{pmatrix} \tag{9}$$

Items 1. and 2. are important as they indicate how the closed-loop zeros can be influenced by the controller parameters in the two feedback configurations of Case A and Case B. Item 3. is the familiar Sylvester's equation. Readers will recall that the matrix $[A|B]$ has full row rank if the plant polynomials $n_h(z)$ $d_h(z)$ are coprime and $q \geq (n-1)$.

Item 5. is the basis for our design concept. Note that Equations 8 and 9 can be interpreted in two ways. On the one hand, given a controller, Equations 8 and 9 define the relationship between the plant and controller parameters and the resulting matrix of Markov parameters. On the other hand, one can view H^{cl-A} or H^{cl-B} as a given or desired design matrix, with the goal being to find controller parameters that satisfy Equations 8 or 9 . Unfortunately, because of Item 4. and the fact that we have an over-determined set of equations, these equations will only have a solution if H^{cl-A} or H^{cl-B} are chosen to be in the space of realizable impulse responses for the closed-loop system. However, because we are working on a finite interval (N) and because as the controller order increases, the null-space of the Sylvester matrix $[A|B]$ becomes non-trivial, it is possible to use Equations 8 and 9 as a design tool in the special case of a deadbeat closed-loop response. From Equations 8 and 9 it is clear that if $\boldsymbol{\delta}$ is deadbeat (i.e., all entries except $\boldsymbol{\delta}(1)$ are zero) then the closed-loop system will be FIR, with an impulse response given by the coefficients of $\boldsymbol{\gamma}^A$ or $\boldsymbol{\gamma}^B$. In the former case, this is simply defined by the zeros of the plant. But, in the latter case, the zeros of both the plant and the controller affect the response. This can be exploited as follows. Suppose we seek a deadbeat response and we let the controller order be such that the null space of the Sylvester matrix is non-trivial. Then we may be able to search over that null space to find a $\boldsymbol{\gamma}^B$ that satisfies the conditions of Theorem 1. We illustrate this with an example in the next section.

6 IIR Design Example

Consider again the second-order system of Equation 4:

$$Y_k(z) = \frac{z-0.9}{z^2+0.2z-0.125}U_k(z). \tag{10}$$

Because the plant is second-order, we know that a first-order controller is sufficient for arbitrary pole placement. Indeed, following the lead from the FIR example, we could easily design an IIR controller to make the response deadbeat. However, in this case, using a first-order controller, the zeros of the closed-loop system would be fully-prescribed based on the pole design. In the case of this plant it is clear that such a design would be adequate in Case A. That is, if we place all the poles at the origin, then the closed-loop response will simply be given by

$$H_{cl}^A(z) = \frac{z-0.9}{z^3} = z^{-1} - .9z^{-2}, \tag{11}$$

which satisfies the condition of Theorem 1. However, we would like to see if it is possible to develop more design freedom by increasing the controller order and using the controller zeros appearing in Case B.

Let us consider using a third-order controller for Case B. The deadbeat poles are solved from Equation 7 as

$$\boldsymbol{\delta} = \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix} = [A|B]\begin{pmatrix}\beta(0)\\ \beta(1)\\ \beta(2)\\ \beta(3)\\ \alpha(0)\\ \alpha(1)\\ \alpha(2)\\ \alpha(3)\end{pmatrix} \tag{12}$$

where the Sylvester matrix $[A|B]$ is given by

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0.2 & 1 & 0 & 0\\ -.9 & 1 & 0 & 0 & -0.0125 & 0.2 & 1 & 0\\ 0 & -.9 & 1 & 0 & 0 & -0.0125 & 0.2 & 1\\ 0 & 0 & -.9 & 1 & 0 & 0 & -0.0125 & 0.2\\ 0 & 0 & 0 & -.9 & 0 & 0 & 0 & -0.0125 \end{bmatrix}$$

All solutions of Equation 12 can be parameterized as

$$\begin{pmatrix}\beta(0)\\ \beta(1)\\ \beta(2)\\ \beta(3)\\ \alpha(0)\\ \alpha(1)\\ \alpha(2)\\ \alpha(3)\end{pmatrix} = \begin{pmatrix}-0.0866\\ -0.0169\\ -0.0017\\ 0.0001\\ 1.0\\ -0.1134\\ -0.0260\\ -0.0097\end{pmatrix} + w_1\begin{pmatrix}0.4862\\ -0.1418\\ -0.0539\\ 0.003\\ 0\\ -0.4862\\ 0.6766\\ -0.2151\end{pmatrix} +$$

$$w_2 \begin{pmatrix} 0.3703 \\ 0.6366 \\ 0.1079 \\ -0.007 \\ 0 \\ -0.3703 \\ -0.2292 \\ 0.5063 \end{pmatrix}$$

The first vector on the left hand side of the equation produces the deadbeat response. The second two vectors form a basis for the null space of the Sylvester equation. Thus, w_1 and w_2 parameterize all possible deadbeat responses for the closed-loop system for Case B. To use this to determine a controller that meets the conditions of Theorem 1, we note that if the response is deadbeat then we can write the expression for the numerator coefficients as:

$$\begin{aligned} \boldsymbol{\gamma}^B &= (\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5) \\ &= (h_1^{cl-B}h_2^{cl-B}h_3^{cl-B}h_4^{cl-B}h_5^{cl-B}) \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.9 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.9 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta(0) \\ \beta(1) \\ \beta(2) \\ \beta(3) \\ \alpha(0) \\ \alpha(1) \\ \alpha(2) \\ \alpha(3) \end{pmatrix} \end{aligned}$$

Or, substituting the parameterization of solutions of Equation 12

$$\begin{pmatrix} h_1^{cl-B} \\ h_2^{cl-B} \\ h_3^{cl-B} \\ h_4^{cl-B} \\ h_5^{cl-B} \end{pmatrix} = \begin{pmatrix} -0.0866 \\ 0.0611 \\ 0.0135 \\ 0.0016 \\ -0.0001 \end{pmatrix} + w_1 \begin{pmatrix} 0.4862 \\ -0.5794 \\ 0.0737 \\ 0.0515 \\ -0.0027 \end{pmatrix} + w_2 \begin{pmatrix} 0.3707 \\ 0.3033 \\ -0.4650 \\ -0.1041 \\ 0.0063 \end{pmatrix}$$

If we pick $w_1 = w_2 = 1$, for example, the resulting closed-loop system seen by the ILC algorithm is

$$\begin{aligned} H_{cl}^B &= 0.7699z^{-1} - 0.2150z^{-2} - \\ &\quad 0.3778z^{-3} - 0.0510z^{-4} + 0.0035^{-5}. \end{aligned}$$

It is easily checked that this system satisfies the conditions of Theorem 1.

Unfortunately, the method is not completely developed. Simply changing the zero from $z = -0.9$ to $z = -1.1$ results in an example in which it is not possible to meet the conditions of Theorem 1 when using a third-order controller and designing for a deadbeat response. More research is needed to understand this problem.

7 Conclusion

In this paper we presented two design approaches for specifying a current cycle feedback controller that will produce a closed-loop system for which all P-type learning control algorithms converge. One approach results in an FIR controller while the other gives an IIR controller. Both yield deadbeat responses. The FIR approach is always achievable, but may suffer from robustness and implementation problems. The IIR approach does not always give a solution. Further research will continue to explore these issues.

Ackowledgement

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