Discretization Schemes for Fractional-Order Differentiators and Integrators

Yang Quan Chen and Kevin L. Moore

***Abstract*—For fractional-order differentiator s^r where r is a real number, its discretization is a key step in digital implementation. Two discretization methods are presented. The first scheme is a direct recursive discretization of the Tustin operator. The second one is a direct discretization method using the Al-Alaoui operator via continued fraction expansion (CFE). The approximate discretization is minimum phase and stable. Detailed discretization procedures and short MATLAB scripts are given. Examples are included for illustration.**

***Index Terms*—Al-Alaoui operator, discretization, fractional differentiator, fractional-order differentiator, fractional-order dynamic systems, recursive, Tustin operator.**

I. INTRODUCTION

Fractional calculus is a 300-year-old topic. The theory of fractional-order derivative was developed mainly in the 19th century. Recent books [1]–[4] provide a good source of references on fractional calculus. However, applying fractional-order calculus to dynamic systems control is just a recent focus of interest [5]–[9]. For pioneering work on this regard, we cite [10]–[13].

In theory, the control systems can include both the fractional order dynamic system or plant to be controlled and the fractional-order controller. However, in control practice it is more common to consider the fractional-order controller. This is due to the fact that the plant model may have already been obtained as an integer order model in the classical sense. In most cases, our objective is to apply fractional-order control (FOC) to enhance the system control performance. For example, as in the CRONE[1] control [14], [7], [8], *fractal robustness* is pursued. The desired frequency template leads to fractional transmittance [15], [16] on which the CRONE controller synthesis is based. In the CRONE controller, the major ingredient is the fractional-order derivative s^r, where r is a real number and s is the Laplacian operator. Another example is the PI$^\lambda$D$^\mu$ controller [6], [17], an extension of PID controller. In general form, the transfer function of PI$^\lambda$D$^\mu$ is given by $K_p + T_i s^{-\lambda} + T_d s^{\mu}$, where λ and μ are positive real numbers; K_p is the proportional gain, T_i the integration constant and T_d the differentiation constant. Clearly, taking $\lambda = 1$ and $\mu = 1$, we obtain a classical PID controller. If $\lambda = 0$ $(T_i = 0)$ we obtain a PD$^\mu$ controller, etc. All these types of controllers are particular cases of the PI$^\lambda$D$^\mu$ controller. It can be expected that the PI$^\lambda$D$^\mu$ controller may enhance the systems control performance due to more tuning knobs introduced. Actually, in theory, PI$^\lambda$D$^\mu$ itself is an infinite dimensional linear filter due to the fractional order in the differentiator or integrator. It should be pointed out that a band-limit implementation of FOC is important in practice, i.e., the finite-dimensional approximation of the FOC should be done in a proper range of frequencies of practical interest [18], [16]. Moreover, the fractional order can be a complex number as discussed in [18]. In this paper, we focus on the case when the fractional order is a real number.

The key step in digital implementation of a FOC is the numerical evaluation or discretization of the fractional-order differentiator. In general, there are two discretization methods: *direct discretization* and *indirect discretization*. In *indirect discretization* methods [18], two steps are required, i.e., frequency domain fitting in continuous time domain first and then discretizing the fit s-transfer function. Other frequency-domain fitting methods can also be used but without guaranteeing the stable minimum-phase discretization. Existing *direct discretization* methods, e.g., [19], [20], applied the direct power series expansion (PSE) of the Euler operator, continuous fractional expansion (CFE) of the Tustin operator and numerical integration based method.

In this brief, we first focus on the *direct discretization* method using the well known Tustin operator which is a straightforward scheme to discretize the fractional-order derivative [19], [20]. The major contribution of this brief is to introduce a recursive formula for discretization with different order of approximation which simplifies the programming efforts. Moreover, the discretized transfer function is stable and minimum phase. However, as pointed out in [21], [22], the Tustin operator based discretization scheme exhibits large errors in high frequency range. A new mixed scheme of Euler and Tustin operators is proposed which yields the Al-Alaoui operator [21]. Using the continued fraction expansion of the Al-Alaoui operator, this paper contributes a new direct discretization scheme with a very good magnitude fit to that of the original continuous fractional differentiator.

This brief is organized as follows. In Section II, we present a new recursive formula for Tustin discretization of the fractional-order derivative with a symbolic validation and an illustrative example. Section III presents another new direct discretization scheme by continued fraction expansion of the Al-Alaoui operator with an illustrative example, too. Section IV concludes this paper.

II. RECURSIVE TUSTIN DISCRETIZATION OF FRACTIONAL-ORDER DERIVATIVE

A. Generating Function

In general, the discretization of the fractional-order differentiator s^r (r is a real number) can be expressed by the so-called generating function $s = \omega(z^{-1})$. This generating function and its expansion determine both the form of the approximation and the coefficients [23]. For example, when a backward difference rule is used, i.e., $\omega(z^{-1}) = (1 - z^{-1})/T$, performing the power series expansion (PSE) of $(1 - z^{-1})^{\pm r}$ gives the discretization formula.

In this brief, we first consider the trapezoidal (Tustin) rule as a generating function

$$\left(\omega(z^{-1})\right)^{\pm r} = \left(\frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}}\right)^{\pm r} \tag{1}$$

for obtaining the coefficients and the form of the approximation.

B. Recursive Tustin Discretization

The key point of the Tustin discretization of fractional-order differentiator is how to get a recursive formula. Here we introduce the so-called Muir-recursion originally used in geophysical data processing with applications to petroleum prospecting [24]. The Muir-recursion motivated in computing the vertical plane wave

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[1] CRONE is a French abbreviation for "*Contrôle Robuste d'Ordre Non Entier*" (which means noninteger order robust control).

TABLE I
TABLE OF FORMULAS $A_n(z^{-1}, r)$ FOR $n = 1, \ldots, 9$

n	$A_n(z^{-1}, r)$
0	1
1	$-rz^{-1} + 1$
3	$-\frac{1}{3}rz^{-3} + \frac{1}{3}r^2z^{-2} - rz^{-1} + 1$
5	$-\frac{r}{5}z^{-5} + \frac{r^2}{5}z^{-4} - \left(\frac{r}{3} + \frac{r^3}{15}\right)z^3 + \frac{2}{5}r^2z^{-2} - rz^{-1} + 1$
7	$-\frac{r}{7}z^{-7} + \frac{r^2}{7}z^{-6} - \left(\frac{r}{5} + \frac{2r^3}{35}\right)z^{-5} + (\frac{26}{105}r^2 + \frac{r^4}{105})z^{-4}$ $-\left(\frac{1}{3}r + \frac{2}{21}r^3\right)z^{-3} + \frac{3}{7}r^2z^{-2} - rz^{-1} + 1$
9	$-\frac{r}{9}z^{-9} + \frac{r^2}{9}z^{-8} - \left(\frac{r}{7} + \frac{r^3}{21}\right)z^{-7} + \left(\frac{34r^2}{189} + \frac{2r^4}{189}\right)z^{-6}$ $-\left(\frac{1}{5}r + \frac{16}{189}r^3 + \frac{1}{945}r^5\right)z^{-5} + \left(\frac{17}{63}r^2 + \frac{1}{63}r^4\right)z^{-4}$ $-\left(\frac{1}{3}r + \frac{1}{9}r^3\right)z^{-3} + \frac{4}{9}r^2z^{-2} - rz^{-1} + 1$

reflection response via the impedance of a stack of n-layered earth can be used in recursive discretization of fractional-order differentiator of the Tustin generating function. In the following, without loss of generality, assume that $r \in [-1, 1]$. Moreover, without complicating the presentation, we only give the recursive formula for positive r

$$\begin{aligned}\left(\omega(z^{-1})\right)^r &= \left(\frac{2}{T}\right)^r \left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^r \\ &= \left(\frac{2}{T}\right)^r \lim_{n\to\infty} \frac{A_n(z^{-1}, r)}{A_n(z^{-1}, -r)} \end{aligned} \tag{2}$$

where $A_0(z^{-1}, r) = 1$, and

$$A_n(z^{-1}, r) = A_{n-1}(z^{-1}, r) - c_n z^n A_{n-1}(z, r), \tag{3}$$

and

$$c_n = \begin{cases} r/n; & n \text{ is odd;} \\ 0; & n \text{ is even.} \end{cases} \tag{4}$$

Therefore,

$$s^r \approx \left(\frac{2}{T}\right)^r \frac{A_n(z^{-1}, r)}{A_n(z^{-1}, -r)}.$$

Symbolically, Table I lists the expressions of $A_n(z^{-1}, r)$ for $n = 1, \ldots, 9$.

C. Symbolic Validation

We now examine the correctness of the recursive discretization of the fractional-order derivative operator by using MATLAB Symbolic Toolbox. We can get the symbolic Taylor expansion for (1) for the first 9 terms. The result is summarized as follows:

$$\begin{aligned} &\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^r \\ &\approx 1 - (2r)z^{-1} + (2r^2)z^{-2} - \left(\frac{2}{3}r + \frac{4}{3}r^3\right)z^{-3} \\ &+ \left(\frac{4}{3}r^2 + \frac{2}{3}r^4\right)z^{-4} - \left(\frac{2}{5}r + \frac{4}{3}r^3 + \frac{4}{15}r^5\right)z^{-5} \\ &+ \left(\frac{46}{45}r^2 + \frac{8}{9}r^4 + \frac{4}{45}r^6\right)z^{-6} \\ &- \left(\frac{2}{7}r + \frac{56}{45}r^3 + \frac{4}{9}r^5 + \frac{8}{315}r^7\right)z^{-7} \\ &+ \left(\frac{88}{105}r^2 + \frac{44}{45}r^4 + \frac{8}{45}r^6 + \frac{2}{315}r^8\right)z^{-8} \\ &- \left(\frac{2}{9}r + \frac{3272}{2835}r^3 + \frac{76}{135}r^5 + \frac{8}{135}r^7 + \frac{4}{2835}r^9\right)z^{-9}. \end{aligned} \tag{5}$$

We can do the similar Taylor expansion for $(A_n(z^{-1}, r)/A_n(z^{-1}, -r))$. Firstly, refer to the formulas $A_n(z^{-1}, r)$ for $n = 1, \ldots, 9$ listed in Table I. We now examine the case when $n = 9$. By collecting the terms in each coefficient, we found that Taylor expansion coefficients are all the same with those given in (5) until z^{-9}. Therefore, it is verified that the proposed recursive formula is as correct as Taylor series expansion till the order of approximation.

D. An Illustrative Example

Using the recursive method mentioned in this subsection, the discretization of $s^{0.5}$ sampled at 0.001 s. is studied numerically, and the approximate models are

$$G_1(z) = \frac{44.72z - 22.36}{z + 0.5}$$

$$G_3(z) = \frac{44.72z^3 - 22.36z^2 + 3.727z - 7.454}{z^3 + 0.5z^2 + 0.083\,33z + 0.1667}$$

$$G_7(z) = \frac{\begin{array}{c}44.72z^7 - 22.36z^6 + 4.792z^5 - 7.986z^4 + 2.795z^3 \\ -4.792z^2 + 1.597z - 3.194\end{array}}{\begin{array}{c}z^7 + 0.5z^6 + 0.1071z^5 + 0.1786z^4 + 0.0625z^3 \\ +0.1071z^2 + 0.0357z + 0.071\,43\end{array}}$$

$$G_9(z) = \frac{\begin{array}{c}44.72z^9 - 22.36z^8 + 4.969z^7 - 8.075z^6 \\ +3.061z^5 - 4.947z^4 + 2.041z^3 - 3.461z^2 \\ +1.242z - 2.485\end{array}}{\begin{array}{c}z^9 + 0.5z^8 + 0.1111z^7 + 0.1806z^6 \\ +0.068\,45z^5 + 0.1106z^4 + 0.045\,63z^3 \\ +0.077\,38z^2 + 0.027\,78z + 0.055\,56.\end{array}}$$

We present four plots as shown in Fig. 1 to show the effectiveness of the approximate discretization.

It should be pointed out that the direct discretization method introduced above always gives a Z-transfer function with stable minimum phase characteristics. The convergence of the recursive scheme [24] is clearly demonstrated by Fig. 1.

III. DIRECT DISCRETIZATION USING CONTINUED FRACTION EXPANSION (CFE) OF AL-ALAOUI OPERATOR

We see from Fig. 1 that the Bode plot of the discretized model based on the recursive expansion of the Tustin operator or trapezoidal rule is with large error in the high frequency range. To have a better approximation in the high frequency range, here we use the Al-Alaoui operator [21] which is a weighted sum of the rectangular rule or Euler operator and the trapezoidal rule. This will give a new straightforward scheme using continued fraction expansion (CFE) to discretize the fractional-order derivative with the aid of a symbolic computation tool.

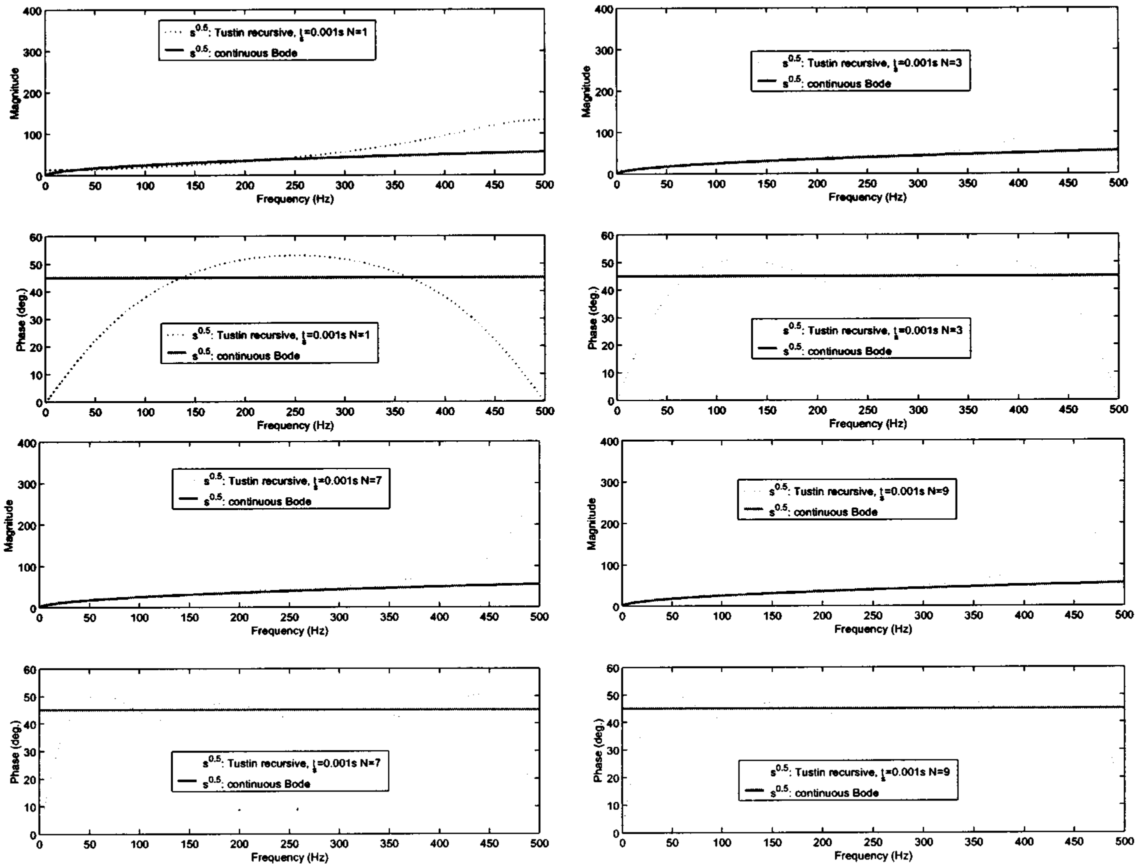

Fig. 1. Recursive Tustin discretization of $s^{0.5}$ at $T = 0.001$ s.

A. Al-Alaoui Operator Based Discretization

The Al-Alaoui operator is a mixed scheme of Euler and Tustin operators. Based on the Al-Alaoui operator, the generating function for discretization is

$$\left(\omega(z^{-1})\right)^{\pm r} = \left(\frac{8}{7T}\,\frac{1-z^{-1}}{1+z^{-1}/7}\right)^{\pm r}. \tag{6}$$

Clearly, (6) is an infinite order of rational discrete-time transfer function. To approximate it with a finite order rational one, continued fraction expansion (CFE) is an efficient way. In general, any function $G(z)$ can be represented by continued fractions in the form of

$$G(z) \simeq a_0(z) + \cfrac{b_1(z)}{a_1(z) + \cfrac{b_2(z)}{a_2(z) + \cfrac{b_3(z)}{a_3(z) + \cdots}}}$$

where the coefficients a_i and b_i are either rational functions of the variable z or constants. By truncation, an approximate rational function, $\hat{G}(z)$, can be obtained.

The resulting discrete transfer function, approximating fractional-order operators, can be expressed as:

$$\begin{aligned} D^{\pm r}(z) &\approx \left(\frac{8}{7T}\right)^{\pm r} \mathrm{CFE}\left\{\left(\frac{1-z^{-1}}{1+z^{-1}/7}\right)^{\pm r}\right\}_{p,q} \\ &= \left(\frac{8}{7T}\right)^{\pm r} \frac{P_p(z^{-1})}{Q_q(z^{-1})} \end{aligned} \tag{7}$$

where CFE$\{u\}$ denotes the continued fraction expansion of u; p and q are the orders of the approximation and P and Q are polynomials of degrees p and q. Normally, we can set $p = q = n$. In MATLAB Symbolic Toolbox, we can easily get the approximate direct discretization of the fractional order derivative by the following script, for a given n (replace 14 by $2n$):

```
clear all; close all; syms x z r
%Al - Alouoi's scheme
x = ((1 - z)/(1 + z/7))^r;
[RESULT, STATUS] = maple (“with (numtheory)”)
%7-th order; put 2 * 7 here.
h7 = maple (“cfrac”, x, z, 14);
h7n = maple (“nthnumer (%%, 14)”);
h7d = maple (“nthdenom (%%%, 14)”);
h7ns = sym (h7n); h7ds = (h7d);
num7 = collect (h7ns, z); den7 = collect (h7ds, z);
fn7 = subs (num7, z, 1/z), fd7 = subs (den7, z, 1/z).
```

Remark III.1: The CFE scheme presented in the above contains two tuning parameters, namely p and q, while in the recursive Tustin method introduced in Section II, there is only one tuning parameter, n. The optimal choice of these parameters is possible based on a quantitative measure. One possibility is the use of the least squares (LS) error between the continuous-frequency response and discretized frequency response. Note that in practice, p and q can usually be set equal. So, in the discretization methods presented in this paper, there is only one tuning knob which will be more attractive in practice. Compared to

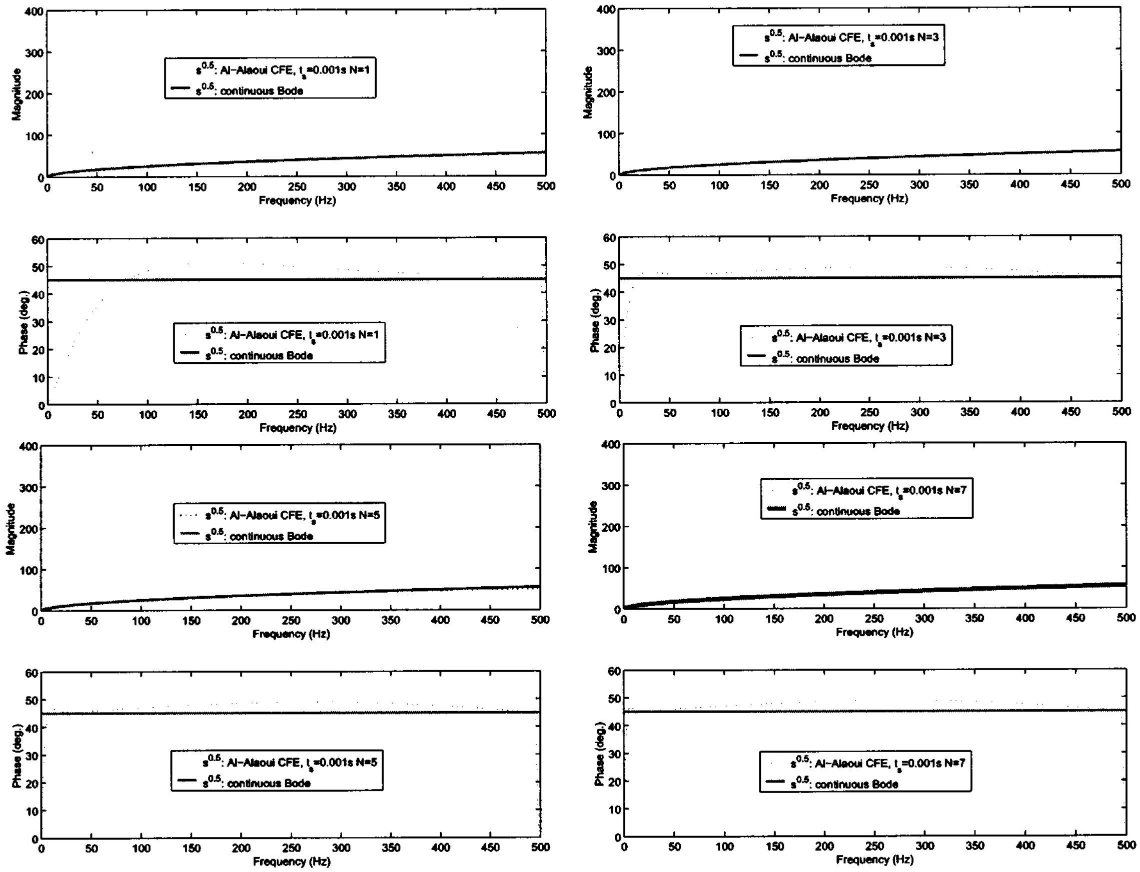

Fig. 2. CFE(Al-Alaoui) discretization of $s^{0.5}$ at $T = 0.001$ s.

the frequency-banded fitting methods, e.g., [18], [16], the direct discretization methods presented in this paper is straightforward and no band-pass weighting filter is assumed.

B. An Illustrative Example

The discretization of the half-differentiator $s^{0.5}$ sampled at 0.001 s. is studied numerically, and the approximate models are

$$G_1(z) = \frac{236.6z - 169}{7z - 1}$$

$$G_3(z) = \frac{1657z^3 - 2603z^2 + 1048z - 62.78}{49z^3 - 49z^2 + 7z + 1}$$

$$G_5(z) = \frac{\begin{array}{c}2.47e04z^5 - 5.999e04z^4 + 4.941e04z^3\\ -1.512e04z^2 + 956.9z + 98.48\end{array}}{730.7z^5 - 1357z^4 + 745.7z^3 - 89.48z^2 - 15.52z + 1}$$

$$G_7(z) = \frac{\begin{array}{c}3.128e05z^7 - 1.028e06z^6 + 1.283e06z^5 - 7.433e05z^4\\ +1.87e05z^3 - 9772z^2 - 2140z + 104.5\end{array}}{\begin{array}{c}9253z^7 - 2.512e004z^6 + 2.436e004z^5 - 9577z^4\\ +905.7z^3 + 219.7z^2 - 23.67z - 1\end{array}}.$$

We present four plots as shown in Fig. 1 to show the effectiveness of the approximate discretization. We can observe from Fig. 2 that the new scheme introduced in this letter is much better than the Tustin scheme in magnitude fit to the original s^r. After the linear phase compensation, the maximum phase error of the Al-Alaoui operator based discretization scheme is around $r \times 8.25^\circ$ at 55% of the Nyquist frequency (around 275 Hz in this example) as shown in Fig. 2. To compensate the linear phase drop, a half sample phase advance is used which means that we should cascade $z^{0.5r}$ to the obtained approximately discretized transfer function $G(z)$. In this example, the phase compensator is $z^{0.25}$ which is noncausal. In implementation, we can simply use $z^{-0.75}/z^{-1}$ instead. Note that the linear phase drop compensation does not affect the magnitude fit. When multi-rate sampling is possible, the phase-drop compensation can be implemented with a smaller error which deserves further investigation. It should also be pointed out that, by checking the pole-zero map, the direct discretization method introduced above always gives a Z-transfer function with stable minimum phase characteristics.

IV. CONCLUDING REMARKS

This brief presents two discretization methods for the fractional-order differentiator s^r where r is a real number. The discretization is a key step in digital implementation of the fractional-order controller containing s^r. The first scheme is a direct recursive discretization of the Tustin operator. The second one is a direct discretization method using the Al-Alaoui operator via CFE. The approximate discretization schemes give minimum phase and stable finite dimensional discrete transfer functions. Detailed discretization procedures and short MATLAB scripts are given. Examples are included for illustration.

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Nonlinear Effects of Radio-Frequency Interference in Operational Amplifiers

Franco Fiori and Paolo S. Crovetti

***Abstract*—In this brief, the susceptibility of operational amplifiers to radio frequency interference (RFI) is studied by a new analytical model. The proposed model, in particular, points out the dependence of the RFI induced dc offset voltage shift in operational amplifiers on design parameters and parasitics, giving both a good insight into the nonlinear mechanisms involved in the phenomenon and a support to integrated circuit designers in order to develop high immunity operational amplifiers. The validity of the proposed approach is discussed comparing model predictions with the results of computer simulations and experimental measurements.**

***Index Terms*—Electromagnetic compatibility (EMC), electromagnetic interference (EMI), harmonic distortion, operational amplifier.**

I. INTRODUCTION

The increased level of environmental electromagnetic pollution makes mandatory the design of electronic systems which are immune to radio frequency interference (RFI). In fact, metal interconnections (wires and printed circuit board traces) of electronic systems translate electromagnetic interference (EMI) into voltages and currents, which result superimposed on nominal system signals. Such an interference reaches integrated circuit (IC) terminals and drives distortion phenomena in active nonlinear devices, generating temporary or definitive errors in IC and electronic system operations [1]–[3].

Analog circuits are particularly susceptible to RFI, as they lack the regenerative effect which is typical of digital circuits. This is one of the main reasons why digital signal processing is preferable to analog signal processing. However, analog signal processing cannot be replaced in many applications, among which at least the analog-to-digital (A/D) and the digital-to-analog (D/A) conversions must be cited, which are necessary in digital signal processing because of the intrinsically analog nature of the information in the physical world. Among analog circuits, operational amplifiers (opamps) are extremely susceptible to RF disturbances. They demodulate RFI added on nominal input signals and so the nominal output signal is corrupted by in-band interference. In particular, the presence of continuous-wave (CW) interference generates an output offset voltage.

This opamp behavior was originally observed in aeronautic electronic systems and subsequently studied performing immunity tests on commercial opamps. In particular, measurements of the output offset voltage induced by RFI conveyed on the input terminals of several feedback opamps were performed varying interference frequency and amplitude [4]. These experimental characterization is useful in the design of EMI filters. More recently, this behavior has been studied by time domain [5]–[7] and frequency domain (harmonic balance) [8] computer simulations: several efforts have been expended on research in order to derive both RFI-oriented numerical models of active devices and opamp macromodels intended to reduce computer simulation time. Although these models allow one to predict efficiently and accurately the RFI-induced upset in opamps, they do not grant a relationship with circuit parameters and parasitics and so they cannot be directly applied to derive design criteria.

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