On Fractional Order Disturbance Observer

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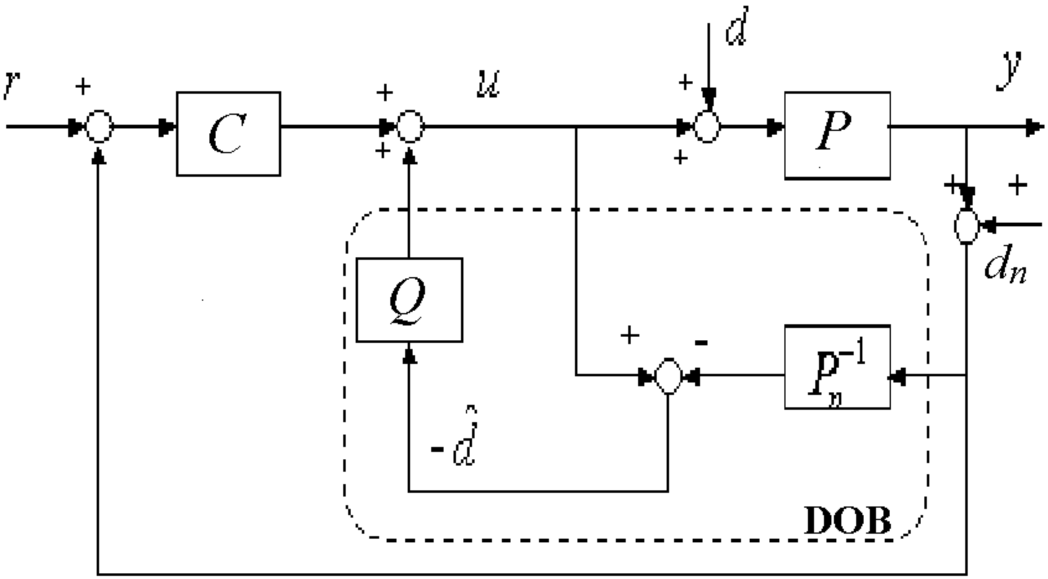
Outline of the presentation

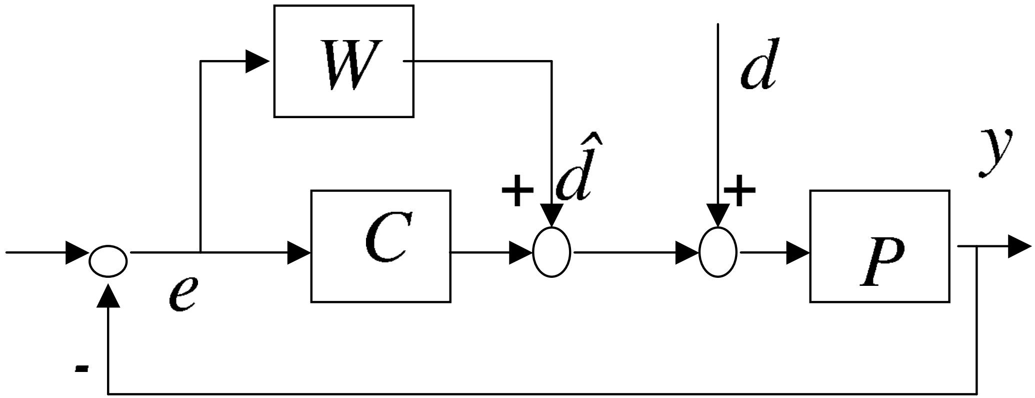
- **What is DOB (Disturbance Observer)**?
- Why Fractional Order DOB?
- A Proposed Solution Using Fractional Order Q-Filter
- Concluding Remarks

What is DOB (Disturbance Observer)

Ohnishi, K. (1987). "A new servo method in mechatronics". *Trans. Japanese Soc. Elect. Eng.*



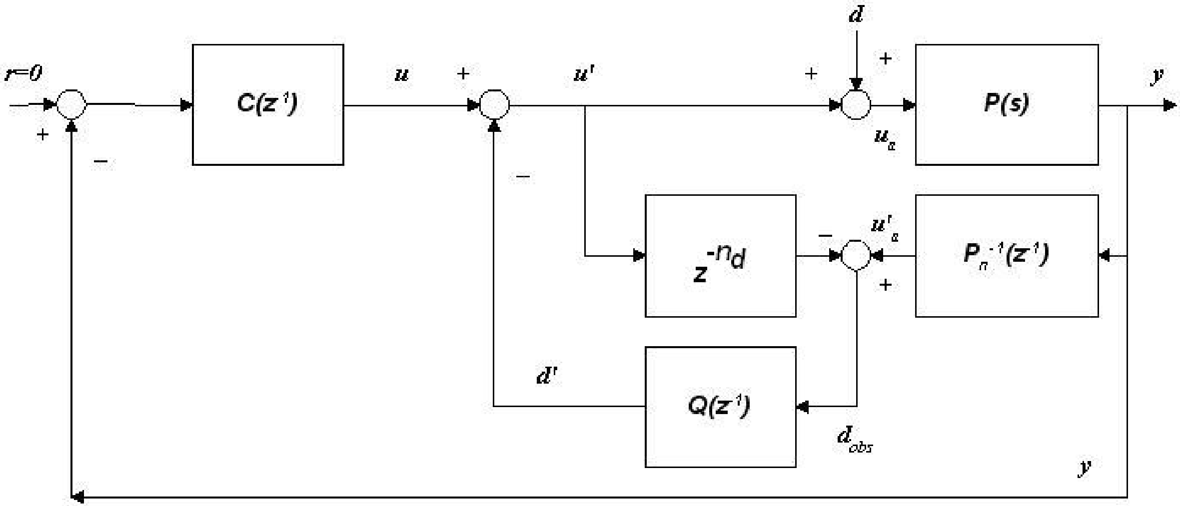
Disturbance Observer Block-diagram - The Conventional Form.



Disturbance Observer Block-diagram - The General Form.

- DOB is **nothing but** another form of loopshaping to add more attenuation in the lower frequency range at the cost of the reduced phase margin and the possible amplification of disturbances at other medium and high frequency bands due to the waterbed effect in the sensitivity function.

- It is **implicitly implied** in DOB that the spectrum of the disturbance d has more low frequency contents than the high frequency ones.



Disturbance Observer Block-diagram - Digital Form.

- d' is the "observed" disturbance for d
- $P_n^{-1}(z^{-1})$ is the stable inverse of P_n, the nominal model of the actual plant P;
- n_d is the number of pure delays of the control signal u', the compensated signal of the controller signal u generated by the controller C;
- Q is a low pass filter with the relative degree n_Q and the cutoff frequency ω_Q.

Three key parameters in DOB design:

- n_d: the number of pure delays of the control signal u';
- n_Q: the relative degree of Q-filter and
- ω_Q: the cutoff frequency of Q-filter.

Error transfer function (ETF) $S(j\omega)$ and the disturbance response transfer function (from d to y) $G_{dy}(j\omega)$.

- With no DOB,

$$S(j\omega) = \frac{1}{1+PC}, \quad G_{dy}(j\omega) = \frac{P}{1+PC} \tag{1}$$

- with DOB,

$$S(j\omega) = \frac{1}{1+PC+\delta_{PC}}, \quad G_{dy}(j\omega) = \frac{P}{1+PC+\delta_{PC}} \tag{2}$$

where

$$\delta_{PC} = \frac{PP_n^{-1}Q + z^{-n_d}QPC}{1-z^{-n_d}Q} = P\frac{z^{-n_d}Q}{1-z^{-n_d}Q}(P_n^{-1}z^{n_d}+C). \tag{3}$$

Clearly, the disturbance observer cannot be implemented if $Q = 1$.

Realization issue of P_n^{-1}

Remark 0.1 *In many motion control systems, the nominal plant is in the form of* $\frac{K}{(\tau s+1)s}$ *and* P_n^{-1} *in this case can be approximated by an FIR (finite impulse response) filter which is always realizable by itself. Therefore, in this case, no constraint is to be put to the relative degree of* Q*. In contrary, in the literature,* QP_n^{-1} *have to be made realizable by letting the relative degree of* Q *be equal to or greater than that of* P_n*.*

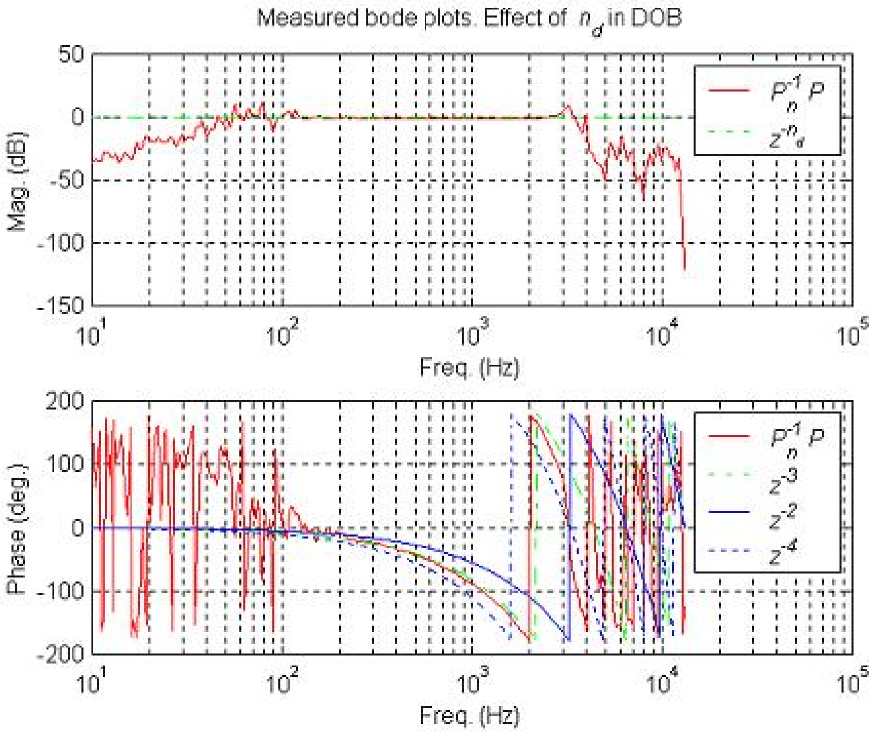


Illustration of the effect of n_d in DOB

- In this case, it is found that $n_d = 3$ is the best choice for a high TPI (tracks per inch) disk drive servo system.
- In general, for different applications, n_d should be different and n_d should also include the delay effect of the plant P.

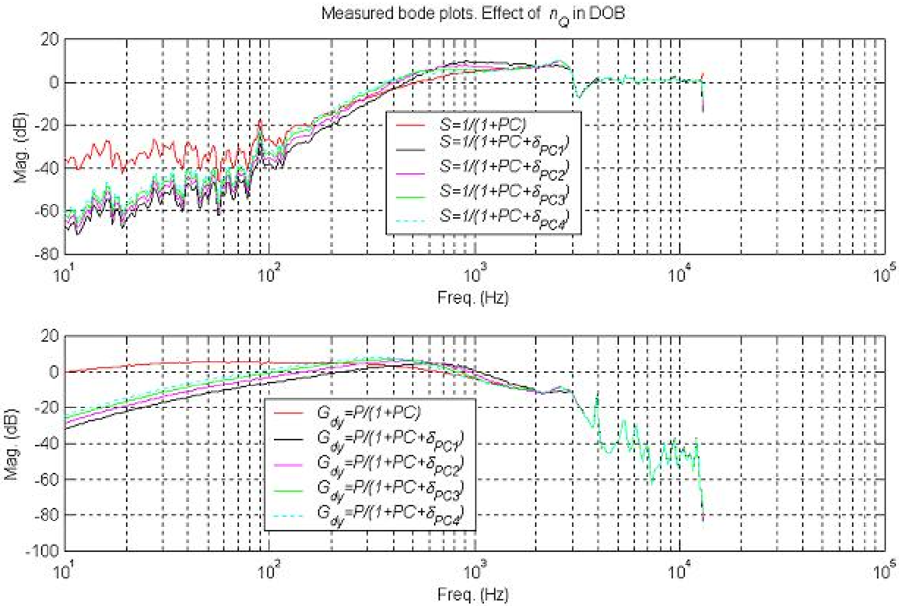


Illustration of the effect of n_q in DOB

Observation:

- For the lowest relative degree (n_Q =1, note again, as pointed out earlier, n_Q cannot be 0), the best disturbance attenuation is achieved but the waterbed effect shows up with the largest amplification of mid-band frequency contents. Therefore, when the disturbance is not presented or is small, n_Q =1 is **not a preferred choice**.

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- A Proposed Solution Using Fractional Order Q-Filter
- Concluding Remarks

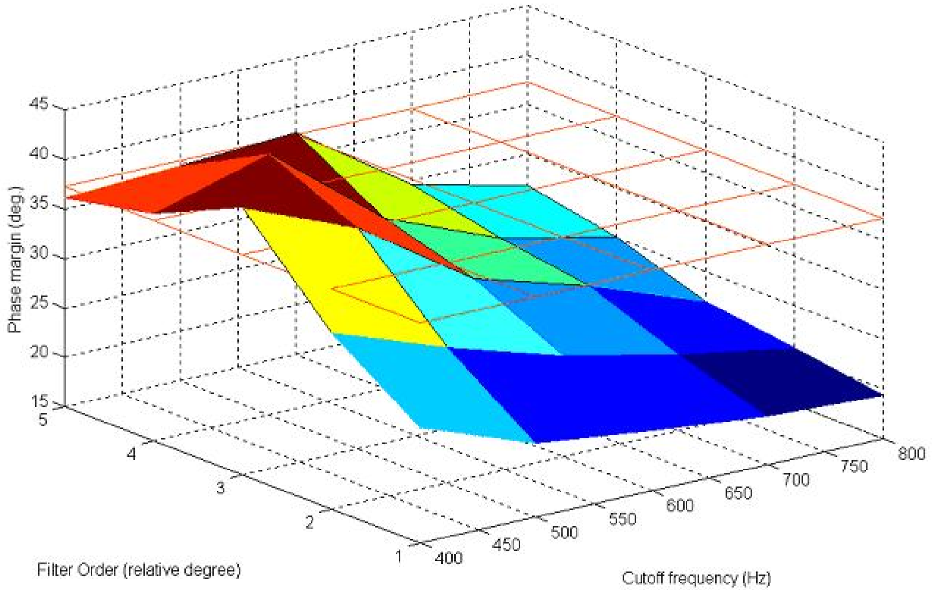

Illustration of phase margin (PM) as a function of n_Q and ω_Q in DOB

- PM is in fact a function of ω_Q as well as n_Q. The basic trend is that the higher the ω_Q the more PM loss; the larger the n_Q the less PM loss for a fixed ω_Q.
- A compromise must be made between the disturbance attenuation performance and the robustness of the original system. **Sticking on integer order Q-filter, the trade-off may not be optimal**.

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In practice, ω_Q is usually specified by the disturbance attenuation requirement. Moreover, the phase margin of the overall compensated system with DOB is also specified. By $\mathbf{PM}(\omega_Q, n_Q)$, we may find that the required n_Q usually lies between two adjacent integers.

For example, from the DOB design, it may turn out that Q-filter should be of the following form

$$Q(s) = \frac{1}{(\tau s + 1)^{n_Q}}, \quad n_Q = 3.25 \tag{4}$$

which is a fractional order low pass filter (FO-LPF).

When we use a fractional order Q-filter in DOB, we call it "fractional order disturbance observer".

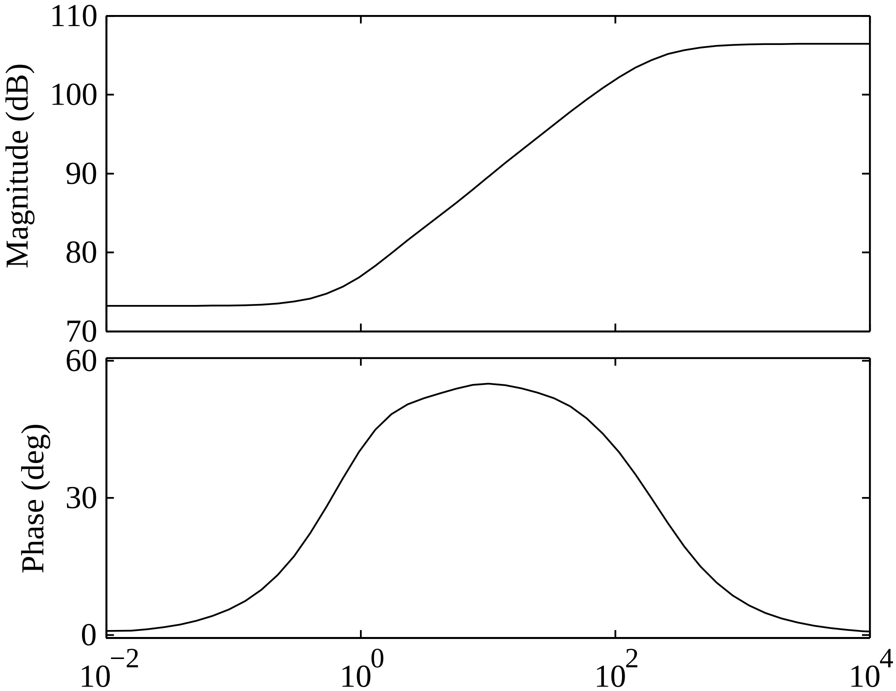
Now, n_Q can be a non integer. The design procedures will be as follows:

- Given the desired ω_Q and the desired PM
- There exist a unique real number n_Q as the relative degree of the Q-filter
- Implement the FO LPF, fine tune the n_Q using iterative feedback tuning (IFT) idea.

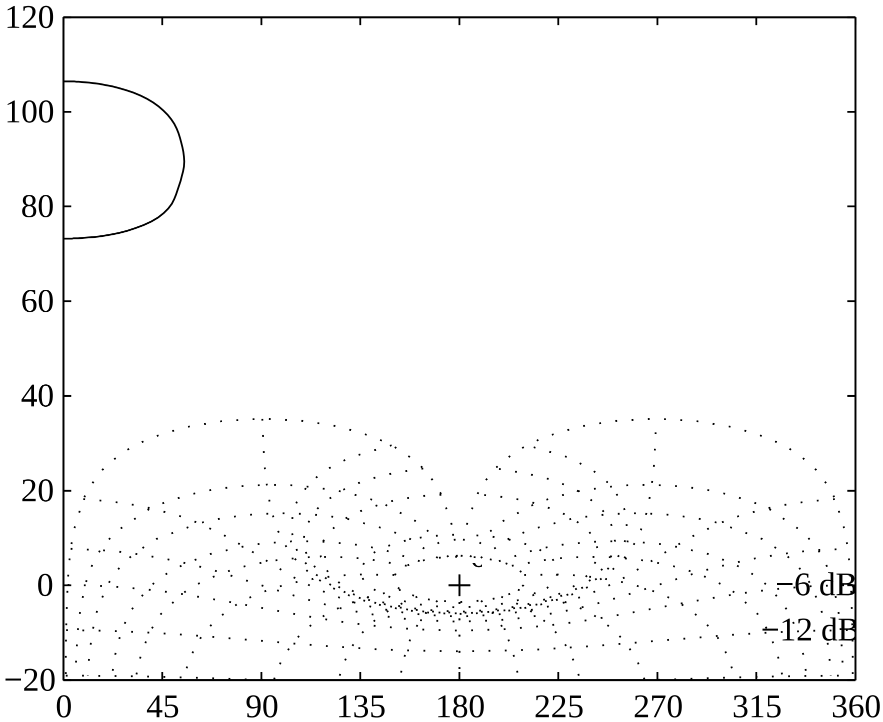
How to approximate the FO LPF? Another idea: **Stable minimum phase fitting using Matlab Toolbox `FDIDENT`** sample code:

```
%Istvan Kollar's stable transfer function fitting
f=logspace(-1,3,200)'; % frequency band of practical interest
Y=1./(j*2*pi*f).^(1/2); % the desired frequency response /s^0.5
U=ones(size(Y));    % set input to 1 to get I/O data
d=fiddata(Y,U,f);   % build the FIDdata
d.variance=[0,1e-6]; % artificial variance
if ~exist('order'), order=9; end order=yesinput('Order of
model',order,[1,inf]);
%First search for best cost function with stabilization:
disp('First search for best fit among trials...')
[m,finf]=elis(d,'s',order,order,...
    struct('stabilization','r','forceminimumphase','r',...
    'plotdens',-inf,'plot0','off'),...
    struct('displaymessages','off'));
[cfm,itmax]=min(finf.cfv); itmax=itmax-1;
fprintf('Now iterate until best fit, itmax = %.0f ...\n',itmax)
figure(1), clf
iterctrl % allow manual finishing iter. on fig. (select 'Finish')
% return the model object 'm', order/order, forcing stabilization:
m=elis(d,'s',order,order,...
struct('stabilization','r','forceminimumphase','r','itmax',itmax));
figure(2), plotelpz(m), zoom on % plot the pole-zero distribution
if ~exist('bi'), bi=2; end, if order==4, bi=2; end, bi=bi+1;
figure(bi), plot(m), zoom on  % plot the Bode magnitude
xlabel(sprintf('Order: %.0f/%.0f',order,order));drawnow
```

Refer to **FDIDENT** home page **http://elecwww.vub.ac.be/fdident**.



Bode plot for order 4/4 - Stable frequency domain fitting of $C_r(s) = C_0 \left(\frac{1+s/\omega_b}{1+s/\omega_h}\right)^r$, $C_0 = 4280.1$, $\omega_b = 0.5$, $\omega_h = 200$, $r = 0.65$.



Nichols chart for order 4/4 - Stable frequency domain fitting of $C_r(s) = C_0\left(\frac{1+s/\omega_b}{1+s/\omega_h}\right)^r$, $C_0 = 4280.1$, $\omega_b = 0.5$, $\omega_h = 200$, $r = 0.65$.

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Concluding Remarks

- With FOC, the proposed FO DOB idea is logically straightforward and should be widely accepted by control practitioners.
- Need to use experimental data to make the FO DOB convincing. However, it is obviously worth to apply.
- A simple and practical tuning procedure for n_Q, as in PID controller relay feedback tuning, is required. An iterative tuning procedure which can take into consideration of the FO LPF (Q-filter) truncation/realization errors is also acceptable.

Thank you!

Q/A session

FOC web pages:
http://mechatronics.ece.usu.edu/foc

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