A New Boundary Control Method for Beam Equation With Delayed Boundary Measurement Using Modified Smith Predictors

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Outline of Presentation

- Delays in boundary control of the beam equation: an unsolved problem

- Introduction to the Smith Predictor

- Problem formulation and a new simulation method

- Results of boundary control of the beam equation using (modified) Smith Predictors

- Concluding remarks

Delays in boundary control of the beam equation: an unsolved problem

- Control of partial differential equations (PDE)
- Boundary control of the beam equation
- The introduction of *unavoidable* small delays
  - Arbitrary small delays cause instability
  - There exists no robust controller (has been proved)
- Conclusion: *any* boundary controllers suffer from arbitrary small delays!

- Dilemma No. 1: why people are still studying the boundary control of the beam equation?
  - In reality, not *arbitrary* small delays cause problems.

    Reason: the beam equation is not a perfect abstraction
  - Then why we are still using the beam equation?

    Answer: the beam equation works well (in most cases)

- Dilemma No. 2: why we are still studying the delays?
  - Small delays *do* cause problems
  - No solutions yet

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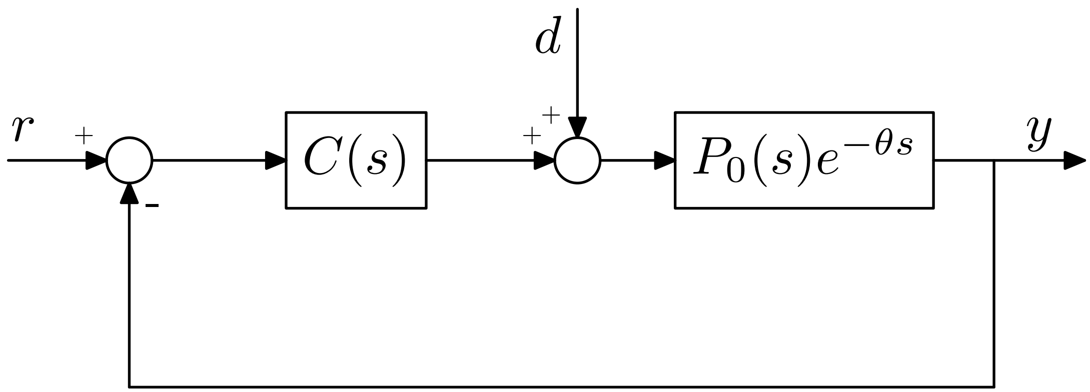
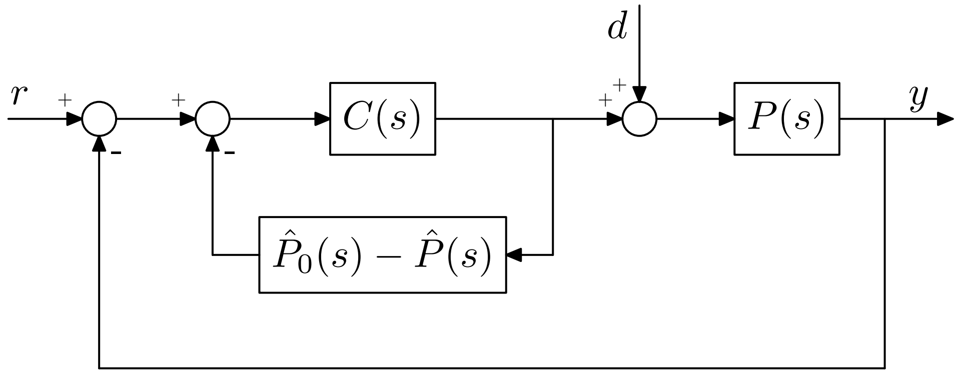
Introduction to the Smith Predictor



A feedback control system with a time delay

$$\frac{y(s)}{r(s)} = \frac{C(s)P_0(s)e^{-\theta s}}{1 + C(s)P_0(s)e^{-\theta s}}.$$

The time delay θ changes the closed-loop poles, which usually reduces the stability margin, or more seriously, destabilizes the system.



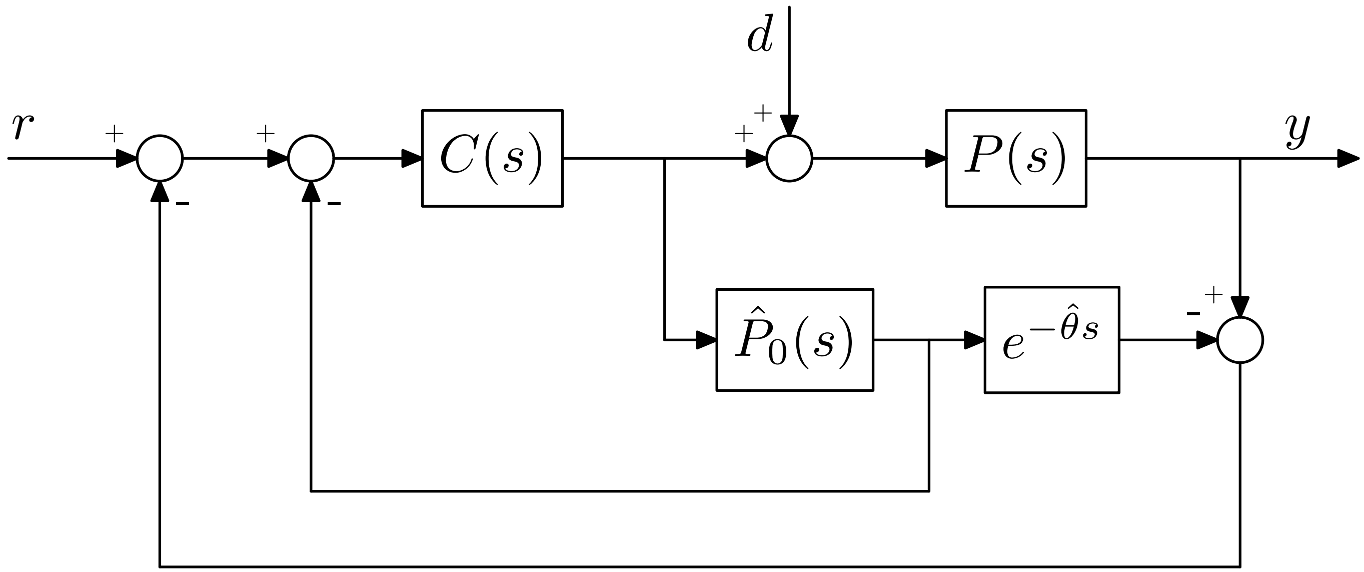
The Smith Predictor

$P(s) \doteq P_0(s)e^{-\theta s}$. $\widehat{P}_0(s)$ and $\widehat{P}(s)$ are nominal models of $P_0(s)$ and $P(s)$, respectively.

Assuming the perfect model matching, *i.e.*, $\widehat{P}(s) = P(s)$,

$$\frac{y(s)}{r(s)} = \frac{C(s)P(s)}{1 + C(s)P_0(s)}.$$

θ is removed from the denominator of the transfer function, making the closed-loop stability irrelevant to the time delays.



An alternative Smith predictor implementation

This implementation of the Smith Predictor makes the design more convenient.

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Problem formulation and a new simulation method

Consider a flexible beam clamped at one end and is free at the other end. The beam is controlled by a boundary control force at the free end.

$$
\begin{aligned}
u_{tt} + u_{xxxx} &= 0, \ x \in (0,1), \ t \geq 0 \\
u(0,t) &= 0, \\
u_x(0,t) &= 0, \\
u_{xx}(1,t) &= 0, \\
-u_{xxx}(1,t) &= f(t),
\end{aligned}
$$

where $u(x,t)$ is the displacement of the beam and $f(t)$ is the boundary control force.

- It is well-known that the following boundary controller stabilizes the system

$$f(t) = -k_d u_t(1, t),$$

where $k_d > 0$ is the constant gain and the suffix d shows it is a derivative gain.

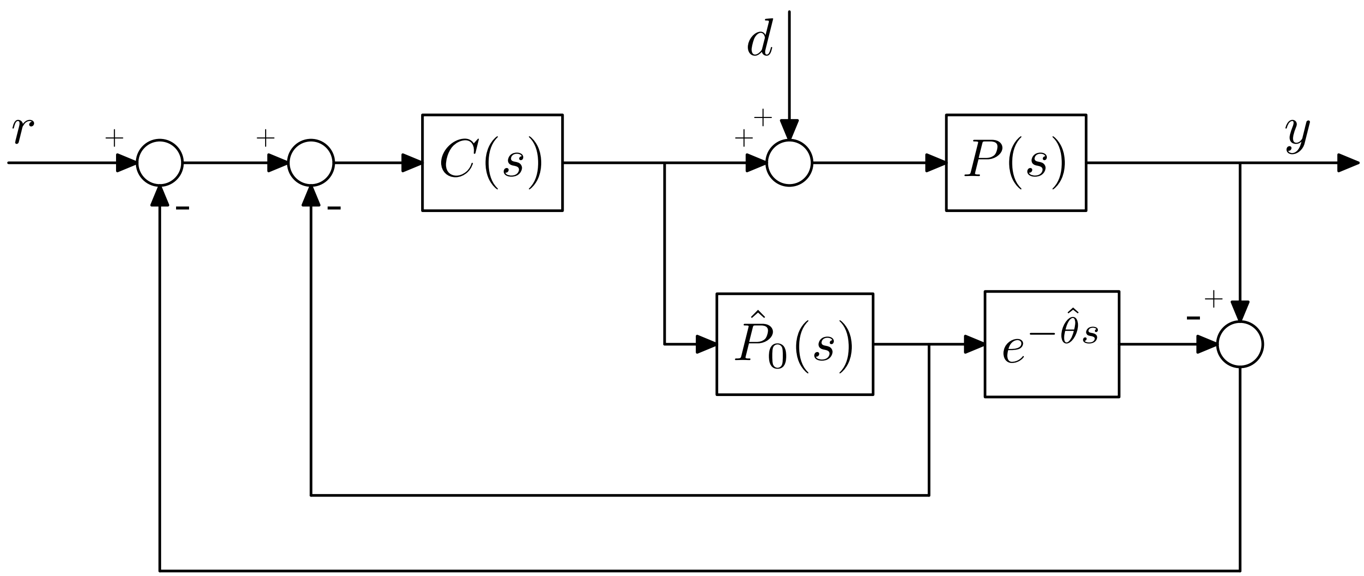
- When an arbitrary small time delay θ is introduced into the feedback loop,

$$f(t) = -k_d u_t(1, t - \theta),$$

the system becomes unstable.

- The Smith Predictor to be developed is based on the above controller.

Applying the Smith Predictor to boundary control of the beam equation



implementation of the Smith Predictor

- What is $C(s)$? $C(s) = k_d$

- What is $P_0(s)$? The transfer function from $f(t)$ to $u_t(1,t)$.

- How to get $P_0(s)$? Talk about it later.

- Expression of $P_0(s)$:

$$P_0(s)=\frac{(1-i)\sqrt[4]{-s^2}\left(1+ie^{2i\sqrt[4]{-s^2}}-ie^{-2\sqrt[4]{-s^2}}-e^{(-2+2i)\sqrt[4]{-s^2}}\right)}{s\left(e^{2i\sqrt[4]{-s^2}}+e^{(-2+2i)\sqrt[4]{-s^2}}+1+e^{-2\sqrt[4]{-s^2}}+4e^{(-1+i)\sqrt[4]{-s^2}}\right)}$$

- Expression of the final controller (Smith Predictor)? Too complicated to be shown within a slide

Where are we now?

- We have obtained a very complicated boundary controller to control a PDE.
- When $f(t)$ becomes this complicated, it is extremely hard to study the problem analytically
- There is even no readily available simulation method (to our best knowledge) for boundary control of PDEs. Matlab, Mathematica, Mathcad, Maple, and FEMLAB do not cover this kind of problems.
- We developed a simulation method combining the symbolic algebra and numerical computation.

General steps to do the simulation

- Take Laplace transform of all equations with respect to t to convert the original PDE of $u(x,t)$ with initial and boundary conditions to an ODE of $U(x,s)$ with boundary conditions.

- Call Matlab Symbolic Math Toolbox function `dsolve()` to symbolically solve the ODE and get the general solution $U(x,s)$ with four arbitrary constants.

- Call Matlab Symbolic Math Toolbox function `diff()` to get the first order, the second order, and the third order derivatives of $U(x,s)$. Now we have four equations with four unknowns.

General steps to do the simulation (cont.)

- Call Matlab Symbolic Math Toolbox function `solve()` to determine the four unknown constants. Now, we have obtained the explicit expression of $U(x, s)$.

- Take inverse Laplace transform of $U(x, s)$ numerically to get the final solution $u(x, t)$.

- $P_0(s)$ can be obtained in similar way

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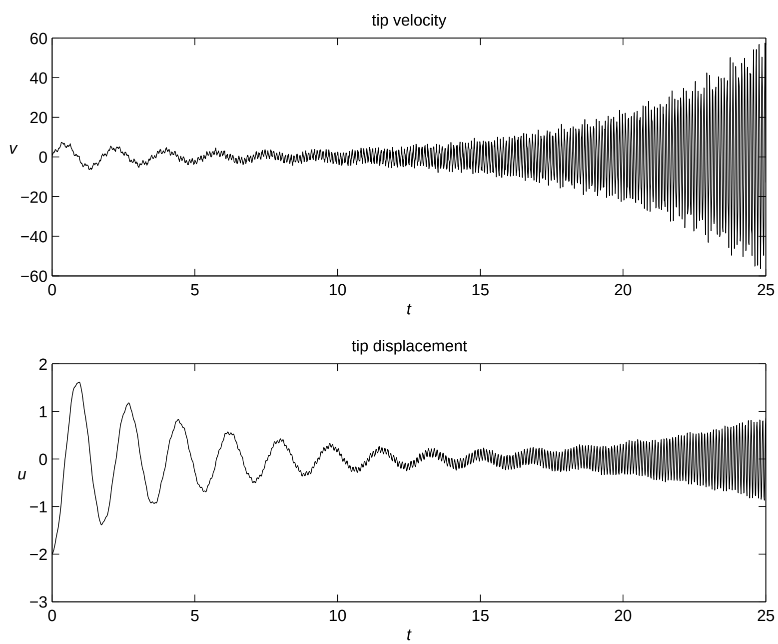
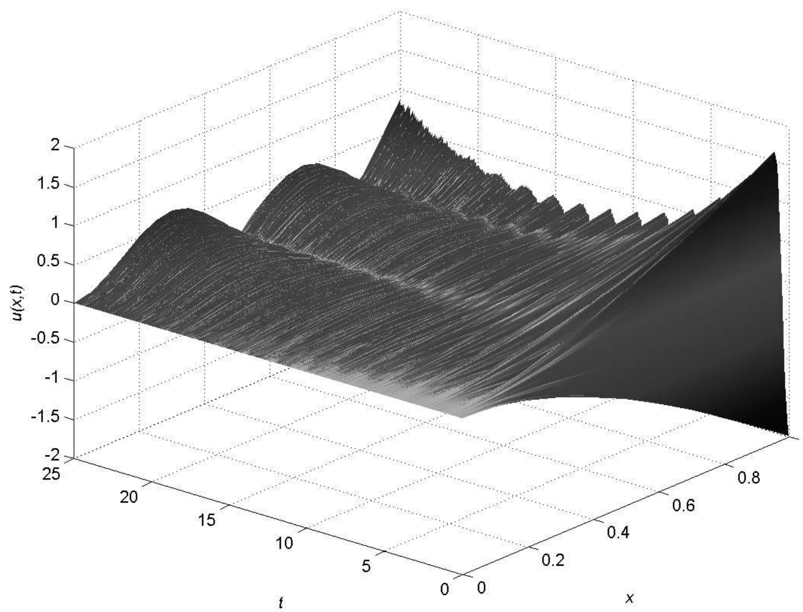
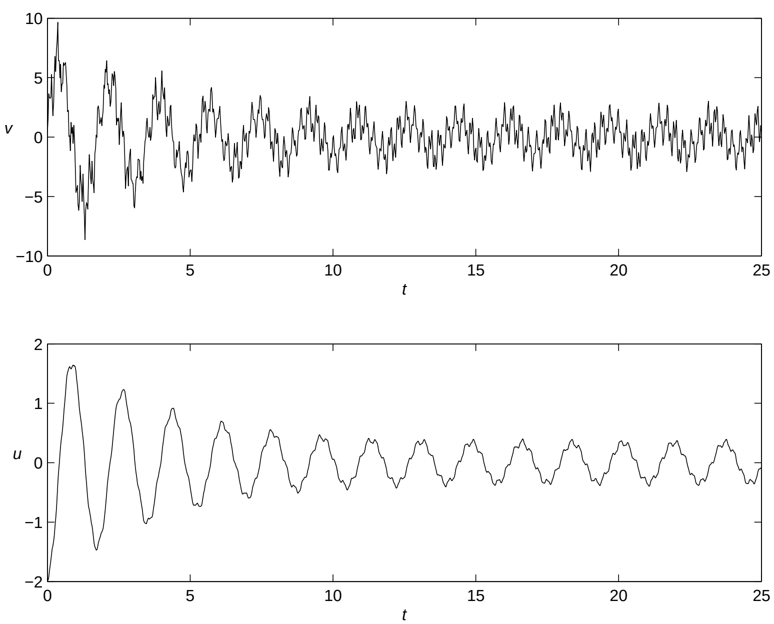
Results of boundary control of the beam equation using (modified) Smith Predictors

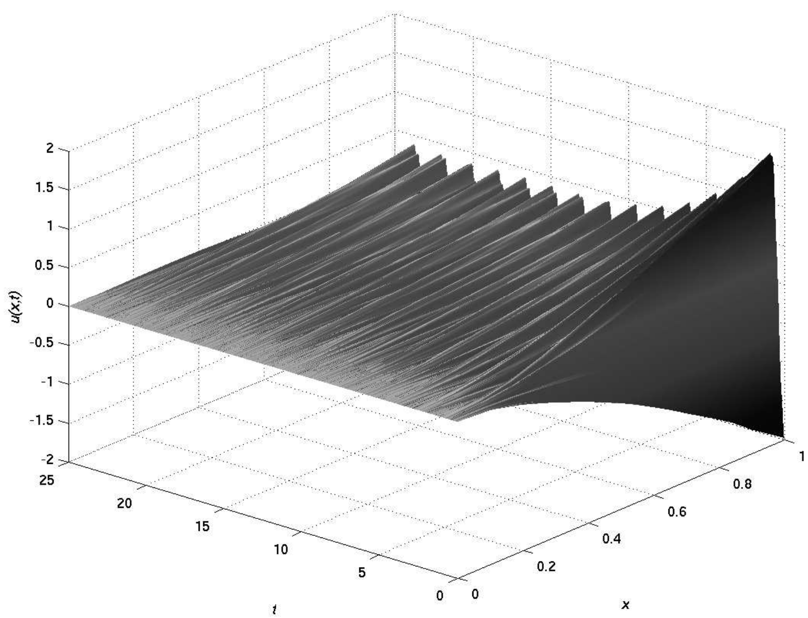
Results without using the Smith Predictor:





Results using the Smith Predictor





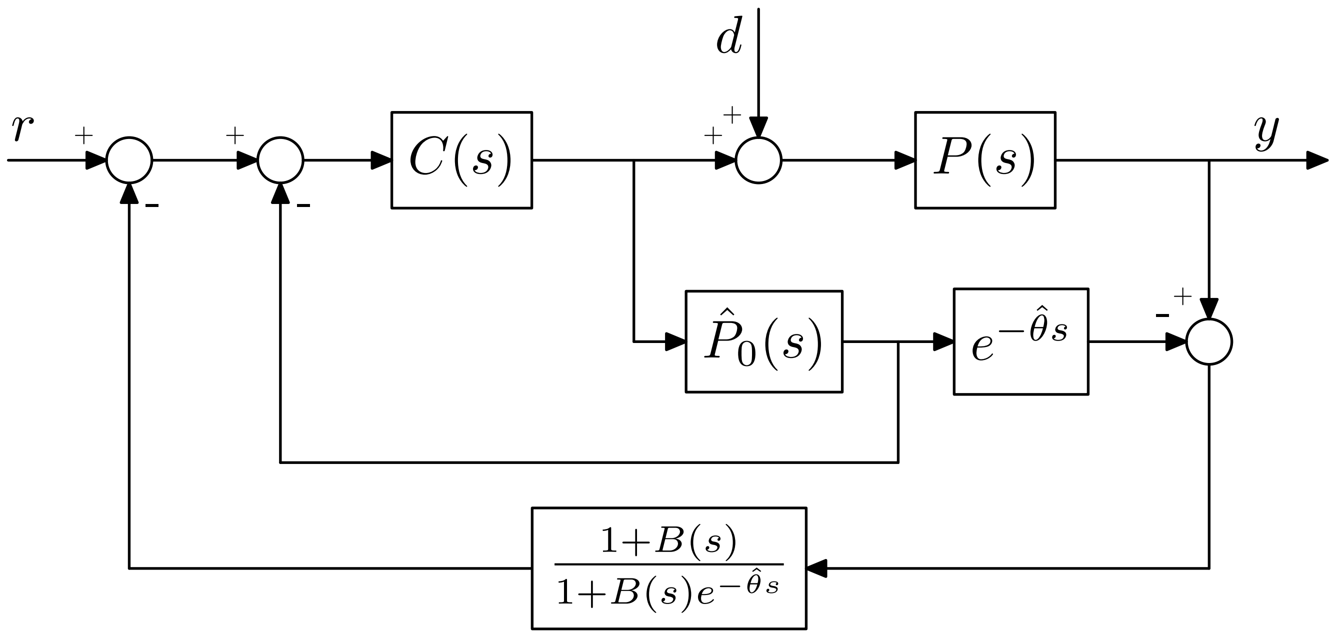
- Improvement: the velocity and displacement are not increasing to infinity any more

- However, the beam vibrates with a relatively large yet non-decreasing magnitude, rather than converging to zero position as we expected.

- The open-loop poles are presented in G_d, the transfer function from the response y to the disturbance d. These poles are excited by input disturbances but not by the reference. Depending on their locations relative to the closed-loop poles, these poles may dominate the response.

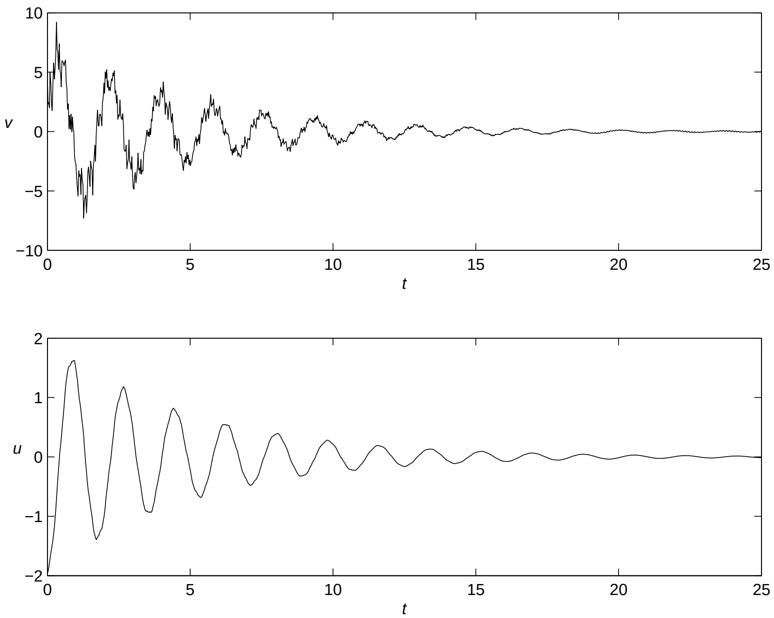
- The non-zero initial conditions act as disturbances, which deteriorate severely the regulation performance of the Smith predictor.

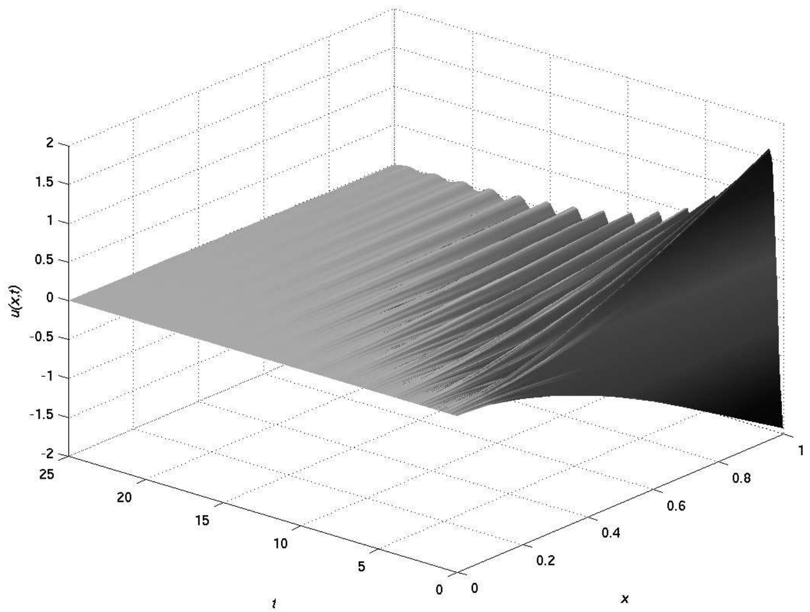
A modified Smith Predictor



$$B(s) = \frac{k}{1 + \tau s}.$$

Results using the modified Smith Predictor





Concluding remarks:

- The instability problem caused by arbitrary small delays is solved.

- Intuitively, it should be more effective when the Smith Predictor is applied to real beams (to be proved).

- The same method can be applied to boundary control of the wave equation.

Thank you!

Questions or comments?