Reliability Study of a Multiple Sensor Network System

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Abstract

Systems can only operate reliably and safely if their sensors work properly. Nowadays, more sensors are mounted to the system. However, the probability of success for each type of sensor is different. Determining the reliability of the whole sensor system is very important. Adding more sensors will increase the reliability of the sensor system, however, the price will also increase. Obtaining the highest reliability of the system at the lowest cost is also very important for each system. This paper describes an approach that uses search algorithms to maximize system reliability and minimize the sensor cost. Search algorithms such as tabu search, genetic search, and simulated annealing are used to determine the optimized solution. A new search algorithm is proposed and is very efficient. Numeric examples are also included in this paper.

1 Introduction

Multiple sensors are used for intelligent systems applications to counter their inaccuracy and uncertainty. During the past decade, computer technology has already produced hardware with switch density as high as 10^6 per cubic centimeters which is approximately the neuron density in the human brain [1], but sensory system development lags far behind. For example, cameras can be produced with 10^6 pixel elements, but human eyes have $250 * 10^6$ pixel elements. To compensate for the lag in sensory technology development, multiple sensors are used in the system design, not only because of their information gathering, but also for their failsafe capability.

Commonly used sensors for systems are video cameras, range finders, sonar, and tactile and infrared sensors. By employing such sensors, duplicate sensors can be employed for fault tolerance and to achieve tasks that can't be performed by a single sensor. For these reasons, multiple sensors have attracted much attention in recent years.

For example, assume there are N sensors, each component is independent and has an identical probability of success constant p. Let $q = 1 - p$, the assumption of statistical independence allows us to use Bernoulli's law, which finds the probability of i out of N components working at the same time as:

$$\binom{N}{i} p^i * q^{N-i} \tag{1}$$

If q is a function of t, say $q(t)$, then $q(t) = 1 - p(t)$, by using Bernoulli's law, the probability of i out of N components working at the same time t is:

$$\binom{N}{i} p(t)^i * q(t)^{N-i} \tag{2}$$

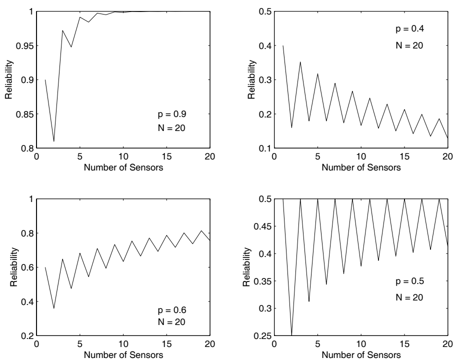

Figure 1: *The relationship between the number of sensors and the realibility*

It is guaranteed that a system will function correctly as long as more than half (in the case of a one dimensional sensor fusion) or more than two-thirds (

in the case of Byzantine Agreement) of the components function correctly [2]. So the reliability of the system is the summation of the terms with i varying from N to $[N/2]+1$. Let $N = 1, 2, \ldots, 20$, we will get the result shown in Figure 1. The maximum number of the sensors is set to 20, and the possibility of success of each sensor is set to 0.9, 0.6, 0.5, 0.4 respectively. It is easy to find out that for $p = 0.9$, only ten sensors will let the system have the possibility of success close to 1. For $p = 0.6$, the system reliability is increasing with respect to the number of the sensors, but not as fast as the curve of $p = 0.9$. For $p = 0.5$, the system reliability is oscillating with respect to the number of the sensors, the reliability is always less than the single sensor reliability. It is interesting to see that for $p = 0.4$, the system reliability is decreasing with respect to the number of sensors. That means if we want to mount multiple sensors onto the system, the reliability of each sensor should be more than 0.5, otherwise, the reliability of the system will decrease with more sensors.

2 The reliability model of multiple types of sensors

In the introduction, we considered only one type of sensor. In reality, there will be many types of sensors used in one system. We can first develop the reliability model for two types of sensors, and then extend the number of types of sensors.

For two types of sensors, assume that there are N_1 sensors in type one, and N_2 sensors in type two, respectively. Each sensor is independent. For the same type of sensor, the probability of success constant is the same. Let's assign p_1 and p_2 to type one and type two as the probability of success constant. Let $q_1 = 1 - p_1$, $q_2 = 1 - p_2$, and $N = N_1 + N_2$. The assumption of statistical independence allows us to use Bernoulli's law, which finds the probability of i_1 out of N components working at the same time as:

$$\sum_{i_2=0}^{i1} \left(\begin{array}{c} N_1 \\ i_2 \end{array} \right) {p_1}^{i_2} * {q_1}^{N_1-i_2} \left(\begin{array}{c} N_2 \\ i_1 - i_2 \end{array} \right) p_2^{i_1-i_2} * q_2^{N_2-(i_1-i_2)} \tag{3}$$

Let $r(i_1)$ denote the above equation, and define:

$$ben(p, n, i) = \left(\begin{array}{c} n \\ i \end{array} \right) p^i * (1-p)^{n-i} \tag{4}$$

Then equation 3 can be rewritten as:

$$r(i_1) = \sum_{i_2=0}^{min(i1,N_2)} BEN_2(i_1, i_2)) \tag{5}$$

where

$$BEN_2(i_1, i_2) = ben(p_1, n_1, i_1) * ben(p_2, n_2, i_1 - i_2) \tag{6}$$

The reliability of the system is the summation of the terms of equation 5 with i_1 varying from N to $[N/2]+1$. Use D_2 denote the reliability of the system, then

$$D_2 = \sum_{i_1=[N/2]+1}^{N} \sum_{i_2=0}^{min(i1,N_2)} ben(p_1, n_1, i_1) * ben(p_2, n_2, i_1 - i_2) \tag{7}$$

It is easy to extend the type of sensors from 1, 2, to m. Each type of sensor has N_i components, $[N_1, N_2, \ldots, N_m]$, and the probability of success p_i, $[p_1, p_2, \ldots, p_m]$. Let $N = \sum_{i=1}^{m} N_i$, and the multiple variable Bernoulli term be $BEN_m(i_1, i_2, \ldots, i_m)$

$$BEN_m(i_1, i_2, \ldots,_m) = (\prod_{j=i_2}^{i_m} ben(p_1, n_1, i)) * ben(p_m, n_m, i_1 - \sum_{k=i_2}^{i_m} i) \tag{8}$$

So the probability of i_1 out of N components working at the same time is:

$$r(i_1) = \sum_{i_2=0}^{i1} \sum_{i_3=0}^{i1-i_2} \cdots \sum_{i_m=0}^{i1-\sum_{k=i_2}^{i_m} i} BEN_m(i_1, i_2, \ldots, i_m) \tag{9}$$

The reliability of this multi type component system is the summation of the terms of equation 9 with i_1 varying from N to $[N/2] + 1$. Use D_m denote the reliability of the system, then

$$D_m = \sum_{i_1=[N/2]+1}^{N} r(i_1) \tag{10}$$

2.1 Cost constrains in redundancy systems

Multiple sensors are used for the systems to increase the reliability of the system, however, they also raise the issue of the cost of the system. There is a tradeoff between system cost and reliability. If the reliability and cost of each component are known, then the problem is to find out the reliability of the whole system subject to the cost limit. We can state the problem as follows:

Assume there are m sensors. For each sensor, the cost and reliability of the sensor are c_1 and p_1 respectively. Let $[N_1, N_2, \ldots, N_m]$ be the combinations of m types of sensors, such that

$$\sum_{i=1}^{m} N_i * c_i < Maximum\_Cost \qquad (11)$$

The problem is to find out the optimal combination of sensors such that the reliability of the system is maximized. Because the reliability equation is nonlinear, it cannot be solved by mathematical programming techniques. Some heuristic approaches such as tabu search, genetic algorithms, and simulated annealing can be found in [2]. However, these methods cannot guarantee the solution is optimal. In the next section, we discuss the above three algorithms, and we provide improved algorithms for this problem.

2.2 Optimization methods

- Tabu search

  A tabu search creates a list of nodes in the search space, which were visited by the search algorithm [3, 4]. These points then become "tabu" for the algorithm, where "tabu" means that these points are not revisited as long as they are on the list. This will allow the search algorithm to climb out of a shallow local minimal in the search processing.

- Genetic Algorithms

  The genetic algorithm attempts to apply the concept of "survival of the fittest" to optimization problems [4, 5, 6]. Biological systems adapt themselves to fit into an ecological niche. The same evolutionary process will lead to series fit answers. This approach has been shown to be experimentally useful for solving optimization problems. Possible solutions to a problem are called chromosomes, which are grouped into gene pool. The next generation is formed via crossover and mutation.

- Simulated annealing

  Simulated annealing tries to find the optimal solution to a problem in a process like the formation of cooling crystals [7]. The idea is that the melted crystals will cool down to a minimal energy state. The strategy is follows: Given a configuration of the elements of the system, randomly displace the elements, one at a time, by a small amount, and calculate the resulting change in energy, ΔE. If $\Delta E < 0$ then accept the displacement and use the resulting configuration as the starting point for the next generation. If $\Delta E > 0$ then the displacement is accepted with probability $p(\Delta E) = exp(-\Delta E/k_b T)$ where T is temperature and k_b is Boltzmann's constant.

The above three algorithms each has its advantages and disadvantages. The tabu search is more sensitive to local minimal than the genetic algorithms, since it focuses searching its very vicinity. It will quickly converge to local minimum and take a long time to climb out of the local minimum. Simulated annealing has the same drawbacks as tabu search. The genetic algorithm doesn't look for the local minimum. None of the three algorithms can guarantee the solution is best. There are also other algorithms. Such as neural network and fuzzy set approach [8, 9] and other heurist search algorithms [10].

Our improved algorithm guarantees the optimal solution without an exhaustive search. Through studying the equation

$$\sum_{i=1}^{m} N_i * c_i < Maximum\_cost, \qquad (12)$$

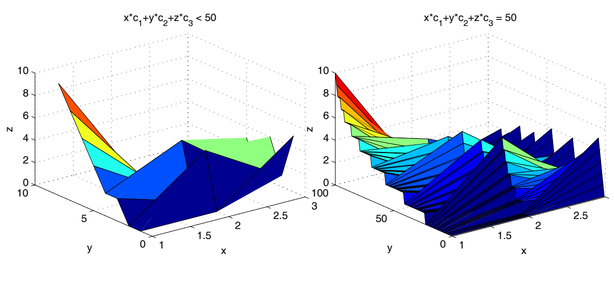

Figure 2: *3 types of sensor example*

we find there is a very large number of solutions. We need to minimize this solution space as far as possible. From the introduction, it is easy to see that more sensors will result in a more reliable system. Inspired by this idea, we tried to find the sets of N_i, such that the $cost = \sum_{i=1}^{m} N_i * c_i$ is less than and closest to the $Maximum\_cost$. The final optimal solution is guaranteed in these sets. Here is a numeric example. Assume that there are three types of sensors, their unit price are 5\$, 8\$ and 10\$, and the $Maximum\_cost = 50$. The whole solution space for $5 * N_1 + 8 * N_2 + 10 * N_3 < 50$ is show on the right side of figure 2, while the optimal solution space is shown on the left side of Figure 2. The number of original sets is 100 and the number of optimal sets is 8. The size of the solution space is greatly reduced.

Here is the new search method:

step1: Sort cost vector $[c_1, c_2, \ldots, c_m]$ in ascending order.
step2: make the type $cc = [0, 0, \ldots, N_m]$, $N_m = int(Maximum\_cost/c_m)$ the start candidate.
step3: add cc to the optimal list.
step4: reduce N_m by 1, $maxprice = maxprice - c_m$, add cc to the optimal list.
step5: Solve $[c_1, c_2, \ldots, c_{m-1}]$, maxprice problem .
step6: Add result of step 5 to optimal list.
step7: if $N_m \neq 0$, goto step 4.

The method is a recursive method. It always finds the configuration of the components such that the reliability is maximized with the price limit. From the above search method, the optimal list is much shorter than the original list. However, the optimal list will still be very large, if the number of the component type is very large. Through experiment, we find out that if we use $averge = \sum N_i * p_i / \sum N_i$ as a criteria, where p_i is the probability of success of each type, and then set the threshold as middle point of the average set. We will have a far more reduced set of list, which can be called final list.

Because the final list is very small, it is easy to calculate the reliability of each type from the final list and obtain the optimal value. The simulation results are included in the next section.

3 Numerical experiment

Tables $1, 2$ and 3 are the results from numerical example, which is based on reference [2]. Eleven possible component types are used to construct the sensor system. Table 1 shows the sensor cost and probability of success.

Sensor	cost	success
1	\$20	0.8333
2	\$10	0.6667
3	\$20	0.8617
4	\$5	0.6552
5	\$25	0.8911
6	\$15	0.7241
7	\$7	0.5882
8	\$8	0.7284
9	\$20	0.8750
10	\$6.8	0.6481
11	\$7	0.6452

Table1: Table of sensor cost and possibility of success

Table two gives the best sensor configuration found by all three heuristics and the improved search method when cost limits of \$58 and \$52 are used. From these two examples, we can see that only tabu search find the single sensor configuration, however, it is not the best configuration. The entire best configuration is the combination of the sensors.

	cost 52				cost 58			
S	Ta	Sa	Ga	new	Ta	Sa	Ga	new
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0
5	0	0	0	0	0	2	1	2
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	3	1	1	1	7	1	1	1
9	1	2	2	2	0	0	1	0
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0

Table2: Sensor configuration under cost limit

Table 3 shows the differences among the methods. Our search method, which is labeled "new" in the table, guarantees that the final solution is optimal. The three heuristics are very close to optimal. In fact, the simulated annealing method also reaches optimal value in this case.

Because the dimension of this problem is low, it is possible to plot the relationship between the reliability and the average probability of success for the space after application of the improved search method. It is apparent that the reliability of the system tends to be high, when the average probability of success is high.

Through a numerical experiment, we found that if we use the middle point as a threshold, we will always have the optimal configuration in the final list.

cost limit	algor	Total Sensor	cost	reli
52	Ta	5	49.70	89%
52	Sa	3	49.40	92%
52	Ga	3	49.40	92%
52	New	3	49.40	92%
58	Ta	7	56.00	91%
58	Sa	3	58.00	94%
58	Ga	3	53.70	93%
58	New	3	58.00	94%

Table3: Difference among the methods

From the table 4, we see that the new search method is very efficient. When the cost limit is 52, the solo configuration is $[2, 5, 2, 10, 2, 3, 7, 6, 2, 7, 7]$, and the number of configurations under the cost constraint is 4939200. After we apply the search method, the number of configuration is reduced to 1599. The number is 87 by the threshold method. The total reduce rate is 56772. Similar for the cost limit is 58.

cost	whole space	new	Threshold	ratio
52	4939200	1599	87	56772
58	9461760	2768	371	25503

Table4: Efficiency of the method

Because the dimension is low for this case, the reliability of all the configurations can be calculated, and the relationship between the reliability and the average probability of success can be derived, as shown in figure 3.

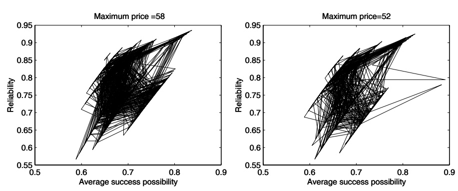

Figure 3: *Relationship between the reliability and the average probability of success*

We can use following third order polynomial to fit the data, here $\tilde{y}$ denotes the estimation of y:

$$\tilde{y} = ax^3 + bx^2 + cx + d \quad (13)$$

Using the LMS method, it is easy to find the coefficients for each case. The linear recursive curve is shown in figure 4.

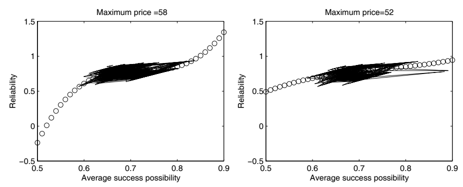

Figure 4: *Linear recursive curve fit*

We find that the curves fit well in figure 4. To find the characteristic of the original data, we use the following model to describe the system.

$$y = \tilde{y} + e = ax^3 + bx^2 + cx + d + e \quad (14)$$

e denotes the error between the original data and the fitted data. The distribution of the error can be obtained by a Q-Q plot [11]. The steps for the Q-Q plot are:

- 1. Order the errors to get $e_1, e_2, \ldots, e_N$, and their probability values $(1-1/2)/N$, $(2-1/2)/N$, ..., $(N-1/2)/N$;
- 2. Calculate the standard normal quantiles q_1, q_2, ..., q_N.
- 3. Plot the pairs of observations (q_i, e_i)

The correlation coefficient for the Q-Q plot is defined by

$$r_Q = \frac{\sum_{j=1}^{N}(e_j - \bar{e})(q_j - \bar{q})}{\sqrt{\sum_{j=1}^{N}(x_j - \bar{x})^2}\sqrt{\sum_{j=1}^{N}(q_j - \bar{q})^2}} \quad (15)$$

The Q-Q plot is shown in figure 5. From the correlation, we find that the error is close to a normal distribution.

4 Conclusion

In this paper, several search methods are reviewed, and a new search method is developed for the problem addressed in section 2. We found out that the three heuristics tried can find a close to optimal configuration. Among them, the simulated annealing method can find solutions, which are as good as, or better than

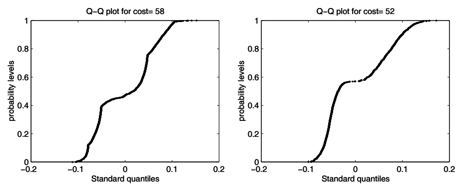

Figure 5: *Q-Q plot for data in figure 4*

the other two heuristics. None of the three heuristics is guaranteed to find the optimal configuration. Our new search method can find the optimal solution for this case. However, in this paper, the assumptions used for the problem are too restrictive to allow generalization. It assumes that each component is independent, in reality that may not be true. There are some approaches for this problem, such as PDOP [12]. These should also be considered in further research.

The proposed algorithms can be used to evaluate the reliability of the robot system. The sensors can be the multiple sensors arrays, such as linear infrared sensor array, sonar sensor array and so on. Further research will extend the reliability of the system with multiple hybrid sensor and sensor arrays.

References

[1] Julius T. Tou and Jens G. Balchen, Highly Redundant Sensing in Robotic Systems. NATO ASI Series

[2] R. R. Brooks and S. S. Iyengar, "Maximizing Multi_sensor System Dependability", Proceedings of the 1996 IEEE/SICE/RSJ International Conference on Multi sensor Fusion and Integration for Intelligent Systems, pp 1-8 1996

[3] R. Battiti and G. Tecchioli, "The Reactive Tabu Search", ORSA Journal on Computing 6(2) pp. 126-140, spring 1994

[4] R.R. Brooks, S.S. Iyengar and Jianhua Chen, "Self-calibration of a noisy multiple sensor system with genetic algorithms", SPIE Vol. 2594, pp 20-30. 1996

[5] J. C. Bean, "Genetic Algorithms and Random Keys for sequencing and Optimization", ORSA Journal on Computing 6(2) pp. 154-160, spring 1994

[6] David W. Coit and Alice E. Smith, "Reliability Optimization of Series-Parallel Systems Using a Genetic Algorithm", IEEE Transactions on Reliability Vol. 45 No. 2, pp 254-260. 1996

[7] P. J. M. van Laarhoven and E.H.L. Aarts, "Simulated Annealing: Theory and Applications", D. Reidel Publishing Co., Dordrecht, 1987

[8] J.D. Wang and T.S. Liu, "Fuzzy Reliability Using a Discrete Stress-Strength Interface Model", IEEE Transactions on Reliability Vol. 45 No. 1, pp 145-149. 1996

[9] P. V. Suresh, "Fuzzy-set Approach to select Maintenance Strategies for Multistate Equipment", IEEE Transactions on Reliability Vol. 43 No. 3, pp 451-456. 1994

[10] Ming J. Zuo, L. F. Choy, and Richard C. M. Yam, "A Model for Optimal Design of Multi-State Parallel-Series Systems", Proceedings of the 1999 IEEE Canadian Conference on Electrical and Computer Engineering, pp 1770-1773. 1999

[11] Richard A. Johnson and Dean W. Wichern, "Applied Multivariate Statistical Analysis", Prentice Hall, Englewood Cliffs, New Jersey. 1988

[12] Joris W. M. van Dam, Ben J.A. Krose and Franciscus C.A. Groen, "Adaptive Sensor Models", Proceedings of the 1996 IEEE/SICE/RSJ International Conference on Multisensor Fusion and Integration for Intelligent Systems, pp 705-712. 1996