

Stability of Linear Time Invariant Systems with Interval Fractional Orders and Interval Coefficients

Ivo Petrás, YangQuan Chen, Blas M. Vinagre and Igor Podlubny

Abstract— This paper considers the stability of the fractional-order linear time-invariant systems with interval type uncertainties in both fractional orders and the relevant coefficients. It is an extension of our previous result on the robust stability analysis of the interval type uncertain fractional order LTI systems with crisply known fractional orders. We have proposed a stability check procedure for uncertain LTI systems with interval fractional orders and interval coefficients. The procedure is demonstrated with some simple examples.

Keywords— Fractional order dynamic systems, robust stability check, interval uncertain systems, interval uncertain orders, interval uncertain dynamics.

I. INTRODUCTION

Recently, a robust stability test procedure is proposed for linear time-invariant fractional order systems (LTI FOS) of commensurate orders with parametric interval uncertainties [1]. The proposed robust stability test method is based on the combination of the argument principle method [2] for LTI FOS and the celebrated Kharitonov's edge theorem [3]. In general, an LTI FOS can be described by the differential equation or the corresponding transfer function of non-commensurate real orders [4] of the following form:

$$G(s) = \frac{b_m s^{\beta_m} + \dots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \dots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}} = \frac{Q(s^{\beta_k})}{P(s^{\alpha_k})}, \quad (1)$$

where α_k, β_k ($k = 0, 1, 2, \dots$) are real numbers and without loss of generality they can be arranged as $\alpha_n > \dots > \alpha_1 > \alpha_0$, $\beta_m > \dots > \beta_1 > \beta_0$. The coefficients a_k, b_k ($k = 0, 1, 2, \dots$) are uncertain constants within a known interval.

It is well-known that an integer order LTI system is stable if all the roots of the characteristic polynomial $P(s)$ are negative or have negative real parts if they are complex conjugate (e.g. [5]). This means that they are located on the left of the imaginary axis of the complex s -plane. When dealing with non-commensurate order systems (or, in general, with fractional order systems) it is important to bear in mind that $P(s^\alpha)$, $\alpha \in \mathbb{R}$ is a multivalued function of s ,

the domain of which can be viewed as a Riemann surface (see e.g. [7]). For distributed parameter systems with a distributed delay, [8] provided a stability analysis method which may be used to test the stability of fractional order differential equations. In [9], the co-prime factorization method is used for stability analysis of fractional differential systems. However, the robust stability test procedure and proof of the validity for the general type of the LTI FOS described by (1) is still open as discussed in [6].

In this paper, we consider an even more complicated situation where the fractional orders in (1) are not exactly known but interval real numbers with known interval bounds. For example, consider the following simple case (a linear time invariant system with interval fractional order α , interval gain k and interval parameter a):

$$G(s) = \frac{k}{s^\alpha + a} \quad (2)$$

where α , a and k are *not* exactly known. It is known, however, that $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $a \in [\underline{a}, \bar{a}]$ and $k \in [\underline{k}, \bar{k}]$ with the bounds $\underline{\alpha}$, \underline{a} , \underline{k} , $\bar{\alpha}$, \bar{a} , and \bar{k} precisely known.

A question related to controlling the above simple plant is: can we find a simple proportional control law to stabilize it? This amounts to checking the stability property of the following characteristic "polynomial":

$$p(s) = s^\alpha + a + kK_p \quad (3)$$

where K_p is the controller gain.

The motivation of this problem can be from the modelling of the dynamics of nonferrous materials and/or their composites such as the nonlaminated magnetic bearing. The fractional order of the LTI model may be prompted by the mean slope of the Bode plot. The variance of the slopes of the measured Bode plots can be practically described by using an interval of width $\pm 3\sigma$ centered at the mean slope.

II. STABILITY CHECK WHEN THE FRACTIONAL ORDERS ARE CRISP AND COMMENSURATE

For the LTI FOS with no uncertainty, the existing stability test (or check) methods for dynamic systems with integer-orders such as Routh table technique, cannot be directly applied. This is due to the fact that the characteristic equation of the LTI FOS is, in general, not a polynomial but a pseudo-polynomial function of the fractional powers of the complex variable s .

Of course, being the characteristic equation a function of a complex variable, stability test based on the argument principle can be applied. On the other hand, it has been

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shown, by several authors and by using several methods, that for the case of LTI FOS of commensurate order, a geometrical method based on the argument of the roots of the characteristic equation (a polynomial in this particular case) can be used for the stability check in the BIBO (bounded-input bounded-output) sense (see e.g. [10]).

In the particular case of *commensurate order* systems, it holds that, $\alpha_k = \alpha k, \beta_k = \alpha k, (0 < \alpha < 1), \forall k \in \mathbb{Z}$, and the transfer function has the following form:

$$G(s) = K_0 \frac{\sum_{k=0}^M b_k (s^\alpha)^k}{\sum_{k=0}^N a_k (s^\alpha)^k} = K_0 \frac{Q(s^\alpha)}{P(s^\alpha)} \quad (4)$$

With $N > M$, the function $G(s)$ becomes a proper rational function in the complex variable s^α which can be expanded in partial fractions of the following form:

$$G(s) = K_0 \left[\sum_{i=1}^N \frac{A_i}{s^\alpha + \lambda_i} \right] \quad (5)$$

where $\lambda_i (i = 1, 2, \dots, N)$ are the roots of the polynomial $P(s^\alpha)$ or the system poles which are assumed to be simple without loss of generality. The stability condition can then be stated as follows [2], [10]:

A commensurate order system described by a rational transfer function (4) is stable if $|\arg(\lambda_i)| > \alpha \frac{\pi}{2}$, with λ_i the i -th root of $P(s^\alpha)$.

For the LTI FOS with commensurate order where the system poles are in general complex conjugate, the stability condition can be expressed as follows [2], [10]:

A commensurate order system described by a rational transfer function $G(\sigma) = \frac{Q(\sigma)}{P(\sigma)}$, where $\sigma = s^\alpha, \alpha \in \mathbb{R}^+, (0 < \alpha < 1)$, is stable if $|\arg(\sigma_i)| > \alpha \frac{\pi}{2}$, with σ_i the i -th root of $P(\sigma)$.

The *robust stability* test procedure for the LTI FOS of commensurate orders with interval parametric uncertainties can be divided into the following steps [1]:

- **step1:** Transcribe the LTI FOS $G(s)$ of the commensurate order α , to the equivalent system $H(\sigma)$, where the transformation is: $s^\alpha \rightarrow \sigma, \alpha \in \mathbb{R}^+$;
- **step2:** Write the interval polynomial $P(\sigma, q)$ of the equivalence system $H(\sigma)$, where interval polynomial is defined as

$$P(\sigma, q) = \sum_{i=0}^n [q_i^-, q_i^+] \sigma^i;$$

- **step3:** For interval polynomial $P(\sigma, q)$, construct four Kharitonov's polynomials: $p^{--}(\sigma), p^{-+}(\sigma), p^{+-}(\sigma), p^{++}(\sigma)$;
 - **step4:** Test the four Kharitonov's polynomials whether they satisfy the stability condition: $|\arg(\sigma_i)| > \alpha \frac{\pi}{2}, \forall \sigma \in \mathbb{C}$, with σ_i the i -th root of $P(\sigma)$;
- Note that for low-degree polynomials, less Kharitonov's polynomials are to be tested:
- Degree 5: $p^{--}(\sigma), p^{-+}(\sigma), p^{+-}(\sigma)$;
 - Degree 4: $p^{+-}(\sigma), p^{++}(\sigma)$;
 - Degree 3: $p^{+-}(\sigma)$.

We demonstrated this technique for the robust stability check for the LTI FOS with parametric interval uncertainties through some worked-out illustrative examples in

[1]. In [1], the time-domain analytical expressions are also available and therefore the time-domain and the frequency-domain stability test results (see also [11]) can be cross-validated.

III. STABILITY CHECK WHEN THE FRACTIONAL ORDERS ARE ALSO INTERVAL REAL NUMBERS

In the general case of *fractional order* systems the transfer function has the form (1). Here, we consider a more specific form of (1) with interval uncertainties in the fractional orders and parameters. Then, the Kharitonov polynomial can be expressed in the following "new" general form:

$$P(s, o, q) = \sum_{i=0}^n [q_i^-, q_i^+] s_i^{[o^-, o^+]}, \quad (6)$$

where q^\pm is parametric interval uncertainties and o^\pm is interval order uncertainties.

Now the question is: can we extend the Kharitonov theorem to interval polynomial with interval orders as in (6) as well? Our conjecture is "yes" for some situations. Intuitively, we should have 16 edge polynomials: $p^{--}(s), p^{-+}(s), p_{+-}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s), p_{++}(s)$, where the upper index is for the order intervals and the lower index is for parameter intervals.

The general form above is not very appropriate for stability investigation because there is a large number of polynomials and we need to reduce this number in practice. Let us consider the combination approach. For examples, if we have two intervals (one for parameter and one for order), we can write four new Kharitonov polynomials; if we have three intervals (two for parameters and one for order), we can write eight polynomial, etc. In generally, we can write 2^n polynomials where n is the total number of intervals in the polynomial (6).

Fortunately, in practice, we usually use two or three terms at most in the fractional order mathematical models (see e.g. [4]). In this case, we will have to investigate four or eight edge polynomials.

In what follows, we consider a special case where the non-integer order intervals are assumed to have commensurate limits for orders. This assumption is not unreasonable in the sense that, we can slightly fine adjust (enlarge) the order intervals so that the bounds are commensurate. Then, we can apply the transform method $s^\alpha \rightarrow \sigma$ to check the stability in σ -complex plane.

Therefore, the *robust stability* test procedure for the LTI FOS of commensurate orders with parametric interval and order uncertainties is similar to the previous one presented in Sec. II for crisp orders. The robust stability test procedures are summarized in the following:

- **step1:** Rewrite the LTI FOS $G(s)$ of the commensurate order α , to the equivalence system $H(\sigma)$, where transformation is: $s^\alpha \rightarrow \sigma, \alpha \in \mathbb{R}^+$;
- **step2:** Write the interval polynomial $P(\sigma, o, q)$ of the equivalence system $H(\sigma)$, where interval characteristic

polynomial is defined as

$$P(\sigma, o, q) = \sum_{i=0}^n [q_i^-, q_i^+] \sigma_i^{[o^-, o^+]};$$

- **step3:** For interval polynomial $P(\sigma, o, q)$, construct 2^n Kharitonov polynomials;
- **step4:** Test the 2^n Kharitonov's polynomials whether they satisfy the stability condition: $|\arg(\sigma_i)| > \alpha \frac{\pi}{2}$, $\forall \sigma \in \mathbb{C}$, with σ_i the i -th root of $P(\sigma)$;

The *robust stability* test procedure for the LTI of fractional orders with parametric interval and order uncertainties, in general form (1), can be divided into the following steps:

- **step1:** Write the interval polynomial $P(s, o, q)$ of the system $G(s)$, where interval polynomial is defined as (6);
- **step3:** For interval polynomial $P(s, o, q)$, construct 2^n Kharitonov polynomials;
- **step4:** Test the 2^n Kharitonov's polynomials whether they satisfy the stability condition: $|\arg(s_i)| > \frac{\pi}{2}$, $\forall s \in \mathbb{C}$, with s_i the i -th root of $P(s)$;

Note that for pseudo-polynomials of fractional power we can effectively use Matlab Symbolic Math Toolbox. We will demonstrate this technique for the robust stability check for the LTI FOS with parametric and order interval uncertainties through the following illustrative examples.

IV. ILLUSTRATIVE EXAMPLES

A. A Simple Motivating Example

Consider the simple case mentioned in Sec. I where $k = 1$, $a = 1$ and fractional order α is within the interval $\alpha \in [1, 2)$:

$$G(s) = \frac{1}{s^\alpha + 1} \quad (7)$$

We assume that the above simple plant will stabilize with simple proportional controller with the transfer function $C(s) = K_p$. To investigate the stability property is necessary to check out the following characteristic pseudo-polynomial:

$$p(s) = s^\alpha + 1 + K_p \quad (8)$$

where K_p is the controller gain. In this particular case we will use the controller gain $K_p = 2$ and then we obtain the characteristic pseudo-polynomial in the form: $p(s) = s^\alpha + 3$, where α is within the considered interval $\alpha \in [1, 2)$.

The resulting transfer function of closed control loop has the following form:

$$G_c(s) = \frac{2}{s^\alpha + 3} \quad (9)$$

B. Graphical Method

Figure 1 shows the pole positions in the complex plane for characteristic pseudo-polynomial with various values of α .

Figure 2 shows the unit step responses for various values of order $\alpha \in [1, 2)$ and for time $T = 7$ sec.

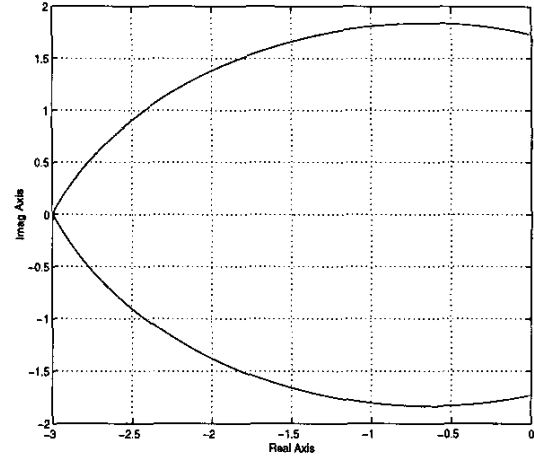


Fig. 1. Pole locations for characteristic pseudo-polynomial $p(s) = s^\alpha + 3$ for $\alpha \in [1, 2)$

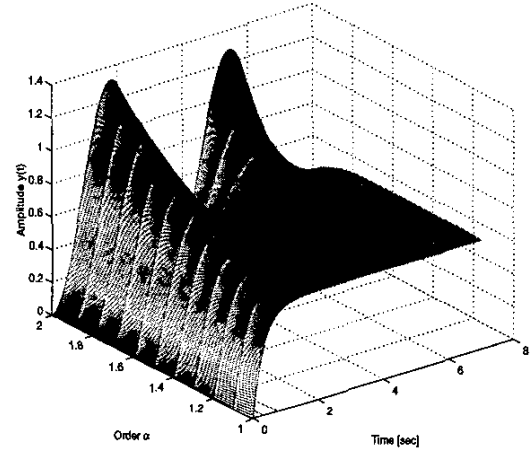


Fig. 2. Transient unit step responses for (9) for various order $\alpha \in [1, 2)$.

Remark IV.1: If we compare the unit step responses depicted in Fig.2 and the pole distribution depicted in Fig.1, we can see the correlation. For the single pole $[-3]$ case when $\alpha = 1$ (lower limit of α), a monotonous unit step response results while for the complex conjugate roots $[\pm 1.73]$ case when $\alpha = 2$ (upper limit of α), an oscillating unit step response results.

Consider the other unknown parameter a which is assumed to lie in a known interval $a \in [1, 2]$. The transfer function of controlled system is

$$G(s) = \frac{1}{s^\alpha + a} \quad (10)$$

and the characteristic pseudo-polynomial of closed control loop has the form $p(s) = s^\alpha + a + 2$.

The resulted closed-loop transfer function has the following form:

$$G_c(s) = \frac{2}{s^\alpha + a + 2}. \quad (11)$$

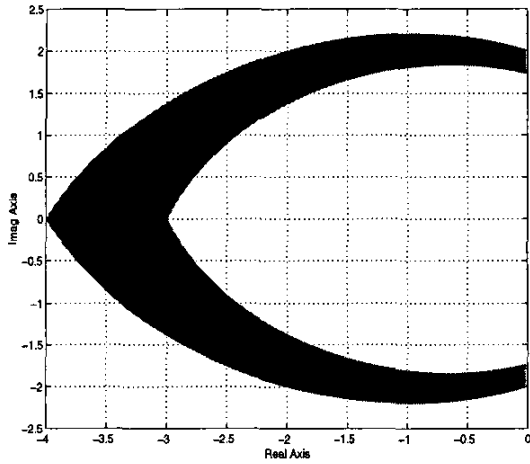


Fig. 3. Poles location for characteristic pseudo-polynomial $p(s) = s^\alpha + a + 2$ for $\alpha \in [1, 2)$ and $a \in [1, 2]$.

The pole locations in the complex plane for the characteristic pseudo-polynomial with various values of order α and a are shown in Fig. 3 where we can see that the poles in the stability region of the complex plane has been changed from curves to a plane, which is exactly depends on the values α and a in the pseudo-polynomial.

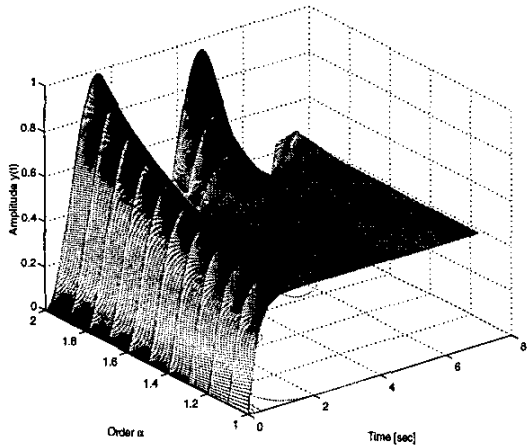


Fig. 4. Transient unit step responses for (11) for various order $\alpha \in [1, 2)$ and for fixed value of a , $a = 2$.

Figure 4 shows the unit step time responses for various values of the order $\alpha \in [1, 2)$ with a fixed $a = 2$.

Remark IV.2: Comparing the unit step responses in Fig.2 and the unit step responses in Fig.4, we can see another correlation. Both responses are for variable values of order $\alpha \in [1, 2)$, but the first one is for the lower limit of parameter $a = 1$ and the second is for the upper limit of parameter $a = 2$. The difference correlate with the poles distribution depicted in Fig.3. With $a = 2$, we have a single pole $[-4]$. When $\alpha = 1$ (lower limit of α), we obtain a monotone unit step response. Similarly, we have a complex conjugate roots $[\pm 2.0]$ when $\alpha = 2$ (upper limit of α) and we obtain an oscillating unit step response.

In both cases, the poles lie in the stability region (left hand half plane) and we can say that the closed-loop system with the mentioned simple plant and simple proportional controller is robustly stable for all parameters α and a within the known considered intervals.

For better visibility of robust stability, Fig. 5 is presented to show the 3D projection of the stable plane for characteristic pseudo-polynomial with various values of the order α and the coefficient a .

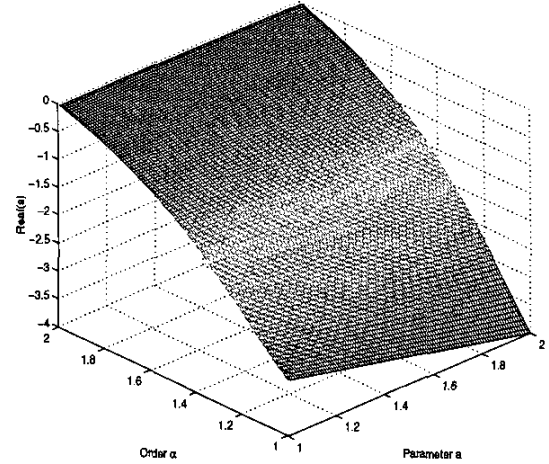


Fig. 5. Real parts of pseudo-polynomial poles projected in 3D view according to parameter a and real order α .

C. Kharitonov Based Method

The graphical method presented in the above is best for the validation of the Kharitonov-based stability check procedure proposed in Sec. III.

From Fig. 3, we can observe that the limit points for the bounded shape in the complex plane: $[-3, -4]$ on the real axis and $[\pm 1.73 i, \pm 2 i]$ on the imaginary axis.

If we return back to characteristic pseudo-polynomial $p(s) = s^\alpha + a + K$ for $\alpha \in [1, 2)$ and $a \in [1, 2]$, we can write the following "new" form for polynomial (for $K = 2$):

$$p(s) = s^{[1,2]} + [1, 2] + 2. \quad (12)$$

We assume that such form of polynomials are continuous and analytical functions and have roots, which can be a single or a complex conjugate (branch points). We can write the following polynomials based on Kharitonov principle but modified for interval orders as well:

$$\begin{aligned} {}^1p(s) &= s + 1 + 2 = s + 3 \rightarrow s = -3, \\ {}^2p(s) &= s + 2 + 2 = s + 4 \rightarrow s = -4, \\ {}^3p(s) &= s^2 + 1 + 2 = s^2 + 3 \rightarrow s_{1,2} = \pm 1.73i, \\ {}^4p(s) &= s^2 + 2 + 2 = s^2 + 4 \rightarrow s_{1,2} = \pm 2. \end{aligned} \quad (13)$$

From the polynomials in (13), we actually can obtain the limit points for shape in the complex plane as depicted in Fig. 3, as roots of each polynomial, respectively.

When the order interval limits are commensurate, we can use the method described in Sec. III. The characteristic

pseudo-polynomial (12) we can be rewritten by using the transform method $s^\alpha \rightarrow \sigma$ as a polynomial in the following form:

$$p(\sigma) = \sigma^{[2,4]} + [1, 2] + 2, \quad (14)$$

where the transformation is that $s^{0.5} \rightarrow \sigma$.

According to the test procedure in Sec. III, we can write the following four polynomials:

$$\begin{aligned} {}^1p(\sigma) &= \sigma^2 + 3 \rightarrow \sigma_{1,2} = \pm 1.732i, \\ {}^2p(\sigma) &= \sigma^2 + 4 \rightarrow \sigma_{1,2} = \pm 2.000i, \\ {}^3p(\sigma) &= \sigma^4 + 3 \rightarrow \sigma_{1,2,3,4} = \pm 0.930 \pm 0.930i, \\ {}^4p(\sigma) &= \sigma^4 + 4 \rightarrow \sigma_{1,2,3,4} = \pm 1.000 \pm 1.000i. \end{aligned} \quad (15)$$

Figure 6 shows the pole locations in σ - complex plane for the characteristic polynomials (15). The closed angular stability limits for $\alpha\pi/2$, for $\alpha = 0.5$ are also drawn in Fig. 6.

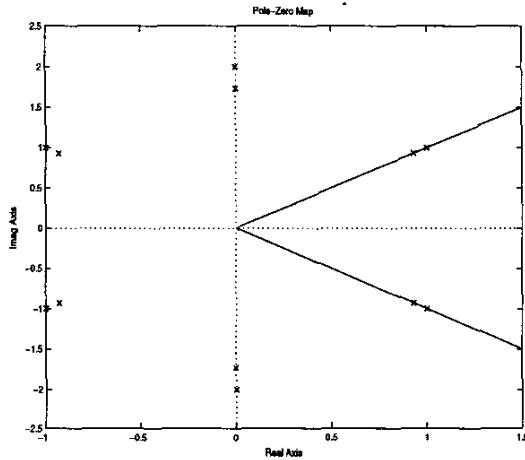


Fig. 6. Pole locations for the polynomials (15) and the stability limits $\pi/4$.

The system is stable because all roots σ_i , $\forall i$ of polynomials (15) have $|\arg(\sigma_i)| > \alpha\frac{\pi}{2}$, for $\alpha = 0.5$.

This result validate the results depicted in Fig. 3 and Fig. 5. Based on these results, we can declare that system (10) is robustly stable for interval parametric uncertainties and interval order uncertainties and can be controlled by a simple proportional controller $C(s) = 2$.

D. An Application Example

The application example considered in this section is based on the work of [12] where the application of fractional order controller for temperature control of heat solid (electrical radiator) was performed.

To obtain the mathematical models of controlled object, various identification methods can be used. In [12], two models were chosen as the nominal models for control system design. The first one is an integer first order model given by

$$G_I(s) = \frac{1}{20.14s + 0.598} \quad (16)$$

The second one is a fractional order one-term model of the following form:

$$G_F(s) = \frac{1}{39.69s^{1.26} + 0.598}. \quad (17)$$

The second one is closer to the reality based on our experience. For comparison purpose, the integer first order model is also used in [12] with the standard control design techniques. However, the reality is probably somewhere *in between* the $G_I(s)$ and $G_F(s)$. Therefore, we have designed (for comparison) two types of controllers, i.e., the classical PD and the PD^δ (δ is a positive real number).

Now, we follow a new idea of using the standard control design method with a PD controller for control of this object but two assumed uncertainties in the mathematical model of controlled object. The new uncertain model has the following form:

$$G(s) = \frac{1}{as^\alpha + 0.598}, \quad (18)$$

where $a \in [15, 45]$ and $\alpha \in [1, 1.3]$.

With the PD controller

$$C(s) = K_p + K_d s,$$

to achieve the desired stability measure $S_t = 2.0$ [12], the following parameters were designed: $K_p = 64.47$ and $K_d = 12.46$, which means that the closed-loop characteristic equation has a simple root $[-2]$.

We now apply the designed PD controller $C(s) = 64.47 + 12.46s$ to the new uncertain model (18). The resulting closed-loop transfer function becomes

$$G_c(s) = \frac{12.46s + 64.47}{as^\alpha + 12.46s + 65.068}, \quad (19)$$

where $a \in [15, 45]$ and $\alpha \in [1, 1.3]$.

The characteristic polynomial of the control system (19) with reference to intervals for order α and parameter a has the following form:

$$p(s) = [15, 45]s^{[1,1.3]} + 12.46s + 65.068 = 0. \quad (20)$$

According to the stability test procedure in Sec. III, we should check the following four polynomials:

$$\begin{aligned} {}^1p(s) &= 15s + 12.46s + 65.068 \\ &= 27.46s + 65.068 \rightarrow s = -2.37, \\ {}^2p(s) &= 15s^{1.3} + 12.46s + 65.068, \rightarrow \\ & \quad s_{1,2} = -1.97 \pm 1.04i, \\ {}^3p(s) &= 45s + 12.46s + 65.068 \\ &= 57.46s + 65.068 \rightarrow s = -1.13, \\ {}^4p(s) &= 45s^{1.3} + 12.46s + 65.068, \rightarrow \\ & \quad s_{1,2} = -0.94 \pm 0.65i. \end{aligned} \quad (21)$$

From the polynomials in (21), we can obtain the limit points for the shape of the pole locations in the complex plane as the roots of each polynomial. Clearly, the shape

is situated in the left hand side of the complex plane which means that the closed-loop system is stable. According to the proposed test procedure, the system is stable as well because all roots $s_i, \forall i$ of polynomials (21) have $|\arg(s_i)| > \frac{\pi}{2}$.

Based on these results, we can declare that system (18) is robustly stable for interval parametric and fractional order uncertainties and can be controlled by the *PD* controller $C(s) = 64.47 + 12.46s$.

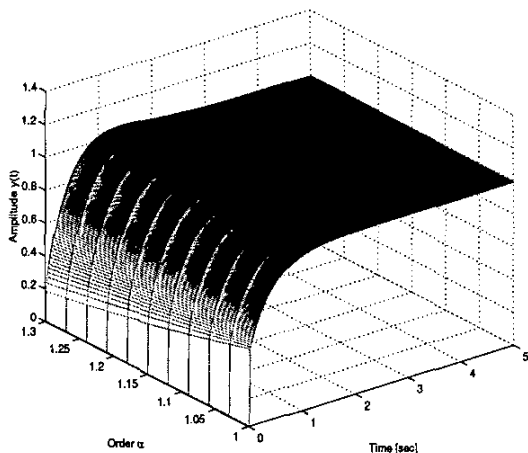


Fig. 7. Transient unit step responses for (19) for various order $\alpha \in [1, 1.3]$ and for fixed value of $a, a = 15$.

Figure 7 shows the unit step responses for various values of order $\alpha \in [1, 1.3]$ with a fixed value of variable $a, a = 15$.

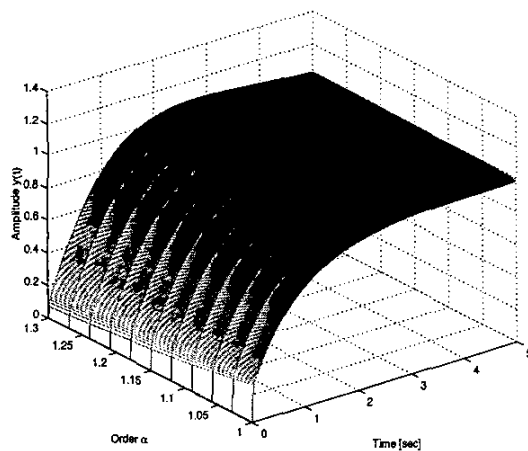


Fig. 8. Transient unit step responses for (19) for various order $\alpha \in [1, 1.3]$ and for fixed value of $a, a = 45$.

Similarly, Fig. 8 shows the unit step responses for various values of order $\alpha \in [1, 1.3]$ with a fixed value of variable $a, a = 45$.

V. CONCLUSION

In this contribution we have presented two illustrative examples for checking the stability of LTI FOS with both order and parametric interval uncertainties which are

known to lie within a known interval. This proposed approach is very simple and good for stability investigation of LTI FOS with small number of uncertain intervals. It is an intuitive extension of the Kharitonov edge theorem for interval polynomials. In our case, instead of the rectangular shape as Kharitonov used, we have to consider more general shapes in the complex plane.

We show only experimental approach to test the robust stability of the LTI FOS with both order and parametric interval uncertainties. However, a clear and exact proof for the proposed stability test procedure is one of the further research topics.

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