Time Periodical Adaptive Friction Compensation

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***Abstract*— In this paper, the adaptive compensation of Coulomb friction is considered where the Coulomb friction coefficient is unknown but can be time-varying with a known period. Detailed stability analysis is given with some demonstrative simulation results. The major contributions of this paper is a periodic adaptive control law for friction compensation in nonlinear electromechanical systems not satisfying local and global Lipschitz condition.**

***Index Terms*— Friction compensation, adaptive control, periodic learning updating, tracking control, servomechanism.**

I. Introduction

Friction, like delay and saturation, is everywhere in many real control systems. That is why friction, delay, and saturation are three constantly hot topics in control community in general and electromechanical systems control in particular. Since the friction compensation concept was introduced in [1], several adaptive friction compensation controllers have been designed in [2], [3], [4], [5], [6]. After the pioneering works by Friedland and Park in [2], [3], several modifications were introduced in [4], [5], [6]. They have focused on designing new adaptation laws by proposing new forms of the tuning function $g(|v|)$ where v is the velocity. Mainly, their considerations were to design more stable nonlinear adaptive controller. In [2], the asymptotically stable controller with restrictive conditions was designed. The contribution in [4] was to relax those restrictive conditions imposed in [2]. Recently, in [5], an exponentially stable adaptive friction controller was developed. So far, in all the existing efforts in attacking friction effects, the Coulomb friction coefficient is assumed to be constant. In other words, the existing stability analysis are all restricted to the time invariant friction coefficient.

As reported in [1], e.g., from Figures 2-3 in [1], the magnitude of friction coefficient depends on velocity which practically is not a constant. So, in many applications, when the reference position and reference velocity are periodically time varying, the friction will also be a periodic variable. This motivates us to consider the design of an new adaptive friction compensation controller with a time varying friction coefficient. In particular, we consider a simple case when the friction coefficient is unknown and periodically time-varying with a known period. Recently, in adaptive control, the periodically time varying adaptive controller was developed in [7]. However, in [7], the local or global Lipschitz condition is necessary. So, the proposed method in [7] cannot be directly applied to the sign dependent friction compensation problem considered in this paper.

The paper is organized as follows: In Section II, a new adaptive friction compensation controller is designed and analyzed. Two different simulation tests are performed in Section III to verify the developed adaptive friction compensation algorithm. Concluding remarks are given in Section IV.

II. Adaptive Friction Compensation

In this section, a new adaptive friction compensator is designed with a time periodic friction coefficient. We also present a rigorous stability analysis using Lyapunov function method. Similar to the system considered in [2], here we consider the following servo control problem:

$$\dot{x}(t) = v(t) \tag{1}$$

$$\dot{v}(t) = -a(t)\mathrm{sgn}(v(t)) + u(t), \tag{2}$$

where the friction coefficient $a(t)$ is periodically time varying as follows

$$a(t-T) = a(t) \tag{3}$$

with a known period T; $x(t)$ is the position state; $v(t)$ is the velocity state; $\mathrm{sgn}(v(t))$ is a sign of velocity; and $u(t)$ is the control input signal.

The control objective is to track or servo the given desired position $x_d(t)$ and the corresponding desired velocity $v_d(t)$ with tracking error as small as possible. In practice, it is reasonable to assume that $x_d(t)$, $v_d(t)$ and $\dot{v}_d(t)$ are all bounded. Our feedback control law is designed as:

$$u = \hat{a}(t)\mathrm{sgn}(v(t)) + \dot{v}_d(t) - \alpha S(t) - \lambda e_v(t), \tag{4}$$

with

$$S(t) = e_v(t) + \lambda e_x(t), \tag{5}$$

where α and λ are positive gains; $\hat{a}(t)$ is an estimated friction coefficient from an adaptation mechanism to be specified later; $\dot{v}_d(t)$ is the desired acceleration; $e_x(t) = x(t) - x_d(t)$ is the position tracking error; and $e_v(t) = v(t) - v_d(t)$ is the velocity tracking error.

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Our adaptation law is designed as follows:

$$\hat{a}(t) = \begin{cases} \hat{a}(t-T) - K\text{sgn}(v)S(t) & \text{if} \quad t \geq T \\ z - g(|v|) & \text{if} \quad t < T \end{cases} \qquad (6)$$

where K is a positive design parameter (we call it the periodic adaptation gain); z will be defined in the following paragraph; and $g(|v|)$ is a tuning function to be selected later based on certain guidelines. In our analysis part, we require the following inequality condition for $g(|v|)$:

$$\frac{1}{4} < g'(|v|) < \infty, \qquad (7)$$

where $g'(\cdot) = \frac{\partial g(\cdot)}{\partial \cdot}$. As will be shown in the sequel, the selection of a $g(|v|)$ to satisfy the above constraint is an easy task.

Denote the estimation error of $a(t)$ as

$$e_a(t) = a(t) - \hat{a}(t).$$

In what follows, we will drop the variable time when no confusion arises. For example, we will use e_a to represent $e_a(t)$, e_v for $e_v(t)$ and so on. We consider two cases: 1) when $0 \leq t < T$ and 2) when $t \geq T$. The key idea is that, for case 1), we only need to show the finite time boundedness. For case 2), we need to show the stability or asymptotic stability in the sense of Lyapunov. Let us investigate the case 2) first. Our major results are summarized in the following theorems.

Theorem 2.1: When $t \geq T$, the control law (4) and the periodic adaptation law (6) guarantee the stability of the equilibrium points e_x, e_v, and e_a as $t \to \infty$.

Proof: Consider the following Lyapunov-like function:

$$V(t) = \frac{1}{2}S^2(t) + \frac{1}{2K}\int_{t-T}^{t} e_a^2(\tau)\text{d}\tau.$$

Then, the difference of the positive Lyapunov-like function at two discrete time points can be calculated as:

$$\begin{aligned} \triangle V(t) &= V(t) - V(t-T) \\ &= \frac{1}{2}S^2(t) - \frac{1}{2}S^2(t-T) \\ &\quad + \frac{1}{2K}\int_{t-T}^{t}[e_a^2(\tau) - e_a^2(\tau - T)]\text{d}\tau \\ &= \int_{t-T}^{t} S(t)\dot{S}(t)\text{d}\tau \\ &\quad + \frac{1}{2K}\int_{t-T}^{t}[e_a^2(\tau) - e_a^2(\tau - T)]\text{d}\tau. \end{aligned} \qquad (8)$$

To simplify our presentation, let the first integral term on the right-hand side be denoted by A and the second integral term by B. That is

$$A = \int_{t-T}^{t} S(t)\dot{S}(t)\text{d}\tau, \quad B = \frac{1}{2K}\int_{t-T}^{t}[e_a^2(\tau) - e_a^2(\tau - T)]\text{d}\tau.$$

Then, by several algebraic calculations and using $a(t-T) = a(t)$, B can be changed as

$$\begin{aligned} B &= \frac{1}{2K}\int_{t-T}^{t}\{[a(\tau) - \hat{a}(\tau)]^2 - [a(\tau - T) \\ &\quad - \hat{a}(\tau - T)]^2\}\text{d}\tau \\ &= \frac{1}{2K}\int_{t-T}^{t}[\hat{a}(\tau - T) - \hat{a}(\tau)][2\{a(\tau) - \hat{a}(\tau)\} \\ &\quad + \{\hat{a}(\tau) - \hat{a}(\tau - T)\}]d\tau \\ &= \frac{1}{2K}\int_{t-T}^{t}\beta(\tau)[2\{a(\tau) - \hat{a}(\tau)\} - \beta(\tau)]\text{d}\tau, \end{aligned} \qquad (9)$$

where

$$\beta(\tau) = \hat{a}(\tau - T) - \hat{a}(\tau).$$

Using the following

$$\dot{e}_x = \dot{x} - \dot{x}_d = e_v,$$
$$\dot{e}_v = \dot{v} - \dot{v}_d = -a(t)\text{sgn}(v) + u - \dot{v}_d$$

we have

$$\dot{S} = \dot{e}_v + \lambda\dot{e}_x = -a(t)\text{sgn}(v) - \dot{v}_d + u + \lambda e_v. \qquad (10)$$

Then, from (4),

$$\dot{S} = -\text{sgn}(v)e_a - \alpha S,$$

and A can be expressed as

$$A = \int_{t-T}^{t}[-\alpha S^2 - \text{sgn}(v)e_a S]\text{d}\tau. \qquad (11)$$

Thus, $\triangle V$ becomes

$$\begin{aligned} \triangle V &= A + B \\ &= \int_{t-T}^{t}[-\alpha S^2 - \text{sgn}(v)e_a S]\text{d}\tau \\ &\quad + \frac{1}{2K}\int_{t-T}^{t}\beta[2\{a(\tau) - \hat{a}(\tau)\} - \beta]\text{d}\tau \\ &= \int_{t-T}^{t}[-\alpha S^2 - \frac{1}{2K}\beta^2]\text{d}\tau \\ &\quad + \int_{t-T}^{t}\frac{e_a}{K}[\beta - K\text{sgn}(v)S]\text{d}\tau, \end{aligned} \qquad (12)$$

where we will denote the first integral term on the right-hand side by C and the second integral term by D. That is

$$C = \int_{t-T}^{t} -\alpha S^2 - \frac{1}{2K}\beta^2\text{d}\tau,$$

$$D = \int_{t-T}^{t}\frac{e_a}{K}[\beta - K\text{sgn}(v)S]\text{d}\tau.$$

Then from (6), $D = 0$, so we have

$$\begin{aligned} \triangle V &= A + B = \int_{t-T}^{t} -\alpha S^2 - \frac{1}{2K}\beta^2\text{d}\tau \\ &= \int_{t-T}^{t} -(\alpha + \frac{K}{2})S^2(\tau)\text{d}\tau. \end{aligned} \qquad (13)$$

Since $\alpha + \frac{K}{2} > 0$, $\Delta V \le 0$, which completes the proof of this theorem. ■

The above theorem only guarantees the stability property in the sense of Lyapunov. To explore the asymptotical stability, the following lemma is needed first.

Lemma 2.1: In the following equation with initial state $x(0) = x_0 = 0$

$$y = \dot{x} + \tau x, \quad \tau > 0,$$

$y \to 0$ as $t \to \infty$ if and only if $x \to 0$ as $t \to \infty$.

Proof: The sufficient condition is immediate because $x = 0$ makes $y = 0$. The necessary condition is proved easily by calculating the solution. When $y = 0$, $x(t)$ is calculated as:

$$x(t) = x_0 + e^{-\tau t}.$$

So, if $x_0 = 0$, as $t \to \infty$, $x(t) \to 0$. ■

Now, we consider the asymptotically stability condition of the equilibrium points e_x, e_v, and e_a in the following theorem.

Theorem 2.2: If the initial position (x_0) is at the desired initial position ($x_d(0)$), i.e., $e_x(0) = 0$, the control law (4) and the periodic adaptation law (6) guarantee the asymptotically stability of the equilibrium points as $t \to \infty$ $(t \ge T)$.

Proof: Here we use LaSalle's invariant set theorem to prove the asymptotical stability. From (13), we know that only $S = 0$ makes $\triangle V = 0$. Using the definition $S = e_v + \lambda e_x$ and relationship $e_v = \dot{e}_x$, we have

$$S = e_v + \lambda e_x = \dot{e}_x + \lambda e_x. \tag{14}$$

So, from Lemma 2.1, if $e_x(0) = 0$, only $e_x = 0$ makes $S = 0$. Also, since $e_x = 0$, we have $e_v = 0$ from $e_v + \lambda e_x = 0$. Therefore, we know that e_x and e_v are asymptotically stable at equilibrium points. Now we think e_a in the following. From $\dot{S} = -\mathrm{sgn}(v)e_a - \alpha S$, we have $\dot{S} = \mathrm{sgn}(v)e_a$ because $S = 0$. Then, by showing that $\dot{S} \to 0$ as $S \to 0$, we can prove $e_a = 0$. Our approaches are as follows. From following definition

$$\dot{S} = \lim_{\Delta t \to 0} \frac{S(t+\Delta t) - S(t)}{\Delta t}, \tag{15}$$

we know that as $t \to \infty$, $S(t + \Delta t) \to 0$ and $S(t) \to 0$. However, from our original assumption of the periodicity such as $\Delta t = T$, if T is not zero, then $\Delta t \ne 0$, while $S(t + \Delta t) - S(t) \to 0$ as $t \to \infty$. Thus, in (15), $\dot{S} \to 0$ as $t \to \infty$, hence if $\mathrm{sgn}(v) \ne 0$, then $e_a = 0$. However, if $e_a \ne 0$, $\dot{S} \ne 0$. Then $S(t + \Delta t) - S(t) \ne 0$. This is a contradiction to $S(t + \Delta t) - S(t) = 0$. Therefore, we conclude that only $e_a = 0$ makes $\dot{S} = 0$ and in the sequel, no trajectory can stay except $e_a = 0$ when $S = 0$. Since we already found that only $e_x = 0$ and $e_v = 0$ make $S = 0$, from the invariant set theorem, the equilibrium points e_x, e_v, and e_a are asymptotically stable. This completes the proof of this theorem. ■

Now, let us consider the case 1) when $t < T$ and the overall stability when $t \ge 0$.

Theorem 2.3: If $|\dot{a}|$ is bounded and $g'(|v|) > \frac{1}{4}$, the equilibrium points of e_x, e_v, and e_a are stable (or asymptotically stable) as $t \to \infty$.

Proof: In this case, we use following Lyapunov function:

$$V(t) = \frac{1}{2}\alpha\lambda e_x^2 + \frac{1}{2}e_v^2 + \frac{1}{2}e_a^2. \tag{16}$$

Then, the derivative of V is expressed as:

$$\begin{aligned}\dot{V}(t) &= \alpha\lambda e_x e_v + e_v(\dot{v} - \dot{v}_d) + e_a[\dot{a} - \dot{z} \\ &\quad + g'(|v|)\dot{v}\mathrm{sgn}(v)] \\ &= \alpha\lambda e_x e_v + e_v[-a\mathrm{sgn}(v) + u - \dot{v}_d] \\ &\quad + e_a[\dot{a} - \dot{z} + g'(|v|)\dot{v}\mathrm{sgn}(v)],\end{aligned} \tag{17}$$

where we used the following substitution:

$$\dot{e}_a = \dot{a} - \dot{\hat{a}} = \dot{a} - \dot{z} + g'(|v|)\dot{v}\mathrm{sgn}(v). \tag{18}$$

By inserting the control input, which is given in (4), to the above equation, the derivative of Lyapunov function is modified as:

$$\begin{aligned}\dot{V} &= -e_v e_a \mathrm{sgn}(v) - (\alpha+\lambda)e_v^2 + e_a\dot{a} \\ &\quad + e_a[g'(|v|)\dot{v}\mathrm{sgn}(v) - \dot{z}].\end{aligned}$$

Then, using one more adaptation law as follows:

$$\dot{z} = g'(|v|)[u - \hat{a}\mathrm{sgn}(v)]\mathrm{sgn}(v) \tag{19}$$

and after several algebraic calculations, $\dot{V}$ can be changed to

$$\dot{V} = -e_v e_a \mathrm{sgn}(v) - (\alpha+\lambda)e_v^2 + e_a\dot{a} - e_a^2 g'(|v|). \tag{20}$$

Finally, using Young's inequality like $a^2 + \frac{b^2}{4} \ge ab$, if $(\alpha + \lambda) > 1$ and $g'(|v|) > \frac{1}{4}$, the following inequality is always true regardless $\mathrm{sgn}(v)$:

$$-e_v e_a \mathrm{sgn}(v) - (\alpha+\lambda)e_v^2 - e_a^2 g'(|v|) < 0. \tag{21}$$

At this moment, $\dot{V}$ is upper bounded by

$$\dot{V} = -[e_v \pm 0.5\mathrm{sgn}(v)e_a]^2 - (\alpha+\lambda-1)e_v^2 - [g'(|v|) - \frac{1}{4}]e_a^2 + e_a\dot{a}. \tag{22}$$

Our argument here is to ensure that $\dot{V}$ is upper bounded. Denote $\eta \equiv g'(|v|) - \frac{1}{4} > 0$. From the above equation, it is easy to see

$$\dot{V} \le -\eta e_a^2 + e_a\dot{a}. \tag{23}$$

If $|\dot{a}| < \Theta$, where Θ is the upper bound of $\dot{a}$, then,

$$\begin{aligned}\dot{V} &\le -\eta e_a^2 + e_a\Theta \\ &\le -\eta\{(e_a - \frac{\Theta}{2\eta})^2 + \frac{\Theta^2}{4\eta}\}.\end{aligned} \tag{24}$$

Clearly, if $\eta > 0$ and $|\dot{a}|$ is bounded, then

$$\dot{V} \le \frac{\Theta^2}{4\eta}. \quad (25)$$

Thus, we conclude that if $g'(|v|) > \frac{1}{4}$, $\dot{V}$ is bounded when $t < T$. Consequently, when V is bounded at $t < T$, e_x, e_v, and e_a are also bounded at $t < T$.

Furthermore, when $t \ge T$, the equilibrium points of e_x, e_v, and e_a are all (asymptotically with $e_x(0) = 0$) stable from equation (13), we conclude that the system (1)-(2) can be (asymptotically with $e_x(0) = 0$) stabilized by the control law (4) and the adaptation law (6) as $t \to \infty$. This completes the proof. ∎

Several remarks follow.

Remark 2.1: On the region of convergence. From (23), when $t < T$, the region of convergence (R_{ROC}) of e_a is as follows:

$$R_{ROC} = \begin{cases} (e_a | e_a < 0 \cup e_a > \frac{\dot{e}_a}{\eta}), & \text{if} \quad \dot{a} > 0 \\ (e_a | e_a < \frac{\dot{e}_a}{\eta} \cup e_a > 0), & \text{if} \quad \dot{a} < 0 \end{cases}. \quad (26)$$

Hence, regardless the sign of $\dot{a}$, if $|e_a| > |\frac{\dot{e}_a}{\eta}|$, then $\dot{V} < 0$. So, as $\eta \to \infty$, the R_{ROC} becomes bigger, because $|\frac{\dot{e}_a}{\eta}| \to 0$. Even if the periodic adaptation law is not used, when $t \ge T$, the control law (4) guarantees the boundedness of e_a.

Remark 2.2: On design of the adaptation function $g(|v|)$. In designing $g(|v|)$, the following function is suggested in order to satisfy the required condition $\frac{1}{4} < g'(|v|) < \infty$:

$$g(|v|) = \xi |v| + e^{-\mu |v|}, \quad \xi > \mu + \frac{1}{4}, \quad (27)$$

where ξ and μ are design parameters for the adaptation law. The derivative of $g(|v|)$ is expressed as:

$$g'(|v|) = [\xi - \mu e^{-\mu |v|}]. \quad (28)$$

Finally, $\hat{a}(t)$ in (6) and $\dot{z}$ in (19) are designed as:

$$\hat{a}(t) = z - \xi |v| - e^{-\mu |v|} \quad (29)$$

$$\dot{z} = [\xi - \mu e^{-\mu |v|}][u - \hat{a}\text{sgn}(v)]\text{sgn}(v) \quad (30)$$

Additionally, we have the following remark regarding our function $g(|v|)$.

Remark 2.3: $g(|v|)$ in (27) is different from the suggested functions in [2], [3], [4], [5], [6] where they did not consider the velocity derivatives and the variation of the friction coefficient. However, in this paper, we have considered the velocity derivative as well as the coefficient variation.

III. Simulation Illustrations

For simulation test, we use the following reference position and velocity signals:

$$\begin{aligned} x_r(t) &= \cos(2\pi f_s t) \\ v_r(t) &= -2\pi f_s \sin(2\pi f_s t) \\ \dot{v}_r(t) &= -(2\pi f_s)^2 \cos(2\pi f_s t) \end{aligned} \quad (31)$$

where $f_s = \frac{1}{T_s}$, and $T_s = 2$ sec. The control gains for e_x and e_v in [2] are 200 and 20 respectively. However from following relationships:

$$\alpha\lambda = 200, \quad \alpha + \lambda = 20, \quad (32)$$

we cannot get the real α and λ, so, without losing comparability we simply choose α as 10 and β as 10. In (6), the periodic adaptation gain K was selected as 10, and, in (27), ξ was selected as 10 and μ was selected as 5. It is assumed that the friction coefficient varies in the following way:

$$a(t) = 50 + 5\sin(2\pi \frac{1}{T} t), \quad (33)$$

where $T = T_s$.Two different simulation tests were performed using adaptation laws in (6).

- Case-I. No periodic adaptation in (6). That is, $T = \infty$.
- Case-II. (6) is used with a finite T.

From the proof of Theorem 2.3, we knew that e_x, e_v, and e_a are all bounded when $t < T$. However, Theorem 2.3 also shows that the control law (4) and the adaptation law, i.e., $z - g(|v|)$) in (6), guarantee the boundedness of e_x, e_v, and e_a even if $t \to \infty$. Therefore, in Case-I, we used only adaptation law of $\hat{a}(t) = z - g(|v|)$ even if $t \to \infty$, i.e., we do not use the periodic adaptation law $\hat{a}(t) = \hat{a}(t-T) - K\text{sgn}(v)S(t)$. This case is also called Simulation 1 in the sequel. Simulation 2 was performed using both adaptation laws in (6). All initial conditions were fixed as zeros. Fig. 1.a shows the reference position/velocity and tracked position/velocity from Simulation 1. Fig. 1.b is the corresponding position/velocity errors of Fig. 1.a. From these figures, we observe that the system is stable, while the desired states are not estimated perfectly. Fig. 1.c shows the position/velocity tracking results from Simulation 2, and Fig. 1.d is the corresponding tracking errors. As shown in Fig. 1.d, the desired states are estimated much more accurately than Simulation 1. As time increases, the state errors go to zero, while there exists periodical state errors in Fig. 1.b when no periodic adaptation is applied. Fig. 2.a shows the true friction coefficient and adapted friction coefficients from Simulation 1 and Simulation 2, respectively. Fig. 2.b is the corresponding friction estimation errors of Fig. 2.a. The friction coefficient was adapted better in Simulation 2 than in Simulation 1. The adapted friction coefficient from Simulation 1 does not converge to the true value, while the adapted friction coefficient from Simulation 2 slowly converges to the true value. Fig. 2.c compares the control inputs from Simulation 1 and Simulation 2. From this figure, we know that there is not much difference in control input for both cases. Only slight difference exists between the control efforts. Fig. 2.d shows the position/velocity tracking errors, when there is no initial position error (this test is for the asymptotical stability). Since there is no initial error, initial error boundaries are smaller than Simulation 1 and Simulation 2. However, the overall performance is almost same to the Simulation 2, in which there is an initial error

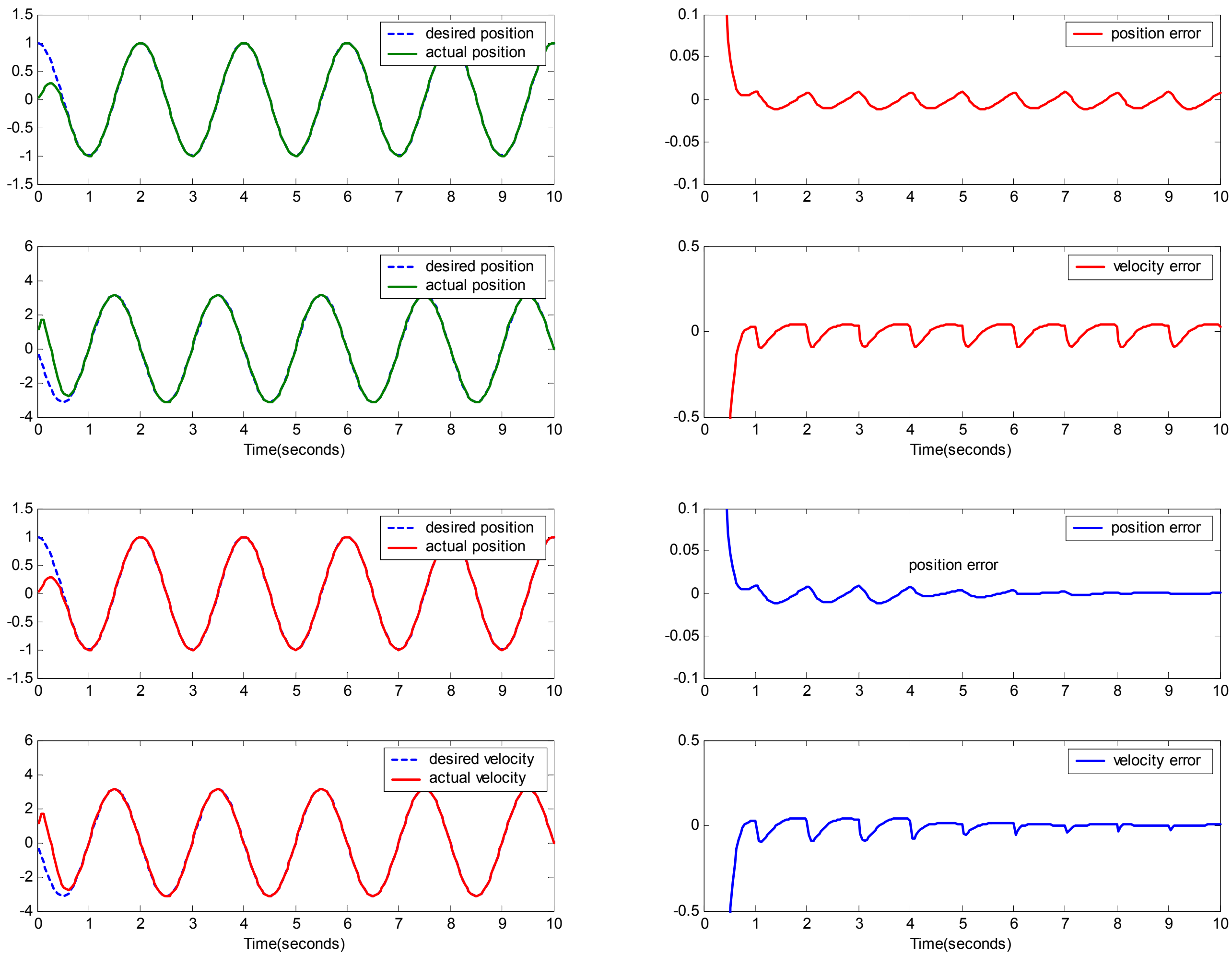

Fig. 1. Upper-left(Fig.1.a): simulation 1 -reference position/velocity and the actual tracking position/velocity; Upper-right(Fig.1.b): simulation 1 -position and velocity tracking errors; Bottom-left(Fig.1.c): simulation 2 -reference position/velocity and the actual tracking position/velocity; Bottom-right(Fig.1.d): simulation 2 -position and velocity tracking errors

(just with the guarantee of the stability).

IV. Conclusion Remarks

In this paper, we developed an adaptive controller for dynamic systems with time varying Coulomb friction. From Theorem 2.3 and our simulation tests, it has been confirmed that the developed friction compensation scheme can be used either to bound the error states without considering periodicity of the friction parameter or to stabilize the system when considering the known periodicity. When the periodicity of the friction coefficient was used, the position and velocity were tracked perfectly as time passes (Figs. 1.c-1.d). Even though the periodicity is not considered, the states are tracked within acceptable error margins (Figs. 1.a-1.b). Moreover, we developed the stability condition and asymptotical stability condition and verified through the simulations. However, there was no performance difference between them. Finally, the time periodic adaptive control technique can be applied to systems with periodically or non-periodically time varying coefficients which are dependent on sign of states. Our future efforts will be in 1) considering the state periodic adaptive friction controller 2) exploring general robust adaptive friction compensation scheme with non-periodic time varying friction coefficient.

V. Acknowledgment

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VI. References

[1] Jr David A. Haessig and Bernard Friedland, "On the modeling and simulation of friction," in *1990 American Control Conference (ACC'90)*, San Diego, Calif, USA., May 23-25 1990, AACC, pp. 1256–1261.

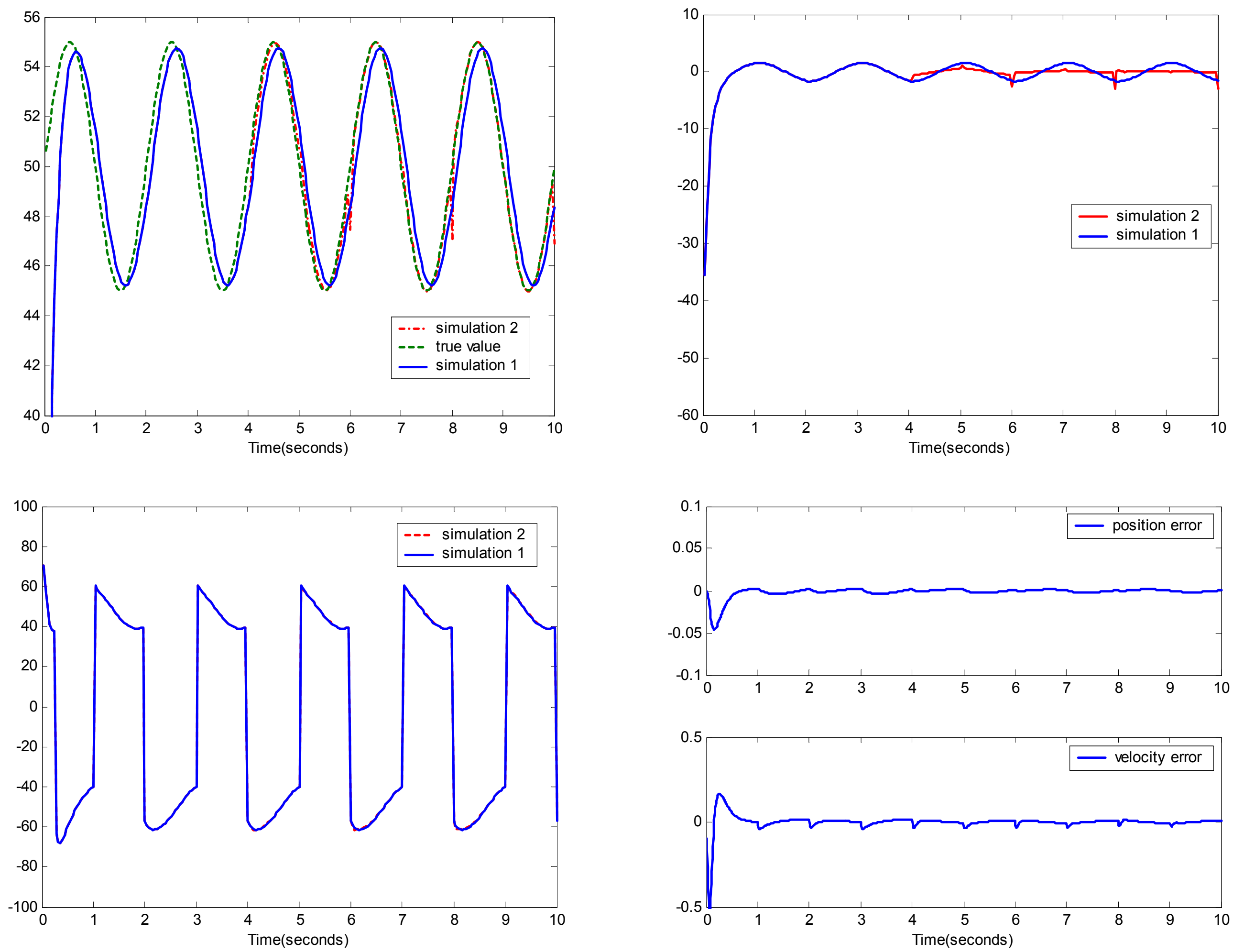

Fig. 2. Upper-left(Fig.2.a): adaptively estimated Coulomb friction coefficients for both cases; Upper-right(Fig.2.b): friction coefficient estimation errors for both cases; Bottom-left(Fig.2.c): control input signals for both cases; Bottom-right(Fig.2.d): with zero initial position error

[2] Bernard Friedland and Young-Jin Park, "On adaptive friction compensation," in *Proceedings of the 30th IEEE Conference on Decision and Control*, Brighton, England, Dec. 1991, IEEE, pp. 2899–2902.

[3] Bernard Friedland and Sophia Mentzelopoulou, "On adaptive friction compensation without velocity measurement," in *Proceedings of the First IEEE International Conference on Control Applications*, Dayton, OH, USA, Sep. 1992, IEEE, pp. 1076–1081.

[4] Yazdizadeh A and Khorasani K, "Adaptive friction compensation based on the lyapunov scheme," in *Proceedings of the 1996 IEEE International Conference on Control Applications*, Dearborn, MI, USA, Sep. 1996, IEEE, pp. 1060–1065.

[5] Teh-Lu Liao and Tsun-I Chien, "An exponentially stable adaptive friction compensator," *IEEE Trans. on Automatic Control*, vol. 45, no. 5, pp. 977–980, 2000.

[6] T. Zhang and M. Guay, "Comments on "an exponentially stable friction compensator," *IEEE Trans. on Automatic Control*, vol. 46, no. 11, pp. 1844–1845, 2001.

[7] Jian-Xin Xu, "A new periodic adaptive control approach for time-varying parameters with known periodicity," *IEEE Trans. on Automatic Control*, vol. 49, no. 4, pp. 579–583, 2004.