Fractional Calculus and Biomimetic Control

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***Abstract*— In this tutorial paper, we introduce the fractional order calculus and fractional order control with possible applications in biomimetic actuator control. Since purely analog type fractional order integrator can be made with certain biomimetic materials or intelligent materials in general, it is natural to advocate the concept of "intelligent control of intelligent materials by using intelligent materials".**

***Index Terms*— Fractional order calculus, fractional order dynamic systems, fractional order control, biomimetic control.**

I. Introduction

Biomimetics (or bionics, biognosis, etc.) is an abstract of "good design from nature" [1]. Roughly speaking, biomimetics is the concept of taking ideas from nature and implementing them in another technology.This concept is actually very old, for example, the Chinese wanted to make artificial silk 3,000 years ago. Some biomimetic processes have been in use for years. An example is the artificial synthesis of certain vitamins and antibiotics. More recently, the biomimetic concepts, ideas and applications are increasingly reported. For example, the latest new biomimetic study reported in the journal Nature, according to the current picks of biomimetic issues [2], is actually from studying how ants avoid traffic jams, which has numerous implications for many scientists to rely on the behavior of ants or other natural systems to give them clues as to how to design computer systems that avoid overcrowded networks. Another biomimetic example, as commented by Philip Ball in the 26 February 2004 issue of Nature on a paper by Liu *et al.* [3], is on the use of microbes in waste-water that could make a handy household battery.

In a more general setting, according to [4], biomimetic refers to human-made processes, substances, devices, or systems that imitate nature. The art and science of designing and building biomimetic apparatus is called biomimetics, and is of special interest to researchers in robotics, artificial intelligence (AI), nanotechnology, the medical industry, and the military. Other possible applications of biomimetics include nanorobot antibodies that seek and destroy disease-causing bacteria, artificial organs, artificial arms, legs, hands, and feet, and various electronic devices. One of the more intriguing ideas is the so-called biochip, a microprocessor that grows from a starter crystal in much the same way that a seed grows into a tree, or a fertilized egg grows into an embryo [4].

Biomimetics is now *not* at the stage in generating new concepts and ideas because the mother nature has already provided numerous models for us to imitate. The key is the implementation and development which is gathering momentum only recently because the science base can cope with the advanced techniques in various areas such as biology, materials, electronics, computing, communication and control etc. The idea of extrapolating designs from nature and copying them has entered into many areas of applied science, most notably the synthesis of new materials. So, it is no surprise that people tend to regard the biomimetics as an interdisciplinary field in materials science, engineering, and biology.

In this paper, we focus on biomimetic actuator control by using fractional order calculus. The topic is within the scope of biomimetic robotics or biomechatronics. Biomimetics is gaining its popularity in the robotics and automation community. As an example, the first World Congress on Biomimetics and Artificial Muscles (Biomimetics 2002) is an important event [5] with as many as 327 participants and more than 160 papers. The dynamic control of biomimetic systems made with biomimetic materials is seldom reported in the literature. Based on the current understanding of the biomimetic material characteristics, we can foresee the challenge of biomimetic control. Here, we do not address the bio-inspired robot control. Instead, we consider the control of biomimetic actuators made with biomimetic materials. We propose to use fractional order calculus as a supplementary mathematical tool for a better control.

This paper is organized as follows. Section II briefly introduces some of the existing biomimetic materials and biomimetic actuators. Section III is a concise introduction of fractional order calculus and fractional order dynamic systems control. Some considerations on biomimetic control using fractional order calculus are presented in Sec. IV. Finally, Sec. V concludes this paper with some remarks on immediate research efforts in fractional order biomimetic control.

II. Biomimetic Materials and Biomimetic Actuators

Many of the contributions at Biomimetics 2002 [5] were focused on artificial muscle - a biomimetic actuator. Other

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papers were mainly on introducing and characterizing alternative biomimetic materials such as EAP (electroactive polymers) [6], [7], ferroelectric and relaxor materials, piezoceramic and piezopolymetric materials [8, e.g.], liquid crystal elastomers [9], [10, e.g.], electro and magnetostrictive materials [11], [12, e.g.], shape memory alloys/polymers [13], [14, e.g.], intelligent gels [15], [16, e.g.], and active materials [5]. However, little has been reported on the controls of actuators made with these biomimetic materials.

Robotics and automation community should be regarded as the end users of biomimetic materials mentioned above, instead of designing and creating these materials. However, when actuators are made with these materials, the external characteristics and limitations must be well understood so as to enable effective actuator control algorithm design. Moreover, as discussed in [17], actuators could be made in an hierarchical way. The hierarchical concept is to build integrated actuators from smaller sub-actuators. The sub-actuators are arranged geometrically and controlled so as to extend the total actuator performance into ranges that are not possible with simple actuator configurations. Hierarchical architecture could be a promising design scheme for high-performance actuators. The concept is to optimize power and energy conversion at a local level and organize to optimize global performance. Custom architectures for different actuator materials piezoelectric, EAP, SMA, molecular motors, and electrostatic may enhance overall biomimetic actuator performance.

Here, we are more interested in several biomimetic materials that can be used as biomimetic actuator such as shape memory materials (SMM), dielectric elastomer and electroactive polymers (EAP), to name a few.

- The shape memory materials (SMM) are materials that can be deformed into a temporary and dormant shape under specific conditions of temperature and stress and will later, under thermal, electrical, or environmental stimuli, relax to their original, stress-free conformation due to the elastic energy stored during the initial deformation [10]. Shape memory alloys (SMA), such as Nickel-Titanium alloy (i.e., Nitanol©), are the primary class of SMM that have been studied and utilized [18]. Some SMAs can even be tailored to exhibit two-way shape memory when suffering a heat cycle [19], thus becoming great candidates for medical device applications. Polymers, which intrinsically show shape memory behavior on the basis of rubber elasticity, appear to be even better candidates than alloys due to their ease of manipulation. Moreover, whereas the shape recovery phenomenon is enthalpy-driven in alloys, it is due to an entropy-driven energy in polymers, which allows for much greater deformations (several hundred percent). A typical shape memory polymer (SMP) can be seen as a rubber, chemically or physically cross-linked, that has the property of super-elasticity above a critical temperature controlled by its glass transition temperature or melting point [10], [19].
- Dielectric elastomer transducers have the unique characteristics which enable a wide range of applications as actuators, generators, sensors, and variable impedance devices. Indeed, many of the applications are under active consideration for use in commercial products. Many more applications of dielectric elastomers are bound to emerge as the technology continues to mature.
- Artificial muscles imitating the true muscles are made from various biomimetic materials mainly by EAPs [5]. For example, polymer gels exhibit abrupt volume changes in response to variations in their external conditions - shrinking or swelling up to 1000 times their original volume. Through the conversion of chemical or electrical energy into mechanical work, a number of devices have already been constructed which produce forces up to 100N/cm^2 and contraction rates on the order of a second. Another example is the Ionic Polymer-Metal Composite (IPMC), a composite of a noble metal, conductive polymer or carbon/graphite, and charged polyelectrolyte membrane. IPMCs have shown considerable progress in producing actuation in electric fields. Furthermore, these composites are capable of sensing motion by producing a voltage difference when deformed by a mechanical force [20].

We will show in Sec. IV that, a good control of a biomimetic actuator made with the above materials is actually a challenging work. We suggest that the fractional order calculus should come into play.

III. Fractional Order Calculus and Fractional Order Dynamic Systems Control

Fractional order calculus is as old as the calculus of differentiation. The theory of fractional-order derivative was developed mainly in the 19-th century. For more references, see [21], [22], [23], [24].

A. Definitions of Fractional-order Differentiator

A fundamental operator ${}_a\mathrm{D}_t^\alpha$, a generalization of differential and integral operators, is introduced as follows:

$$ {}_a\mathrm{D}_t^\alpha = \begin{cases} \mathrm{d}^\alpha/\mathrm{d}t^\alpha, & \Re(\alpha) > 0, \\ 1, & \Re(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha}, & \Re(\alpha) < 0, \end{cases} \quad (1) $$

where α can be a complex number but it is assumed to a real number in this paper, a is a real number related to initial value which can be usually taken as 0. There are two commonly used definitions for the general fractional differentiation and integral, i.e., the Grünwald-Letnikov definition and the Riemann-Liouville definition [21], [23], [24]. The Grünwald-Letnikov definition is that

$$ {}_a\mathrm{D}_t^\alpha f(t) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} \quad (2) $$

where $[\cdot]$ is a flooring-operator while the Riemann-Liouville definition is given by

$$ {}_a\mathrm{D}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau, \quad (3) $$

for $(n-1 < \alpha < n)$ where $\Gamma(x)$ is the well known Euler's `gamma` function. One can observe that by introducing notion of the fractional-order operator ${}_a\mathrm{D}_t^\alpha$, the differentiator and integrator can be unified. Therefore, in this paper, we shall use the term "fractional-order differentiator" or "fractional derivative" alone which should be understood to imply both differentiator and integrator as shown in (1).

B. Some Properties of Fractional-order Differentiator

Here we introduce two general properties of fractional derivative. The first is the composition of fractional with integer-order derivative and the second is the property of linearity. For more properties of fractional derivative, refer to [23], [24].

The fractional-order derivative commutes with integer-order derivation [24],

$$\frac{\mathrm{d}^n}{\mathrm{d}t^n}({}_a\mathrm{D}_t^p f(t)) = {}_a\mathrm{D}_t^p\left(\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}\right) = {}_a\mathrm{D}_t^{p+n} f(t), \qquad (4)$$

under the condition $t = a$ one gets $f^{(k)}(a) = 0, (k = 0, 1, 2, \ldots, n-1)$. The relationship (4) says the operators $\mathrm{d}^n/\mathrm{d}t^n$ and ${}_a\mathrm{D}_t^p$ commute.

Similar to the integer-order differentiation, ${}_a\mathrm{D}_t^\alpha$ is a linear operator

$${}_a\mathrm{D}_t^p\left(\lambda f(t) + \mu g(t)\right) = \lambda\, {}_a\mathrm{D}_t^p f(t) + \mu\, {}_a\mathrm{D}_t^p g(t). \qquad (5)$$

C. Linear Fractional-order Differential Equations (FODEs)

A typical n-term linear FODE in time domain is give by

$$a_n\, \mathrm{D}_t^{\beta_n} y(t) + \cdots + a_1\, \mathrm{D}_t^{\beta_1} y(t) + a_0\, \mathrm{D}_t^{\beta_0} y(t) = 0 \qquad (6)$$

where $a_k (k = 0, 1, \cdots, n)$ are constant coefficients of the FODE; $\beta_k, (k = 0, 1, 2, \cdots, n)$ are real numbers. Without loss of generality, assume that $\beta_n > \beta_{n-1} > \cdots > \beta_1 > \beta_0 \geq 0$.

It is possible to solve the above FODE analytically by using Mittag-Leffler function in two parameters which is a generalization of exponential function e^z. The Mittag-Leffler function in two parameters is defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad (\alpha, \beta > 0). \qquad (7)$$

Clearly, e^z is a particular case of the Mittag-Leffler function [25]

$$E_{1,1}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k+1)} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z.$$

The analytical solution of the n-term FODE (6) is given in general form in [25].

D. Laplace Transformation Method

The Laplace transform formula for the Riemann-Liouville fractional derivative (3) has the form [25]:

$$\int_0^\infty \mathrm{e}^{-st}\, {}_0\mathrm{D}_t^\alpha f(t)\, \mathrm{d}t = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k\ {}_0\mathrm{D}_t^{\alpha-k-1} f(t)\Big|_{t=0}, \qquad (8)$$

for $(n - 1 < \alpha \leq n)$ where $F(s) = \mathcal{L}[f(t)]$ is the normal Laplace transformation.

Consider a control function which acts on the FODE system (6) as follows:

$$a_n\, \mathrm{D}_t^{\beta_n} y(t) + \cdots + a_1\, \mathrm{D}_t^{\beta_1} y(t) + a_0\, \mathrm{D}_t^{\beta_0} y(t) = u(t). \qquad (9)$$

By Laplace transform, we can get a fractional transfer function [26]:

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{1}{a_n s^{\beta_n} + \cdots + a_1 s^{\beta_1} + a_0 s^{\beta_0}}. \qquad (10)$$

In general, a fractional-order dynamic system can be represented by [25], [27], [28]

$$\begin{aligned}\frac{Y(\mathrm{j}\omega)}{U(\mathrm{j}\omega)} &= \frac{b_m(\mathrm{j}\omega)^{\alpha_m} + \cdots + b_1(\mathrm{j}\omega)^{\alpha_1} + b_0(\mathrm{j}\omega)^{\alpha_0}}{a_n(\mathrm{j}\omega)^{\beta_n} + \cdots + a_1(\mathrm{j}\omega)^{\beta_1} + a_0(\mathrm{j}\omega)^{\beta_0}} \\ &= \frac{\sum_{k=0}^{m} b_k(\mathrm{j}\omega)^{\alpha_k}}{\sum_{k=0}^{n} a_k(\mathrm{j}\omega)^{\beta_k}} \qquad (11)\end{aligned}$$

in frequency domain where $s = \mathrm{j}\omega = \mathrm{j}(2\pi f)$ and f is the frequency in Hertz. It should be pointed out that, for fractional-order control, in the literature, discussions in the frequency-domain dominate.

E. Fractional Order Controllers

The early attempts to apply fractional-order derivative to systems control can be found in [29], [30], [31]. In this section, four representative fractional-order controllers in the literature will be briefly introduced, namely, TID (tilted integral derivative) controller, CRONE controller, $\mathrm{PI}^\lambda\mathrm{D}^\mu$ controller and fractional lead-lag compensator. For detailed introduction and comparison, refer to [32] For the latest developments, we refer to [33], [34].

- *TID Controller.* In [35], a feedback control system compensator of the PID type is provided, wherein the proportional component of the compensator is replaced with a **t**ilted component having a transfer function $s^{-\frac{1}{n}}$. The resulting transfer function of the entire compensator more closely approximates an optimal transfer function, thereby achieving improved feedback controller. Further, as compared to conventional PID compensators, the TID compensator allows for simpler tuning, better disturbance rejection ratio, and smaller effects of plant parameter variations on closed loop response.
  The object of TID is to provide an improved feedback loop compensator having the advantages of the conventional PID compensator, but providing a response which is closer to the theoretically optimal response. In TID patent [35], an analog circuit using op-amps plus capacitors and resistors is introduced with a detailed component list which is useful in some cases where the computing power to implementing $T_3(s)$ digitally is not possible. An example is given in [35] to illustrate the benefits from TID over conventional PID in both time and frequency domain.
- *CRONE Controller.* The CRONE control was proposed by Oustaloup in pursuing *fractal robustness* [36], [37]. CRONE is a French abbreviation for "*Contrôle Robuste d'Ordre Non Entier*" (which means non-integer order robust control). In this section, we shall follow the

basic concept of *fractal robustness*, which motivated the CRONE control, and then mainly focus on the second generation CRONE control scheme and its synthesis based on the desired frequency template which leads to fractional transmittance [38], [39].

In [40], "fractal robustness" is used to describe the following two characteristics: the isodamping and the vertical sliding form of frequency template in the Nichols chart. This desired robustness motivated the use of fractional-order controller in classical control systems to enhance their performance.

With a unit negative feedback, for the characteristic equation

$$1 + (\tau s)^{\alpha} = 0,$$

the forward path transfer function, or the open-loop transmittance, is that

$$\beta(s) = \left(\frac{1}{\tau s}\right)^{\alpha} = \left(\frac{\omega_u}{s}\right)^{\alpha}, \qquad (12)$$

which is the transmittance of a *non integer integrator* in which $\omega_u = 1/\tau$ denotes the unit gain (or transitional) frequency.

In controller design, the objective is to achieve such a similar frequency behavior, in a medium frequency range around ω_u, knowing that the closed loop dynamic behavior is exclusively linked to the open loop behavior around ω_u. Synthesizing such a template defines the non-integer approach that the second generation CRONE control uses.

There are a number of real life applications of CRONE controller such as the car suspension control [41], [37], flexible transmission [36], hydraulic actuator [42] etc. CRONE control has been evolved to a powerful non-conventional control design tool with a dedicate MATLAB toolbox for it [43]. For an extensive overview, refer to [44] and the references therein.

- *$PI^{\lambda}D^{\mu}$ Controller.* $PI^{\lambda}D^{\mu}$ controller, also known as $PI^{\lambda}D^{\delta}$ controller, was studied in time domain in [45] and in frequency domain in [46]. In general form, the transfer function of $PI^{\lambda}D^{\delta}$ is given by

$$C(s) = \frac{U(s)}{E(s)} = K_p + T_i s^{-\lambda} + T_d s^{\delta}, \qquad (13)$$

where λ and δ are positive real numbers; K_p is the proportional gain, T_i the integration constant and T_d the differentiation constant. Clearly, taking $\lambda = 1$ and $\delta = 1$, we obtain a classical PID controller. If $\lambda = 0$ $(T_i = 0)$ we obtain a PD^{δ} controller, etc. All these types of controllers are particular cases of the $PI^{\lambda}D^{\delta}$ controller. The time domain formula is that

$$u(t) = K_p e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + T_d \mathrm{D}_t^{\delta} e(t). \qquad (\mathrm{D}_t^{(*)} \equiv {}_0\mathrm{D}_t^{(*)}). \qquad (14)$$

It can be expected that $PI^{\lambda}D^{\delta}$ controller (14) may enhance the systems control performance due to more tuning knobs introduced. Actually, in theory, $PI^{\lambda}D^{\delta}$ itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator. For controller tuning techniques, refer to [47], [48]

- *Fractional Lead-Lag Compensator.* In the above, fractional controllers are directly related to the use of fractional-order differentiator or integrator. It is possible to extend the classical lead-lag compensator to the fractional-order case which was studied in [49]. The fractional lead-lag compensator is given by

$$C_r(s) = C_0 \left(\frac{1 + s/\omega_b}{1 + s/\omega_h}\right)^{r} \qquad (15)$$

where $0 < \omega_b < \omega_h$, $C_0 > 0$ and $r \in (0, 1)$.

IV. Biomimetic Actuator Control Based On Fractional Order Calculus - Some Considerations

As we know that "biomimetics" is an *engineering* discipline learning from nature. Biomimetics is in fact an enabling discipline which looks towards nature for ideas that may be adapted and adopted for solving problems: 'Inspiration rather than imitation'. Although in the current stage, the dominant efforts are on characterizing the biomimetic material properties, eventually, the control will come into play. The typical behaviors of biomimetic actuators include the time-dependent relaxation, anisotropy, elasticity, viscoelasticity, creep and hysteresis and other memoryless nonlinearities. These are tough challenging behaviors for advanced controllers. To appreciate the importance of controls, let us refer to the recent special issue on dynamics and control of smart structures, IEEE Transactions on Control Systems Technology. The Guest Editorial [50] tells that "Materials that are used in the construction of smart structures include, but are not limited to, piezoelectric materials, shape memory alloys, magnetorheological and electrorheological fluids and magnetostrictive materials. Most of these materials have been known for a long time. For example, piezoelectricity was first discovered in the early 1800s. The reason for an explosion of recent interest in this area can be attributed to the availability of powerful computers over the past two decades, which has allowed researchers and engineers to control the behavior of smart structural systems in efficient ways. A key factor in guaranteeing high-performance operation of smart structures is the control algorithm." In this special issue, many papers tried to attack the hysteresis problem [51], [52, e.g.] since hysteresis is an intrinsic characteristic of transducers based on "smart materials," which rely on modifications of stress-strain relationships invariably associated with hysteretic behavior.

We briefly envision the following on fractional calculus for biomimetic control:

- *Fractional order modelling and fractional order system identification.* To better model the dynamics of biomimetic materials, integer order based model will not be enough. Furthermore, the fractional order itself could be varying or state-dependent. Refer to [53] and the references therein for more details.
- *Fractional order controller design.* In Sec. III-E, it has been shown that fractional order controllers can improve the control systems performance. However, the controller design and tuning methods are just of a

recent focus [47], [48]. The fractional order disturbance observer [54] will be useful for vibration suppression in biomimetic control systems.

- *Fractional order controller for nonlinearity compensation.* For memoryless nonlinearities such as deadzone, backlash, static distortion can be compensated by using fractional order control. For example, [55] investigated the use of FOC to suppress the vibration due to backlash in motion control systems. For nonlinearities with memory, the typical one is the hysteresis. It is for sure that FOC can compensate the hysteresis effect in a closed-loop control systems. This is motivate by observing the Fig. 10 of [51] where the so-called "phaser" is nothing but a band-pass approximation of a fractional order differentiator.

Having pointed out the possible use of FOC in biomimetic control, it is now important to have a look of the implementation issue of fractional order controllers. Existing methods are all based on certain approximation schemes in either digital [43], [56] form or analog form [57]. Another important direction for FOC realization is the "authentic application" using "Fractors" [58]. The key idea of Fractor is that we can make use of certain "lossy" dielectrics to make a capacitance with impedance of the form of $Z_F = K/s^\alpha$ with K the lumped gain and $\alpha \in [0, 1]$ the fractional order [58]. For example, the lithium hydrazinium sulphate ($LiN_2H_5SO_4$) has such an impedance characteristic. This idea is now currently under pursuing aiming to mass produce compact FOC elements. We can expect that a revolution may happen when existing PI/PID controllers could be upgraded to fractional order PI/PID controllers (See Sec. III-E) since more than 90% of the industrial controllers are of PI/PID type.

Since the dynamic behaviors of many biomimetic materials exhibit certain "fractionality", this prompts us to check the other suitable implementations of FOC using biomimetic materials similar to $LiN_2H_5SO_4$. Therefore, the big picture for the future is the intelligent control of biomimetic system using biomimetic materials with fractional order calculus embedded. In other words, it is definitely worth to have a look of the notion of "intelligent control of intelligent materials using intelligent materials." Advocating this picture is the major purpose or contribution, if any, of this paper.

V. Concluding Remarks

The primary purpose of this paper is to introduce the notion of "intelligent control of intelligent materials by using intelligent materials". In this tutorial paper, after having briefly reviewed biomimetic materials and the biomimetic actuators, the fractional order calculus and fractional order control with possible applications in biomimetic actuator control is concisely introduced. Since the purely analog type fractional order integrator can be made with certain biomimetic materials or intelligent materials in general, it is logical to expect that the idea of "intelligent control of intelligent materials by using intelligent materials" will be widely adopted in future biomimetic controls with fractional order calculus embedded.

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