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Time-Optimal Magnetic Attitude Control for Small Spacecraft

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***Abstract*— Spacecraft attitude control using only magnetic coils suffers from a slowly varying uncontrollable axis. This lack of controllability results in slow slew maneuvering and convergence to equilibrium positions. In this paper the time-optimal control solution for magnetic attitude control is presented. Nonlinear time-varying models with constrained inputs are considered instead of the linearized model generally used. The time to reach equilibrium is less than that achieved by other conventional design methods, enabling magnetic attitude control to be applied to some time-critical applications.**

***Index Terms*— Attitude control; time-optimal; magnetic control.**

I. INTRODUCTION

Spacecraft attitude control using magnetic coils as the only actuators has become an active research topic in recent years due to its simplicity, low cost, and power efficiency. This technique is especially suitable for small satellites with modest attitude control performance requirements. The concept of magnetic attitude control is that interaction between the magnetic moment generated within a spacecraft and the magnetic field of the Earth produces a torque which can be used to control the attitude of the spacecraft. The main drawback of this method is that the control torque can only be generated perpendicular to the geomagnetic field vector, i.e., at any time instant, the direction parallel to the magnetic field of the Earth is uncontrollable. Therefore, methods applicable for other attitude control actuators, such as reaction wheels and thrusters can not be directly applied to magnetic attitude controller design. Controllability can only be achieved with the help of the changing nature of the magnetic field of the Earth along the spacecraft's orbit. This dependency makes magnetic attitude control inherently a time-varying problem. The uncontrollability problem, time-varying nature, and the nonlinear dynamics make the controller design very difficult.

Observing the periodic nature of the geomagnetic field, several current controller design methods have focused on periodic control theory [1], [2], [3], [4]. First, the controller is assumed to be periodic with respect to its orbit. Then the controller is obtained by solving the linearized time-varying equations. One of the consequences of this method is that the time unit becomes an orbital period, with attitude maneuvers or convergence to set points taking several orbits. In [5], [6], and [7], non-periodic controllers were proposed without relying on the periodic assumption of the controller and linearized model. Simulation results show that one more orbit is still required for a typical attitude maneuver.

Almost all current controller design methods have the following limitations. First, the convergence time to equilibrium is relatively long, which limits the application of magnetic attitude control for some time-critical tasks. Second, the maximum limits on the magnetic moments of the coils is not considered. Third, the nonlinear nature of the system dynamics is not taken advantage of to improve the control performance.

An interesting question raised here is: what is the performance potential for a magnetic attitude controller? Or specifically, how fast can a magnetic attitude controller converge to a desired orientation? To answer this question, we resort to optimal control theory, considering time-varying magnetic field, a nonlinear dynamic model and saturation constraints on the actuator.

The paper is organized as follows. In section II, the problem definition is presented. In section III, a numerical optimal control software package, RIOTS, is introduced to solve the optimal control problem. In section IV, transformation from a free final-time optimal control problem to fixed final-time optimal control problem is introduced. Section V presents the simulation results. In section VI, a model predictive controller based on the open-loop optimal controller is developed to increase the robustness of the control system. Section VII talks about the possibility of combining the open-loop time-optimal controller or the model predictive controller and a PD-like controller. Finally, section VIII concludes the paper.

II. PROBLEM FORMULATION

We assume that the spacecraft is inertially pointing with the following dynamics

$$I_x\dot{\omega}_x + \omega_y\omega_z(I_z - I_y) = T_x, \quad (1)$$

$$I_y\dot{\omega}_y + \omega_x\omega_z(I_x - I_z) = T_y, \quad (2)$$

$$I_z\dot{\omega}_z + \omega_x\omega_y(I_y - I_x) = T_z, \quad (3)$$

$$\dot{q} = \frac{1}{2}[\Omega]q, \quad (4)$$

where equations (1) (2) (3) are the well-known *Euler's moment equation* [8], $[\omega_x, \omega_y, \omega_z]^T$ is the angular velocity vector of spacecraft in the body coordinate system, I_x, I_y, and I_z are the moments of inertia of the spacecraft about the three principal axes. $[T_x, T_y, T_z]^T$ represent the external torque vector in the body coordinate system acting on the

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spacecraft. $q = [q_1, q_2, q_3, q_4]^T$ is the quaternion representing the attitude of the spacecraft [8], [9]. The matrix $[\Omega]$ is defined as

$$[\Omega] = \begin{pmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{pmatrix}, \qquad (5)$$

For the magnetic attitude control problem, the control torque $\vec{T}$ is

$$\vec{T} = \vec{M} \times \vec{B}_{body}(t, q), \qquad (6)$$

$$\vec{B}_{body}(t, q) = R(q) B_{ecf}(t), \qquad (7)$$

where $\vec{M}$ is the magnetic moment generated by three mutually perpendicular coils, $\vec{B}_{body}(t, q)$ is the magnetic field of the Earth in the body coordinate system, and $\vec{B}_{ecf}(t)$ is the magnetic field of the Earth in an Earth-centered coordinate system and can be regarded as a known time-varying parameter, since the magnetic field and the position of a spacecraft relative to the Earth can be predicted using orbit propagator algorithms. The definition of rotation matrix $R(q)$ can be found in [8] and [9]. The International Geomagnetic Reference Field (IGRF 2000) model was used to calculate the Earth's magnetic field [9].

The magnetic moment $\vec{M}$ is subject to the following saturation constraints:

$$|M_i| \leq M, \ i = x, y, z, \qquad (8)$$

where M is the maximal magnetic moment a single coil can generate.

Substituting (5) and (6) into (1), (2), (3) and (4), we have the following state space equations:

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ I_x \dot{\omega}_x \\ I_y \dot{\omega}_y \\ I_z \dot{\omega}_z \end{pmatrix} = \begin{pmatrix} (\omega_z q_2 - \omega_y q_3 + \omega_x q_4)/2 \\ (-\omega_z q_1 + \omega_x q_3 + \omega_y q_4)/2 \\ (\omega_y q_1 - \omega_x q_2 + \omega_z q_4)/2 \\ (-\omega_x q_1 - \omega_y q_2 - \omega_z q_3)/2 \\ -\omega_y \omega_z (I_z - I_y) + M_y B_z(t,q) - M_z B_y(t,q) \\ -\omega_x \omega_z (I_x - I_z) + M_z B_x(t,q) - M_x B_z(t,q) \\ -\omega_x \omega_y (I_y - I_x) + M_x B_y(t,q) - M_y B_x(t,q) \end{pmatrix} \qquad (9)$$

For this time-optimal control problem, we choose the following minimum-time objective function with end-point penalties:

$$J = c_q \sum_{k=1}^{3} (q_{kf} - q_{kd})^2 + c_\omega \sum_{k=1}^{3} (\omega_{kf} - \omega_{kd})^2 + \int_0^T \mathrm{d}t, \qquad (10)$$

where q_{kf} is the final quaternion, $q_{kd}$is the desired quaternion, ω_{kf} is the final angular velocity, ω_{kd} is the desired angular velocity, T, to be determined, is the time period for the attitude maneuver, c_q and c_ω are the penalty costs on attitude and angular velocity end point errors, respectively. Only the first three terms of $[q_{1f}, q_{2f}, q_{3f}, q_{4f}]^T$ are penalized because the fourth term is dependent on the other three terms.

The above formulated problem is a nonlinear, time-varying time-optimal control problem with constrained control inputs. The analytical solution is difficult to determine. Based on the well-known single degree of freedom solution [10], the minimum time optimal controller is expected to be a bang-bang controller. The questions are how small T can be, compared with other control laws, and whether the solution is suitable for practical implementation. We solve this problem using RIOTS, an optimal control software package.

III. Introduction to RIOTS

RIOTS [11], [12] is a group of programs and utilities, written mostly in C, FORTRAN, and M-file scripts and designed as a toolbox for MATLAB, that provides an interactive environment for solving a very broad class of optimal control problems (OCP). RIOTS is able to run under Windows 98/2000/XP and Linux operation systems. The user-OCPs can be prepared purely in M-files and no compiler is needed to solve the OCPs. To speed up the OCP solving process, there are two ways to go: by using the MATLAB compiler or by providing the user-OCP in C which is to be compiled by a C-compiler and then linked with some pre-built linking libraries (currently, Microsoft Visual C++ 6.0 for Windows 98/2000/XP and GNU gcc for Linux are supported).

The numerical methods used by RIOTS are supported by the theory in [13], which uses the approach of consistent approximation as defined in [14]. The following is a list of some of its main features.

- RIOTS solves a very large class of finite-time optimal control problems that includes: trajectory and endpoint constraints, control bounds, variable initial conditions (free initial time problems), and problems with integral and/or endpoint cost functions.
- Systems functions can be supplied by the user as either C-files or M-files.
- Systems dynamics can be integrated with fixed step-size Runge-Kutta integration, a discrete-time solver for a variable step-size method.
- The controls are represented as splines. This allows for a high degree of function approximation accuracy without requiring a large number of control parameters.
- The optimization routines use a coordinate transformation that creates an orthonormal basis for the spline subspace of controls, resulting in a significant reduction in the number of iterations required to solve a problem and an increase in the solution accuracy. It also makes the termination tests independent of the discretization level.
- There are three main optimization routines, each suited for different levels of generality of the optimal control problem. The most general is based on sequential quadratic programming methods. The most restrictive, but most efficient for large discretization levels, is based on the projected descent method. A third algorithm used the projected descent method in conjunction with an augmented Lagrangian formulation
- There are programs that provides estimates of the integration error for the fixed step size Runge-Kutta methods and estimates of the error of the numerically obtained optimal control.

- The algorithms are all founded on rigorous convergence theory.

IV. Transforming a Free Final-Time Problem into a Fixed Final-Time Problem

RIOTS is only able to solve fixed final-time optimal control problems, while the final-time for a time-optimal control problem is inherently free. However, a free final-time optimal control problem can be transformed into a fixed final-time optimal control problem by augmenting the system dynamics with two additional states [13]. The idea is to specify a nominal time interval, $[0, T_f]$, for the problem and to use a scale factor, adjustable by the optimization procedure, to scale the system dynamics and hence scale the duration of the time interval. This scale factor and the scaled time are represented by the extra states. Then RIOTS can minimize the objective function with the addition of these extra states and use their final values to adjust the time scale of the solution.

Suppose the state differential equation is

$$\dot{\mathbf{x}} = \mathbf{h}(t, \mathbf{x}, \mathbf{u}), \tag{11}$$

with the object function

$$J = g(\mathbf{x}(T)) + \int_0^T l(t, \mathbf{x}, \mathbf{u})\,\mathrm{d}t, \tag{12}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state variable vector, $\mathbf{u}(t) \in \mathbb{R}^m$ is the control input vector, $h : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$, $g : \mathbb{R}^n \rightarrow \mathbb{R}$, $l : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$, and T is the free final-time to be optimized.

Introducing two additional state variables, x_{n+1} and x_{n+2}, (11) and (12) can be transformed to the following fixed final-time optimal control problem with the state differential equation

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \dot{x}_{n+1} \\ \dot{x}_{n+2} \end{pmatrix} = \begin{pmatrix} x_{n+2}\mathbf{h}(x_{n+1}, \mathbf{x}, \mathbf{u}) \\ x_{n+2} \\ 0 \end{pmatrix}, \ t \in [0, T_f] \tag{13}$$

and the objective function

$$J = g(\mathbf{x}(x_{n+2}T_f)) + \int_0^{T_f} l(x_{n+1}, \mathbf{x}, \mathbf{u})\,\mathrm{d}t, \ t \in [0, T_f] \tag{14}$$

where x_{n+2} is the scale factor to be optimized, t is the pseudo time, $x_{n+1} = t x_{n+2}$ is the real time, T_f is a fixed positive real number chosen arbitrarily.

Following the above method, (9) and (10) can be transformed to the following fixed final-time optimal control problem with the state differential equation

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ I_x\dot{\omega}_x \\ I_y\dot{\omega}_y \\ I_z\dot{\omega}_z \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} x_9(\omega_z q_2 - \omega_y q_3 + \omega_x q_4)/2 \\ x_9(-\omega_z q_1 + \omega_x q_3 + \omega_y q_4)/2 \\ x_9(\omega_y q_1 - \omega_x q_2 + \omega_z q_4)/2 \\ x_9(-\omega_x q_1 - \omega_y q_2 - \omega_z q_3)/2 \\ x_9(-\omega_y\omega_z(I_z - I_y) + M_y B_z - M_z B_y) \\ x_9(-\omega_x\omega_z(I_x - I_z) + M_z B_x - M_x B_z) \\ x_9(-\omega_x\omega_y(I_y - I_x) + M_x B_y - M_y B_x) \\ x_9 \\ 0 \end{pmatrix} \tag{15}$$

and the objective function becomes

$$J = c_q \sum_{k=1}^{3}(q_{kf} - q_{kd})^2 + c_\omega \sum_{k=1}^{3}(\omega_{kf} - \omega_{kd})^2 + x_9 T_f. \tag{16}$$

V. Numerical Results

Since the order of this time-optimal magnetic attitude control problem is relatively high and the computation of the Earth's magnetic field is a time-consuming recursive process, we developed C-files to interface with RIOTS. The simulation parameters are listed as follows. The spacecraft has an inertia matrix given by $I = diag[0.7, 0.7, 0.7] kg \cdot m^2$, operating in a $51\,^\circ$ inclination circular orbit with an altitude of $450km$. The maximal magnetic moment a coil can generate is $M = 10Am^2$. For a better intuitive understanding of the spacecraft motion, the quaternions are converted to equivalent Z-Y-X Euler angles for numerical inputs and plotting. The initial attitude is $[30°, 35°, 40°]$ in Euler angles. The target attitude is $[0°, 0°, 0°]$. The initial and target angular velocity are both assumed to be $0\,deg/sec$. c_q and c_ω are chosen to be 1×10^5 and 1×10^8 through trial-and-error methods. c_q does not need to be very large, since applications using magnetic attitude control generally have modest accuracy requirements. c_ω is chosen to be large in order to force the final angular velocity to zero. The Earth's magnetic field used in the optimization process is a fourth order spherical harmonic model [9].

The simulation results are shown in Fig.1 and Fig.2. The control input represented in the body coordinate system is shown in Fig. 1, which is a bang-bang controller, as expected. The attitude maneuver is shown in Fig. 2 represented by Euler angle. We can see that a large angle maneuver from $[30°, 35°, 40°]$ to $[0°, 0°, 0°]$ can be performed in only about 150 seconds, about $1/35$ of the period. This is attributed to the bang-bang controller, which makes full use of the ability of the actuators, and the use of nonlinear model.

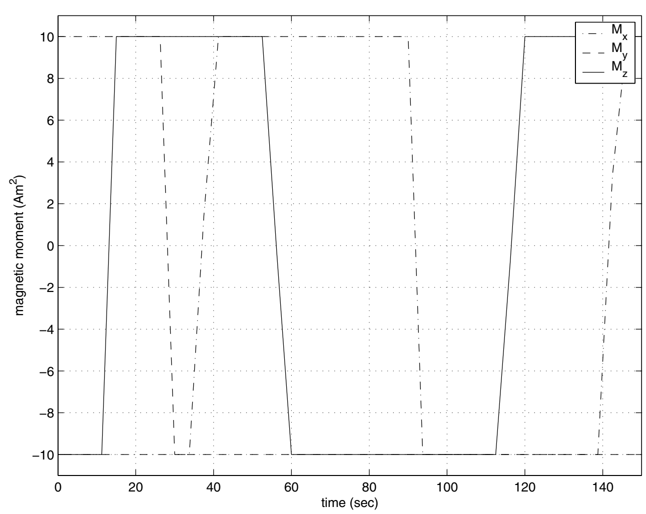

Fig. 1. Magnetic moment in body coordinate system, open-loop time-optimal control

We also simulated the control algorithm proposed in [7], a PD-like controller. The PD-like controller is formulated

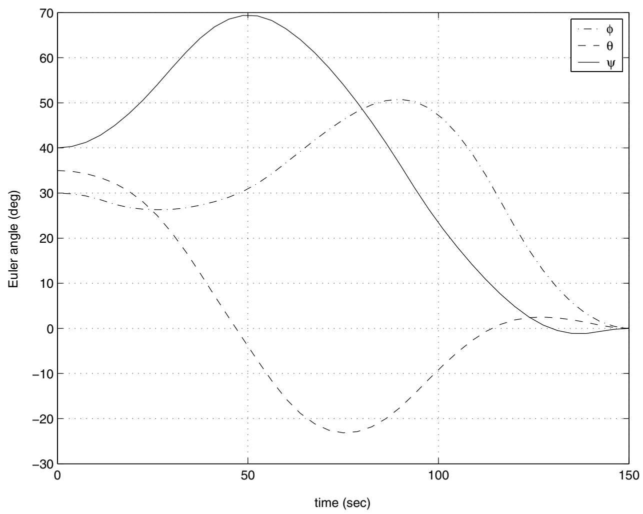

Fig. 2. Euler angle vs. time, open-loop time-optimal control

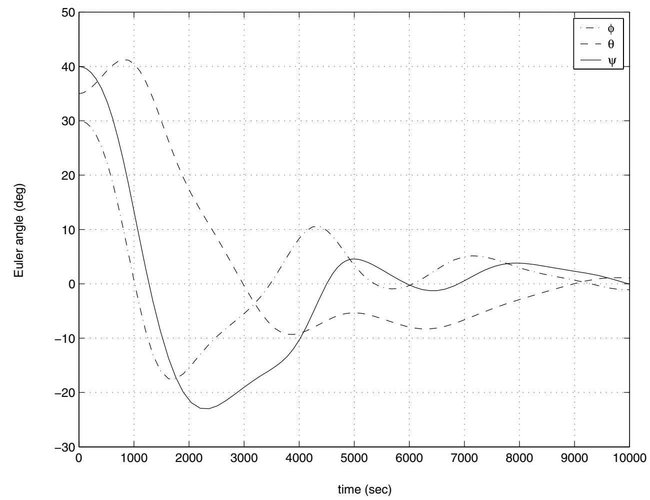

Fig. 3. Euler angle vs. time, time-optimal PD-like controller

as

$$M = B^T(b(t))(-\epsilon^2 k_p q_r - \epsilon k_v w), \qquad (17)$$

where $B^T(b(t))$ is a 3×3 matrix with elements constituted by the current measurement of the magnetic field vector; q_r is the first three elements of the quaternion; w is the angular velocity; ϵ, k_p, and k_v are three positive constants chosen by the designer.

For comparison purposes, we selected the parameters of the PD-like controller to satisfy a similar time minimization criterion

$$\min_{\epsilon,k_p,k_v,x_{sf}} J = c_q \sum_{k=1}^{3}(q_{kf}-q_{kd})^2+c_\omega \sum_{k=1}^{3}(\omega_{kf}-\omega_{kd})^2+x_{sf}T_f, \qquad (18)$$

where x_{xf} is the scale factor to be optimized and other parameters have the same meaning as in (16).

Since this is a parameter optimization problem rather than an optimal control problem, instead of RIOTS, we choose a free software, SolvOpt [15], which is a solver for nonlinear programming problems, to solve it.

The attitude trajectory driven by the time-optimal PD-like controller is plotted in Fig. 3. Comparison between Fig. 2 and Fig. 3 shows that the time-optimal control solution introduced in this paper is about 50 times faster, which makes magnetic attitude control applicable to some time-critical tasks. Also, the short time (a few minutes) required to reach equilibrium point makes this open-loop controller less susceptible to external disturbances. The computational time required to determine the time-optimal solution is about five minutes on an Intel P4 3.0G machine, which is acceptable, especiallyp considering the ever growing CPU speed in the future.

The main drawback of this time-optimal controller is its open-loop nature, which suffers from modeling errors and disturbances. To increase the robustness of the control system, We have developed two closed-loop implementations of the minimum time controller. The continuous optimization method is described in [16]. The method presented in this paper is the model predictive control.

VI. A MODEL PREDICTIVE SUB-OPTIMAL FEEDBACK CONTROLLER

The design of closed-loop optimal controller has been an active research area since the advent of optimal control theory. Unfortunately, except for some simple systems, the design of closed-loop optimal controller needs to solve the Hamilton-Jacobi-Bellman (HJB) equation [10], an almost impossible task for higher order nonlinear systems. A more practical method is perturbation feedback control [10], in which a compensating controller is derived using variational methods based on the error between the current states and the pre-calculated optimal trajectory, and the compensating controller is then added to the pre-calculated optimal controller. However, this method can not be applied to bang-bang controller because bigger inputs can not be generated. To design a closed-loop optimal bang-bang controller, one needs to find the switching surfaces, an unsolved problem for general nonlinear systems, especially if the systems is time-varying. In [17], a Gröber basis approach was proposed to find the switching surfaces. However, practical implementation requires the system dynamics be analytically integrable when the inputs are constants, excluding its application to magnetic attitude control problem. In [18] and the referenced publications within, model predictive controller was used to approximate the optimal controller when the moving horizon is extended. However, the computational burden increases when the moving horizon is extended, making this method hard to be implemented on-line.

In this paper, we propose a model predictive controller [19], as a closed-loop sub-optimal controller. The basic idea is to calculate the optimal open-loop controller first, then try to track the pre-calculated optimal trajectory using a model predictive controller. This can be used as a general method to design a closed-loop sub-optimal controller for nonlinear systems, especially when the input is constrained, which the perturbation feedback control method can not handle.

We use the following excerpt from [20] to introduce model predictive control: "model predictive control (MPC) or receding horizon control (RHC) is a form of control in which the current control action is obtained by solving on-line, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in the sequence is applied to the plant."

We assume the control is constant during the sampling period and model the dynamic system by difference equations

$$x(k+1) = f(x(k), B(k), M(k)), \tag{19}$$

where x, B, and M are the system states, the time-varying Earth's magnetic field vector, and the magnetic moment generated by the magnetic coils, respectively.

Denote the already obtained optimal trajectory as $x^*(k)$, we formulate the following tracking model predictive control problem, in which the receding horizon is equal to the sampling time.

$$\min_{M(k)} \quad (x(k+1)-x^*(k+1))^T Q(x(k+1)-x^*(k+1)) \tag{20}$$

where Q is the weighting matrix and $M(k)$ is subject to the following constraints

$$|M(k)_i| \leq M, \ i = x, y, z, \tag{21}$$

where M is the maximal magnetic moment a single coil can generate.

$M^*(k)$, the optimal value of $M(k)$, is used as the next control.

Following are some comments and implementation-related issues.

- In case of deviation from the pre-calculated optimal trajectory, the model predictive control is no longer globally optimal, because the globally optimal controller should be bang-bang. However, if the pre-calculated optimal trajectory can be followed, the model predictive controller performance should approach the optimal controller.
- This method differs from the usual model predictive control in that we have obtained the open-loop optimal control, which is used as the initial value of $M(k)$ in the optimization process. Assuming small disturbances and modeling errors, the open-loop optimal control should be close to $M^*(k)$, which reduces the computational time.
- Since the sampling time is usually very short (three seconds in our simulation), B_{ecf} can be considered constant during the sampling period. This avoids calling functions for IGRF model during the optimization process, which are recursion-based and time-consuming.
- Because of the short sampling time, second-order Runge-Kutta methods can be accurate enough to predict $x(k+1)$ based on $x(k)$, which again reduces the computation requirement.

TABLE I

COMPARISON OF CONTROLLER PERFORMANCE AND ROBUSTNESS

	Convergence Time (sec)	Final Attitude Error (deg.)	Final Angular Rate (deg./sec)
Open-loop Control Exact Sys. Knowledge	150	0.06	5.4×10^{-3}
Optimal PD-like Control Exact Sys. Knowledge	8500	3.57	8.1×10^{-3}
Open-loop Control with Modeling Errors	150	16.60	1.7×10^{-1}
Model Predic. Control with Modeling Errors	180	4.19	7.7×10^{-2}

- The final static attitude errors can be adjusted by the weighting matrix Q, depending on the attitude precision requirement.

To show the improved robustness against the modeling errors and disturbances, we simulated the controller response with an added aerodynamic torque, an inaccurate inertia matrix, and an inaccurate magnetic field model of the Earth. The assumed elements in the inertia matrix are one percent bigger than the actual. The Earth's real magnetic field is assumed to be the 10th order spheric harmonic model, while the optimal bang-bang controller is derived with the fourth order spheric harmonic model. Since the inertia matrix used is proportional to the identity matrix, the effect of gravity gradient torque is negligible.

The open-loop optimal controller is applied to the mis-modeled system. The trajectory of Euler angle is plotted in Fig. 4. We can see that the open-loop controller performs poorly if the system model is not accurate enough. When the model predictive controller is applied to the mis-modeled system, the trajectory of Euler angle is plotted in Fig. 5. The model predictive controller shows its robustness against the modeling errors and external disturbances. The model predictive controller output is plotted in Fig. 6.

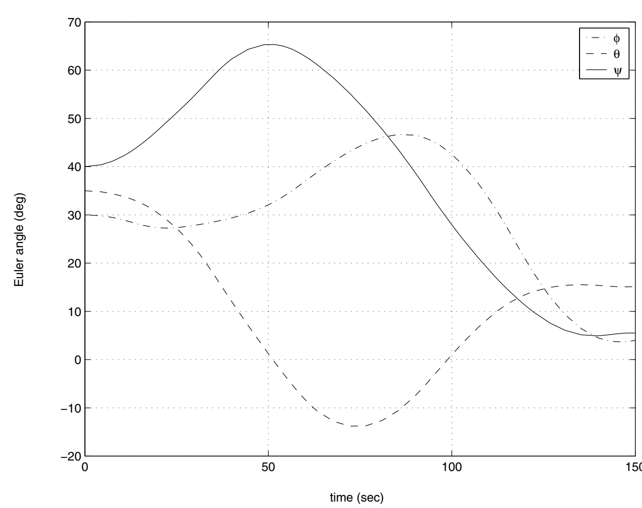

Fig. 4. Euler angle vs. time, open-loop controller

Table I summarizes the performance and the robustness of the controllers simulated in this paper.

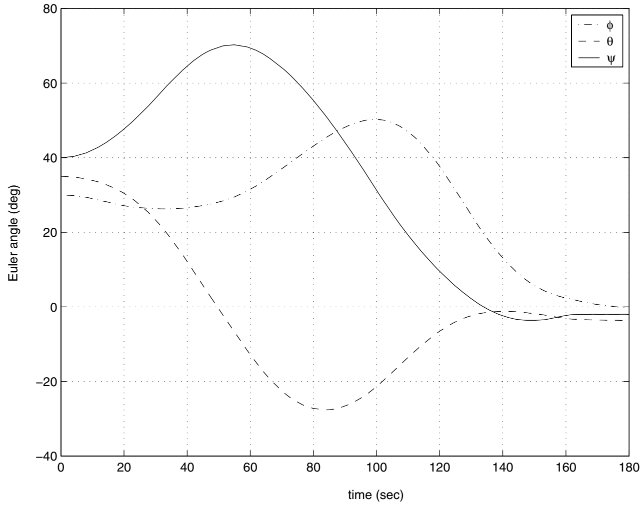

Fig. 5. Euler angle vs. time, model predictive controller

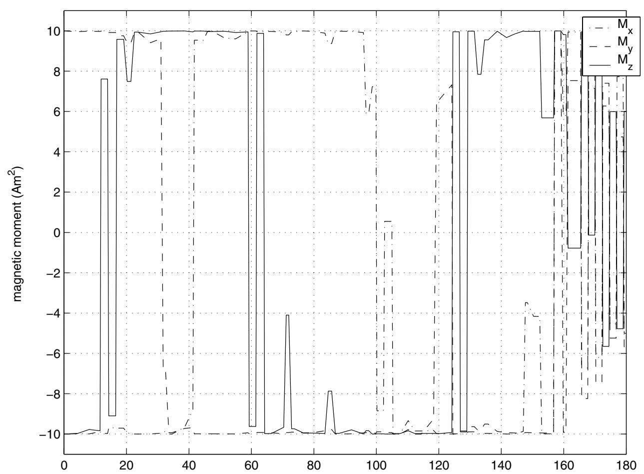

Fig. 6. Model predictive controller output

VII. Combining the Time-Optimal Controller and the PD-like Controller

The advantages of the model predictive controller are its closed-loop nature and fast convergence, while its main disadvantages are the high power cost and the real time computational requirements. The advantages of the PD-like controller are its power efficiency, linearity, and fast computation. The best approach may be to combine the two methods together. For example, the open-loop controller or the model predictive controller is used for large slew maneuvers, with the PD-like controller switched in when the errors reach sufficiently small values for attitude maintenance.

VIII. Concluding Remarks

The open-loop time-optimal solution and the model predictive control solution are presented. The solutions show significant improvement in the time required for slew maneuvers over conventional designs. The model predictive control solution is shown to be robust against modeling errors and disturbances. The model predictive control approach introduced in this paper can be used as a general method to design sub-optimal feedback controller for nonlinear systems with constrained inputs.

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