Proceedings of the IEEE
International Conference on Mechatronics & Automation
Niagara Falls, Canada • July 2005

Monotonic Convergent Iterative Learning Controller Design With Iteration Varying Model Uncertainty

Hyo-Sung Ahn
Center for Self-Organizing and Intelligent Systems (CSOIS)
Dept. of Electrical and Computer Engineering
Utah State University, Logan, UT 84322-4160, USA

Kevin L. Moore
Research and Technology Development Center
Johns Hopkins University Applied Physics Laboratory
11100 Johns Hopkins Road, Laurel, MD 20723-6099, USA

YangQuan Chen
Center for Self-Organizing and Intelligent Systems (CSOIS)
Dept. of Electrical and Computer Engineering
Utah State University, Logan, UT 84322-4160, USA

***Abstract*— In iterative learning control, model uncertainty issue has been studied in various ways. But, in existing works, the model uncertainty has been considered as iteration invariant, that is, it does not change on the iteration axis. Furthermore, in these works, monotone convergence problem has not been properly studied. In this paper, new monotonic convergence condition of iteration varying model uncertain iterative learning control system is derived. So, the main goal of this paper is to guarantee the monotone convergence of the iteration varying model uncertainty system until the tracking error is reduced below the previously calculated base-line error boundary. Through the numerical example, the validity of the suggested method is illustrated.**

***Index Terms*— Iterative learning control, iteration variant model uncertainty, monotone convergence, base-line error.**

I. Introduction

Iterative learning control has a well-established research history as shown in [1], [2], [3]. Recently, monotone convergence property of ILC system has been fully investigated in [4]. With model uncertainty, three existing results are available for monotone convergence test of ILC. In [5], Lyapunov equation was solved to find sufficient condition of interval ILC system; in [6], whole vertex matrices of interval Markov matrix were checked for the monotone convergence, and in [7] monotone convergence ILC was designed based on the monotone convergence condition provided in [6]. However, scopes of these works are restricted to the iteration invariant ILC system. In this paper, monotone convergence condition of an iteration varying interval ILC system is studied. Base-line error of the ILC system has been recently studied in [8], [9], [10]. However, in these works, the base-line error has not been properly related with the monotone convergence property. This paper calculates the base-line error boundary over which the monotone convergence of the iteration varying model uncertain ILC system is guaranteed. Validation of a developed analysis is verified through a numerical simulation. This paper consists of as follows: In Section II, a brief explanation about the super-vector ILC is given and in Section III, the monotonically convergent ILC system is designed with iteration varying model uncertainty. In Section IV, Markov parameters of the k^{th} iteration plant is calculated for $(k+1)^{th}$ iteration update and a numerical example is given in Section V. Final conclusion is given in Section VI.

Corresponding author: Prof. YangQuan Chen, Center for Self-Organizing and Intelligent Systems, Dept. of Electrical and Computer Engineering, 4160 Old Main Hill, Utah State University, Logan, UT 84322-4160. **T**: (435)7970148, **F**: (435)7973054, **W**: `www.csois.usu.edu`, **E**: `yangquan.chen@usu.edu`

II. Basic background materials of model uncertain super-vector ILC

Iterative learning control (ILC) is an effective control tool for improving the transient response and tracking performance of uncertain periodic (dynamic) systems. Basic ILC update rule is shown in Fig. 1, where the next trial control input is calculated from the previous control input and previous transient error. Mathematically, ILC update rule of Fig. 1 can be written as:

$$u_{k+1}(t) = u_k(t) + \gamma_k(y_d(t) - y_k(t)) \qquad (1)$$

where k is the iteration trial, t is the discrete time point, $y_d(t)$ is the desired output, and $y_k(t)$ is the measured output. In this algorithm, by properly choosing the learning gain γ_k, it is easy to make the system converge. Iterative learning control using super-vector analysis approach has been well established in the literatures [4], [11], [12]. The advantage of the super-vector notation is that, if the time axis is considered as 1-dimensional multiple inputs, the 2-dimensional ILC problem is then changed as a 1-dimensional multi-input multi-output (MIMO) problem. As shown in, for example, [2], [11], [13], most discrete-time ILC problems can be expressed in the form

$$Y_k = HU_k$$

where $Y_k, U_k \in R^N$ with trial length N, and H is a lower-triangular Toeplitz matrix whose elements are Markov parameters of the system to be controlled in the linear case. For time-varying systems and some classes of affine nonlinear systems, a similar representation can also be developed, with the key feature that the matrix H is lower triangular. The super-vector approach to ILC is to design a learning gain matrix Γ so the resulting "closed-loop system" with control input $U_{k+1} = U_k + \Gamma E_k$, in the iteration domain, given by

$$E_{k+1} = (I - H\Gamma)E_k$$

where $E_k = Y_d - Y_k$ is the output tracking error, is either asymptotically and/or monotonically convergent along the

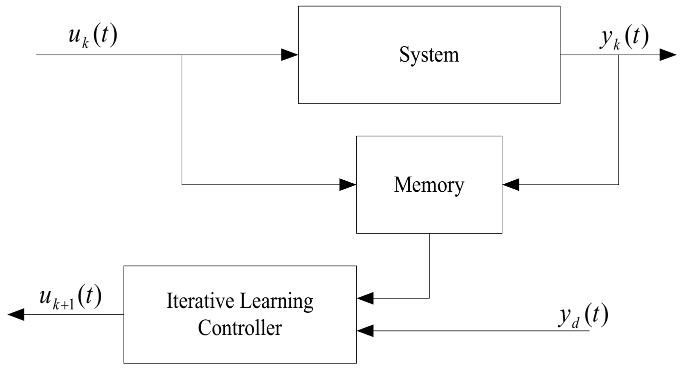

Fig. 1. ILC configuration.

iteration axis in an appropriate norm topology. Such stability conditions have been analyzed in [14], [15], [4]; with design issues considered in [16], [17], [4]. However, in these works, Markov matrix H is considered as iteration invariant, i.e., H is not dependent on iteration k. In this paper, however, it is assumed that H could be iteration dependent. So, for input and output relationship, the following equation is used:

$$Y_k = H_k U_k$$

where H_k is varying along the iteration axis.

Let us consider the following uncertain plant model

$$\begin{aligned} x_k(t+1) &= (A+\Delta_k^A)x_k(t) + (B+\Delta_k^B)u_k(t) + v(k,t) \\ y_k(t) &= (C+\Delta_k^C)x_k(t) + w(k,t) \end{aligned} \tag{2}$$

where Δ_k^A, Δ_k^B, and Δ_k^C are model uncertainties varying with iteration, and $v(k,t)$ and $w(k,t)$ are time and iteration dependent process and measurement noises. As a reminder, t is the discrete time point along the time axis, this means that t is on a finite horizon [1]. That is, at each iteration trial, t has N different discrete time points. Meanwhile, k is on an infinite horizon. So it monotonically increases. Thus, we can consider two different types of model uncertainties. The first type is iteration independent model uncertainty and the second type is iteration variant model uncertainty. In this paper, we are interested in the latter. So, the main research goal of this paper is the monotone convergent ILC system design with iteration varying model uncertainty. Under this research direction, the firstly required task is to convert the model uncertainty of (2) to the Markov matrix H_k according to the model uncertainty type. Let us suppose that the model uncertainty has been converted into Markov matrix plant properly (for the interval model conversion, see [18]) and it is denoted as:

$$H_k^I = H^o + \Delta_k^H, \tag{3}$$

where H^o is Markov matrix corresponding to the nominal plant; Δ_k^H is an additive uncertain Markov matrix corresponding to the uncertainty of the plant, which is bounded as $\Delta_k^H \in \Delta^I := \left[\underline{\Delta}, \overline{\Delta}\right]$ with upper boundary $\overline{\Delta}$ and lower boundary $\underline{\Delta}$, and superscript I is used to denote the interval model uncertainty (for interval concept in ILC, see [6]). Then, using a model uncertain set concept, H_k^I can be bounded as:

$$H_k^I \in \mathcal{H}^I := \left[\underline{H}, \overline{H}\right] \tag{4}$$

where $\mathcal{H}^I$ is an interval uncertain Markov plant set, $\underline{H}$ is the lower boundary of the interval Markov plant, and $\overline{H}$ is the upper boundary of the interval Markov plant. Next, our task is to find the learning gain matrix such that the uncertain super-vector ILC systems defined from (3) are asymptotical stable or monotone convergent against all possible uncertain interval plants $\mathcal{H}^I$. In this paper, we will only consider the monotone convergence property due to the practical reason. Throughout the paper, it is assumed that $\overline{\Delta} = -\underline{\Delta}$ and the model uncertainty is bounded in operator norm topology such as:

$$\|\Delta_k^H\| \le \Delta^*, \ \ \Delta_k^H \in \Delta^I, k = 1, 2, \cdots, \infty$$

Then, for the calculation of Δ^*, the following lemma is used:

Lemma 2.1: The maximum 2-norm of Δ_k^H, i.e., Δ^*, is calculated from $\|\underline{\Delta}\| = \|\overline{\Delta}\| = \Delta^*$.

Proof: See theorem 3.1 of [19]. ∎

In this section, the basic concepts of iteration varying model uncertain ILC system have been described. In next section, iteration varying learning gain ILC system is designed for the monotone convergence of iteration varying model uncertain ILC system.

III. ILC Design with Iteration Varying Learning Gain Matrices for Iteration Varying Model Uncertain System

In this section, the iteration varying learning gain matrices are designed. Note, although it is not impossible to guarantee the monotone convergence of iteration varying model uncertain ILC system with fixed learning gain matrices, we found that it is quite tough to reduce the base-line error using fixed learning gain matrices. So, this paper only focuses on the iteration varying learning gain ILC system.

Using $H_{k+1}^I \equiv H^o + \Delta_{k+1}$ and $H_k^I \equiv H^o + \Delta_k$, and using ILC update rule

$$U_{k+1} = \Lambda_k U_k + \Gamma_k E_k \tag{5}$$

[1] It is important to emphasize that ILC stability analysis along the time axis is basically questionable because the stability property in control system is generally searched on the infinite domain except some input-output stabilities. So, in ILC, stability along the time axis could not be properly investigated. Only, the stability analysis along the iteration axis is analytically right. From this respect, super-vector approach is considered most natural in ILC.

we can derive the following error propagation rule:

$$E_{k+1} = \left(H^I_{k+1}\Lambda_k(H^I_k)^{-1} - H^I_{k+1}\Gamma_k\right)E_k + \left(I - H^I_{k+1}\Lambda_k(H^I_k)^{-1}\right)Y_d. \quad (6)$$

In (5), the ILC control input U_{k+1} is calculated by multiplying Λ_k to U_k and by multiplying Γ_k to E_k. Here, noticing that Y_d is fixed [2] and E_k is known, we can find an intermediary matrix W_k that satisfies $Y_d = W_k E_k$. Furthermore, in calculating U_{k+1} of (5), E_k is calculated from $E_k = Y_d - H_k U_k$. So, since Y_d and U_k are known, it is reasonable to assume that H_k can be found at $(k+1)^{th}$ iteration. Thus, if we use iteration varying learning gain matrices, in (6), only H^I_{k+1} is unknown and uncertain. Therefore, since H^I_k in (6) is no more uncertain, (6) can be written as:

$$E_{k+1} = \left(H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1} - H^I_{k+1}\Gamma_k\right)E_k + \left(I - H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1}\right)W_k E_k \quad (7)$$

where the estimated Markov matrix $\widehat{H}_k$ has been substituted for H_k. Then, we obtain the following theorem.

Theorem 3.1: Let the learning gain matrices be given as $\Lambda_k = (H^o)^{-1}\widehat{H}_k$ and $\Gamma_k = (H^o)^{-1}$, then the iteration varying ILC system is monotonic convergent, i.e., $\|E_{k+1}\| < \|E_k\|$ if

$$\Delta^* < \frac{1}{\alpha}$$

where $\alpha = \min_{W_k}\left\{\|(H^o)^{-1}W_k\|\right\}$ where W_k is determined from $Y_d = W_k E_k$.

Proof: From (7), the monotone convergent condition is given as:

$$\|\mathcal{T}\| < 1 \quad (8)$$

where $\mathcal{T} := H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1} - H^I_{k+1}\Gamma_k + \left(I - H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1}\right)W_k$. Let us substitute $\Gamma_k = \Lambda_k\widehat{H}_k^{-1}$ into (8), then we have

$$\left\|\left(I - H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1}\right)W_k\right\| < 1.$$

From the relationships:

$$\begin{aligned}
&\left\|\left(I - H^I_{k+1}\Lambda_k(\widehat{H}_k)^{-1}\right)W_k\right\| \\
&= \left\|\left(I - (H^o + \Delta_{k+1})\Lambda_k(\widehat{H}_k)^{-1}\right)W_k\right\| \\
&= \left\|\left(I - (H^o + \Delta_{k+1})(H^o)^{-1}\right)W_k\right\| \\
&= \left\|\Delta_{k+1}(H^o)^{-1}W_k\right\| \\
&\le \|\Delta_{k+1}\|\left\|(H^o)^{-1}W_k\right\| \\
&\le \Delta^*\left\|(H^o)^{-1}W_k\right\|
\end{aligned}$$

if

$$\Delta^*\left\|(H^o)^{-1}W_k\right\| < 1 \Leftrightarrow \Delta^* < \frac{1}{\|(H^o)^{-1}W_k\|},$$

then ILC system is monotonic convergent. Thus, we have $\Gamma_k = (H^o)^{-1}$. However, observe that W_k matrix which satisfies the relationship $Y_d = W_k E_k$ is not unique. Therefore, to check the monotone convergence of ILC system at k^{th} iteration without conservatism, we have to find

$$\max_{W_k}\left\{\frac{1}{\|(H^o)^{-1}W_k\|}\right\} \Leftrightarrow \min_{W_k}\left\{\|(H^o)^{-1}W_k\|\right\},$$

with $Y_d = W_k E_k$. Consequently, at k^{th} iteration trial, if

$$\Delta^* < \frac{1}{\alpha}$$

where $\alpha = \min_{W_k}\left\{\|(H^o)^{-1}W_k\|\right\}$ with $Y_d = W_k E_k$, then $\|E_{k+1}\| < \|E_k\|$. ■

If amount of conservatism can be allowed, Theorem 3.1 can be relaxed as follows. From $\|Y_d\| \le \|W_k\|\|E_k\|$ and from

$$\frac{1}{\|(H^o)^{-1}\|\|W_k\|} \le \frac{1}{\|(H^o)^{-1}W_k\|},$$

we derive the following result.

Corollary 3.1: With $\Lambda_k = (H^o)^{-1}\widehat{H}_k$ and $\Gamma_k = (H^o)^{-1}$, the iteration varying ILC system is monotonic convergent if

$$\Delta^* < \frac{\underline{\sigma}(H^o)}{\alpha}$$

where $\alpha = \min\{\|W_k\|\}$ with $Y_d = W_k E_k$.

Proof: Since $\|(H^o)^{-1}\| = \frac{1}{\underline{\sigma}(H^o)}$, the proof is immediate. ■

Remark 3.1: In Theorem 3.1 and Corollary 3.1, the monotone convergence conditions were derived without calculating the inverse interval matrix of (6). So, the results will be much less conservative than a result obtained from using the inverse interval matrix of (6). However, as a disadvantage of this method, we have to solve $\alpha = \min_{W_k}\left\{\|(H^o)^{-1}W_k\|\right\}$ and $\alpha = \min\{\|W_k\|\}$ with constraint $Y_d = W_k E_k$ every iteration trial.

Theorem 3.1 and Corollary 3.1 show the monotone convergence conditions at k^{th} iteration. Using some operator norm properties, we can find the reversed condition: under what condition, the monotone convergence is not guaranteed any more ? The following theorem is for this.

Theorem 3.2: With $\Lambda_k = (H^o)^{-1}\widehat{H}_k$ and $\Gamma_k = (H^o)^{-1}$, defining $E_k = \widehat{E}_k\varrho$ such that $\|\widehat{E}_k\| = 1$, if

$$\frac{\underline{\sigma}(H^o)}{\alpha_{min}} \le \Delta^*$$

where $\alpha_{min} = \frac{\|Y_d\|}{\varrho}$, then the monotone convergence of ILC system is not guaranteed any more.

Proof: From Corollary 3.1, $\alpha = \min\{\|W_k\|\}$ with $Y_d = W_k E_k$. Now, changing this into $Y_d = W_k\widehat{E}_k\varrho$, we find the relationship

$$\|Y_d\| = \|W_k\widehat{E}_k\|\varrho \le \|W_k\|\varrho \quad (9)$$

because $\|W_k\| = \max_{\|x\|=1}\|W_k x\|$. So, $\alpha = \min\{\|W_k\|\} \ge \frac{\|Y_d\|}{\varrho}$ since $\|W_k\| \ge \frac{\|Y_d\|}{\varrho}$. Therefore, the following relationship is always true: $\frac{\underline{\sigma}(H^o)}{\alpha} \le \frac{\underline{\sigma}(H^o)}{\alpha_{min}}$. Consequently, if $\frac{\underline{\sigma}(H^o)}{\alpha_{min}} \le \Delta^*$, then $\frac{\underline{\sigma}(H^o)}{\alpha} \le \Delta^*$, so the monotone convergence is not guaranteed. ■

In above Theorem 3.1 and Corollary 3.1, to calculate α, an optimization scheme can be used. However, in each iteration, it is computationally expensive to perform the optimization

[2] Of course, Y_d could be iteration variant, but in this paper it is assumed iteration invariant.

as commented in Remark 3.1. In what follows, so-called a sub-optimization scheme is suggested to find α. To find $\alpha = \min_{W_k} \left\{ \|(H^o)^{-1} W_k\| \right\}$ with constraint $Y_d = W_k E_k$, let us use the equality:

$$(H^o)^{-1} Y_d = (H^o)^{-1} W_k E_k \tag{10}$$

Now, defining $\beta \equiv \arg_{E_k(j)} \max_{i=1,\cdots,N} |E_k(i)|$ where $E_k = [E_k(1), E_k(2), \cdots, E_k(N)]^T$ and $j \in \{1, 2, \cdots, N\}$, we find $N \times N$ matrix $\mathcal{O}$, whose elements are zeros except its j^{th} column (denoted by $\mathcal{O}^j$), which is defined as:

$$\mathcal{O}^j \equiv [Z(1)/\beta, Z(2)/\beta, \cdots, Z(N)/\beta]^T \tag{11}$$

with notation $Z \equiv (H^o)^{-1} Y_d$. Then, α in Theorem 3.1 and Corollary 3.1 is calculated as $\alpha = \|\mathcal{O}\|$.

Remark 3.2: In Theorem 3.1, we are interested in finding $\min_{W_k} \left\{ \|(H^o)^{-1} W_k\| \right\}$ subject to $(H^o)^{-1} Y_d = (H^o)^{-1} W_k E_k$. Observing that $(H^o)^{-1} Y_d$ is fixed, so if E_k is big, then $(H^o)^{-1} W_k$ is small, or if E_k is small, then $(H^o)^{-1} W_k$ is big. Therefore, by selecting the biggest element of E_k, we can choose smallest $(H^o)^{-1} W_k$ such that $H^{-1} Y_d = (H^o)^{-1} W_k E_k$. So, even though $\alpha = \|\mathcal{O}\|$ is not an optimal value, it will be close to $\min_{W_k} \left\{ \|(H^o)^{-1} W_k\| \right\}$. This approach is computationally very simple, but could be an effective solution.

Now, by simple algebraic manipulation, since the following is true:

$$\|\mathcal{O}\| = \frac{1}{|\beta|} \|Z\|$$

we can obtain a simplified monotone convergent condition:

Corollary 3.2: If $\Delta^* \leq \frac{|\beta|}{\|Z\|}$, learning gain matrices $\Lambda_k = (H^o)^{-1} \widehat{H}_k$ and $\Gamma_k = (H^o)^{-1}$ guarantee the monotonic convergence of iteration varying ILC system.

Corollary 3.2 provides a very simple monotone convergent checking method. Now, using above result, we arrive our final goal for the simplified monotone convergence condition and base-line error boundary.

Theorem 3.3: Learning gain matrices $\Lambda_k = (H^o)^{-1} \widehat{H}_k$ and $\Gamma_k = (H^o)^{-1}$ guarantee the monotone convergent transient if

$$\Delta^* \|Z\| < \|E^b\|_\infty.$$

Proof: From $\Delta^* \leq \frac{|\beta|}{\|Z\|}$, since we have $\Delta^* \|Z\| \leq |\beta|$, due to the fact that $|\beta| = \|E_k\|_\infty$, the proof is straightforward. ■

Theorem 3.3 tells us that if $\|E^b\|_\infty$ is bigger than $\Delta^* \|Z\|$, which can be calculated previously, the ILC system is monotonic convergent. So, the system converges until the tracking error is reduced below $\Delta^* \|Z\|$. Thus, we now define $E^* \equiv \Delta^* \|Z\|$ as base-line error boundary in ∞-norm topology.

IV. Parameter estimation

In this section, a method is developed to estimate Markov parameters, i.e., for $\widehat{H}_k$. From

$$Y_k = H_k U_k + w_k \tag{12}$$

where w_k is the measurement noise, our purpose is to estimate Markov matrix H_k using the known control input U_k and the measured output Y_k. Generally, in Wiener filter or least square estimation, assuming that H_k is known and Y_k is measured, it is desired to estimate U_k. In detail, in least square method, the solution is given as:

$$\widehat{U}_k = (H_k^T H_k)^{-1} H_k^T \widetilde{Y}_k \tag{13}$$

where $\widetilde{Y}_k$ is the measured output and $\widehat{U}_k$ is the estimated input. So, in least square method, the measurement noise is not considered. In Wiener filter, the input is estimated as:

$$\widehat{U}_k = \mathcal{G} \widetilde{Y}_k \tag{14}$$

where Wiener filter $\mathcal{G}$ is calculated as:

$$\mathcal{G} = [H_k^T H_k + \sigma_n^2 \mathbf{R}^{-1}]^{-1} H_k^T \tag{15}$$

where

$$\mathbf{R} = E[U_k U_k^T], \quad \mathbf{R}_n = \text{diag}(\sigma_n^2) = E[w_k w_k^T].$$

However, in ILC, our purpose is to estimate H_k using known U_k and measured output $\widetilde{Y}_k$, so above Wiener filter and least square estimation cannot be used. But, fortunately, from the fact that H_k is lower triangular Toeplitz matrix, we can easily formulate the Wiener filter to our ILC system. The key idea of this approach is to use the following property:

Property 4.1: If $U_k(1) \neq 0$, then the following commutative property is true:

$$H_k U_k = \mathbf{U}_k \mathbf{h}_k$$

where H_k is a lower triangular Toeplitz matrix, U_k is a column vector, $\mathbf{U}_k$ is a Toeplitz matrix defined from U_k such as:

$$\mathbf{U}_k = \begin{bmatrix} U_k(1) & 0 & 0 & \cdots & 0 \\ U_k(2) & U_k(1) & 0 & \cdots & 0 \\ U_k(3) & U_k(2) & U_k(1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_k(N) & U_k(N-1) & U_k(N-2) & \cdots & U_k(1) \end{bmatrix}$$

and new Markov column vector $\mathbf{h}_k$ is defined as:

$$\mathbf{h}_k = [H_k(1,1), H_k(2,1), H_k(3,1), \cdots, H_k(N,1)]^T. \tag{16}$$

Thus, using Property 4.1, (12) can be rewritten as:

$$Y_k = H_k U_k + w_k = \mathbf{U}_k \mathbf{h}_k + w_k \tag{17}$$

Finally, by estimating $\mathbf{h}_k$ from Wiener filter or from least square method, we can optimally estimate Markov matrix H_k at each iteration trial. From the least square method, $\mathbf{h}_k$ is estimated as:

$$\widehat{\mathbf{h}}_k = (\mathbf{U}_k^T \mathbf{U}_k)^{-1} \mathbf{U}_k^T \widetilde{Y}_k. \tag{18}$$

From Wiener filter, Markov parameters are estimated as:

$$\widehat{\mathbf{h}}_k = \mathcal{G} \widetilde{Y}_k \tag{19}$$

where Wiener filter $\mathcal{G}$ is calculated by

$$\mathcal{G} = [\mathbf{U}_k^T \mathbf{U}_k + \sigma_n^2 \mathbf{R}^{-1}]^{-1} \mathbf{U}_k^T \tag{20}$$

where

$$\mathbf{R} = E[\mathbf{h}_k \mathbf{h}_k^T], \quad \mathbf{R}_n = E[w_k w_k^T].$$

So, if the least square method is used, then we do not need to know the stochastic characteristics of the uncertain Markov matrix and measurement noise, while in Wiener filter, we need to know these stochastic characteristics. It is required to pay attention for the calculation of $E[\mathbf{h}_k\mathbf{h}_k^T]$ because $\mathbf{h}_k$ is an interval vector defined as: $\mathbf{h}_k = \mathbf{h}_k^o + \Delta\mathbf{h}_k$ or $\mathbf{h}_k = [\underline{\mathbf{h}_k}, \overline{\mathbf{h}_k}]$. Here, simply by assuming the uniformly distributed interval uncertainty, we calculate the expectation value such as: $E[\mathbf{h}_k] = \mathbf{h}_k^o$. Thus, $\mathbf{R}$ is simply calculated as:

$$\mathbf{R} = E[\mathbf{h}_k\mathbf{h}_k^T] = \begin{bmatrix} h_1^o h_1^o & h_1^o h_2^o & \cdots & h_1^o h_N^o \\ h_2^o h_1^o & h_2^o h_2^o & \cdots & h_2^o h_N^o \\ \vdots & \vdots & \ddots & \vdots \\ h_N^o h_1^o & h_N^o h_2^o & \cdots & h_N^o h_N^o \end{bmatrix} \quad (21)$$

Remark 4.1: In Wiener filter, for the calculation of $\mathcal{G}$ we need to find the inverse of $\mathbf{R}$. But, as shown in (21), rank$(\mathbf{R}) = 1$, so the inverse of $\mathbf{R}$ does not exist. To overcome this singularity, we give a bit of interval uncertainty to each element of $\mathbf{R}$.

Remark 4.2: In the parameter estimation, it is assumed that $U_k(1) \neq 0$ in order to avoid the singularity of $\mathbf{U}_k$. So, at initial iteration, the first discrete time control input is enforced as $U_1(1) \neq 0$ while $U_1(i) = 0$ with $i = 2, 3, \cdots, N$.

V. EXAMPLES

The following discrete system is used, which was given in [5]:

$$x_{k+1} = \begin{bmatrix} -0.50 & 0.00 & 0.00 \\ 1.00 & 1.24 & -0.87 \\ 0.00 & 0.87 & 0.00 \end{bmatrix} x_k + \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} u_k \quad (22)$$

$$y_k = [\,2.0 \quad 2.6 \quad -2.8\,]\, x_k, \quad (23)$$

with poles at $[\,0.62 + j0.62, 0.62 - j0.62, -0.50\,]$ and zeros at $[\,0.65, -0.71\,]$. The following signal is used as a desired trajectory: $Y_d(j) = \sin(8.0(j-1)/N)$ where $N = 10$. To avoid the singularity, the initial control input at the first iteration is given as 1, i.e., $U_1(1) = 1$, and $U_1(i) = 0$, $i = 2, 3, \cdots, N$ and the measured noise covariance is modelled as $\sigma_n^2 = 0.001$. From (22) and (23), the nominal Markov parameters are calculated as: $\mathbf{h}^o = [2.0000, 1.6092, -0.0235, -1.0295, -1.3839, -0.8676, -0.0421, 0.6322, 0.8118, 0.5263]^T$. For the interval model uncertainty, it was simulated that 10 percent interval uncertainty can arise at each iteration. For the interval uncertainty, Matlab command $rand$ was used and for the measured noise, Matlab command $randn$ was used. Fig. 2 shows the test result. In both upper and lower figures, $\Delta^*\|Z\|$ is calculated from Theorem 3.3. The bottom figure is $\|E_k\|_\infty$ and the upper figure is $\|E_k\|_2$. Even though Theorem 3.3 guarantees the base-line error in ∞ norm topology, as shown in the upper figure, $\|E_k\|_2$ is clearly well bounded by the previously calculated the base-line error $\Delta^*\|Z\|$. From this figure, we conclude that the suggested learning gain matrices and parameter estimation method guarantee the base-line error below the previously calculated value. Further, until the base-line error is reduced below the calculated base-line boundary, the tracking error is converging monotonically.

VI. CONCLUSION REMARKS

In this paper, monotonic convergent ILC system was designed with considering iteration varying model uncertainty. Using the suggested method, we calculated the base-line error boundary and tracking error has been reduced below this base-line. Further, the monotonic convergency property of ILC system has been enforced until it arrives the base-line error boundary. Thus, the main contribution of our work is that, for the first time, the monotone convergent ILC system was designed with iteration varying model uncertainty and the tracking error has been successfully reduced below the calculated base-line boundary. Thus, our final conclusion is that although there exists iteration varying model uncertainty, ILC approach suggested in this paper provides a satisfactory transient performance.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] K. L. Moore, *Iterative learning control for deterministic systems*, ser. Advances in Industrial Control. Springer-Verlag, 1993.

[3] Y. Chen and C. Wen, *Iterative Learning Control: Convergence, Robustness and Applications*, ser. Lecture Notes series on Control and Information Science. London: Springer-Verlag, 1999, vol. LNCIS-248.

[4] K. L. Moore, Y. Chen, and V. Bahl, "Monotonically convergent iterative learning control for linear discrete-time systems," *To appear in Automatica*, 2005.

[5] H.-S. Ahn, K. L. Moore, and Y. Chen, "Schur stability radius bounds for robust iterative learning controller design," in *accepted to 2005 American Control Conference*, Portland, Oregon, 2005.

[6] ——, "Stability analysis of iterative learning control system with interval uncertainty," in *accepted to the 15th IFAC World Congress*, 2005.

[7] ——, "Monotonic convergent iterative learning controller design based on interval model conversion," in *accepted to the IEEE International Symposium on Intelligent Control*, 2005.

[8] Y. Q. Chen and K. L. Moore, "Iterative learning control with iteration-domain adaptive feedforward compensation," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii USA, Dec. 2003, pp. 4416 – 4421.

[9] K. L. Moore, Y. Chen, and H.-S. Ahn, "Algebraic H_∞ design of higher-order iterative learning controllers," in *accepted to the IEEE International Symposium on Intelligent Control*, 2005.

[10] H.-S. Ahn, Y. Chen, and K. L. Moore, "Stochastic iterative learning control along the iteration axis," in *submitted to the 2005 IEEE CDC/ECC*, 2005.

[11] K. L. Moore, "Multi-loop control approach to designing iterative learning controllers," in *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, Dec. 1998, pp. 666 – 671.

[12] H.-S. Lee and Z. Bien, "A note on convergence property of iterative learning controller with respect to sup norm," *Automatica*, vol. 33, no. 8, pp. 1591–1593, 1997.

[13] K. L. Moore, "An iterative learning control algorithm for systems with measurement noise," in *Proceedings of the 38th IEEE Conference on Decision and Control*, Phoenix, AZ USA, Dec. 1999, pp. 270 – 275.

[14] K. L. Moore and Y. Chen, "On monotonic convergence of high order iterative learning update laws," in *Invited Session on* High-order Iterative Learning Control *at The 15-th IFAC Congress*. Barcelona, Spain: IFAC, July 21-26 2002.

[15] Y. Q. Chen and K. L. Moore, "An optimal design of PD-type iterative learning control with monotonic convergence," in *Proceedings of the 2002 IEEE International Symposium on Intelligent Control*, Vancouver, Canada, Oct. 2002, pp. 55 – 60.

[16] ——, "On D^α type iterative learning control," in *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, FL USA, Dec. 2001, pp. 4451 – 4456.

[17] ——, "PI-type iterative learning control revisited," in *Proceedings of the 2002 American Control Conference*, May 2002, pp. 2138 – 2143.

[18] H.-S. Ahn, Y. Chen, and K. L. Moore, "On the impulse response of interval uncertain system," in *submitted to the 2005 IEEE CDC/ECC*, 2005.

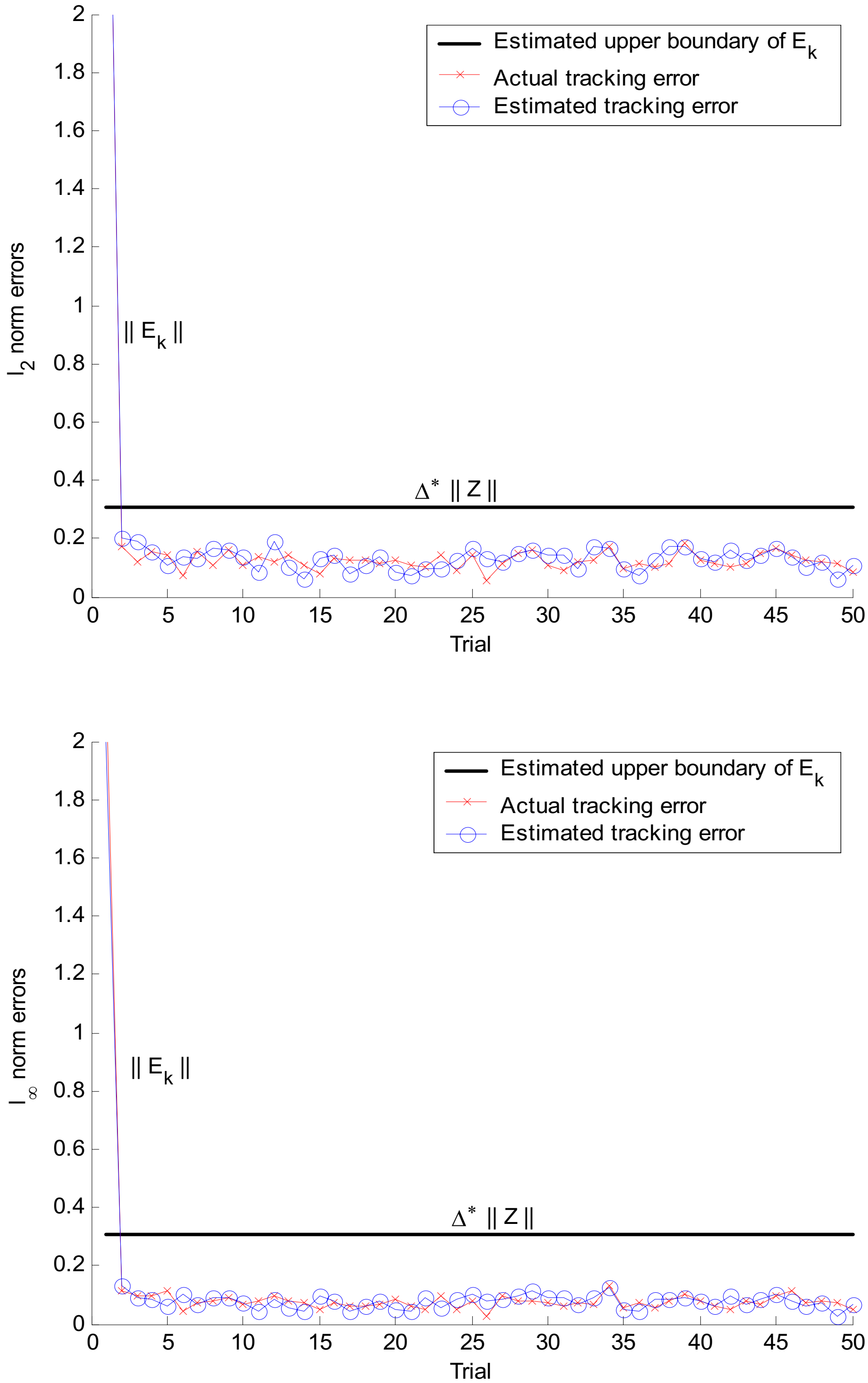

Fig. 2. Test results.

[19] J. A. Rogas and J. M. Collado, "Stability of interval matrices using the distance to the set of unstable matrices," in *Proceedings of the 1994 American Control Conference*, Baltimore, MD, 1994.