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Automatic Dynamic Flocking in Mobile Actuator Sensor Networks by Central Voronoi Tessellations

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***Abstract*— In this paper, we discuss the application of Voronoi diagram and central Voronoi tessellations in the distributed control of mobile robots for area coverage, dynamic target tracking and feedback control of the environment. Based on [1], we extend the application of central Voronoi tessellations (CVT) to a dynamic changing environment, provided that the environment changes relatively slow. Then we describe how automatic flocking behavior of mobile robots can be achieved by using CVT in area coverage control and multiple target tracking. We also discuss the hierarchical structure of CVT-based mobile actuator/sensor network where formation control can be applied for the subgroup of robots within a Voronoi cell. Simulation results show the correctness of our proposed method.**

***Index Terms*— Voronoi diagram, central Voronoi tessellations, coverage control, flocking behavior, formation control, diffusion process.**

I. Introduction

With the advancement of large scale digital circuit design and power efficient wireless communication, wireless sensor network has gained intensive research interest in the recent years [2], [3], [4]. Despite of the vast research efforts, wireless sensor network has its intrinsic disadvantages. First, it cannot apply active control on the environment. Second, it cannot do anything with the hardware faults on the sensor nodes. By introducing intelligent mobile actuators into the wireless sensor network, we can control the environment in an active way. The robustness of the network can also be improved and the life time extended by the replacement/maintenance service that the mobile actuators can provide. Mobile actuator/sensor network combines mobile robots with the wireless sensor networks [5], [6]. Each robot may have limited sensing ability and limited communication ability. They can coordinate with each other to fulfill tasks by temporal-spatial feedback closed-loop controls.

In this paper, we consider the application of mobile actuator/sensor network and discuss how Voronoi diagram and central Voronoi tessellations (CVT) can produce geometric constraints on the mobile robots so that the mobile robots can exhibit desired behaviors automatically in a distributed way. This research is inspired by the aggregating behavior of social insects and animals, e.g., birds flocking, fish schooling and ant foraging for food. Much research has been done for this kind of swarm behaviors [7], [8]. Voronoi diagram and CVT may give some clues in this research area. Many species of animals employ Voronoi tessellations to stake out their territory [9]. In this paper, CVT will be used as a basic tool to achieve automatic flocking behavior of mobile robots.

By introducing mobile robots into the wireless senor network, we can get a hierarchical network structure which is composed of 1) a sparsely distributed, mobile robots which can take intensive computations and 2) a densely populated low-cost static sensors whose function is to collect environment information and send data to the mobile robots. This heterogeneous mobile network can have extra advantages over the traditional wireless sensor network, for example, it can expand the network sensing capability, provide flexibility and robustness of the network, and even provide maintenance for the sensor node. Different from [10], where mobile robots only get service from the sensor network, here the mobile robots are part of the whole network and they work with distributed sensors to fulfill a common mission.

One of the main applications of wireless senor networks can be in environment monitoring. In [1], central Voronoi tessellations are used to determine the optimal position of mobile sensors for active sensing and coverage control of the environment. However in [1], only static environment is considered and the distribution density function that characterizes the concerned area is assumed *a priori*, which is not realistic for many applications. Another closely related research field is the dynamic target tracking in wireless sensor networks [11], [12], [13]. Although sensors can be dynamically clustered for target tracking, the network does not employ any mobility platform. While in [14] mobile robots are used to estimate the state of a target, the target is generally assumed to be static.

In [14], under the assumption that each mobile robots can only observe one target at a time, an interesting problem is how to decide the task assignment for each mobile robot in a decentralized way when there are multiple targets present at the same time. A solution is given in [14] by optimizing of a global cost function on the measurements. In this paper, we introduce how geometric constraints introduced by CVT can lead to the automatic grouping behavior of mobile robots for multiple target tracking and how task assignment problem can be solved with only local information.

As 2-D Voronoi diagram can be extended to high-order Voronoi diagram, we can extend the hierarchy architecture of the mobile actuator/sensor network so that in each Voronoi cell, there can be a group of mobile robots with a leader. While the position of the leader robot is decided by the mass central of CVT, the following robots will keep a rigid formation by using a distributed formation control. Challenges and opportunities of this extended hierarchy architecture will be discussed in this paper.

The remaining part of this paper is organized as follows: In Sec. II, we give a brief introduction of CVT-based mobile actuator/sensor network. Section III is devoted to presenting our proposed algorithm for mobile robot path planning in a dynamic environment by using CVT. Then, in Sec. IV, we describe how automatic flocking behavior can be achieved and how task assignment problem can be solved in a multiple targets tracking scenario by using CVT. In Sec. V, we discuss the extension of hierarchical wireless mobile actuator/sensor networks and introduce the the combination of wireless actuator/sensor network with robot formation control. To show the effectiveness of our proposed method, two simulation results are presented in Sec. VI. Finally, conclusions and future research directions are presented in Sec. VII.

II. CVT-based 2-level hierarchical heterogeneous mobile actuator/sensor network

The CVT based 2-level hierarchical heterogeneous mobile actuators/sensor network is based on the concept of Voronoi diagram and central Voronoi tessellations. Voronoi diagram and central Voronoi tessellations are two geometric approaches to partition a polygon. For details, please refer to [15].

The network is composed of two parts. The first part is a densely distributed sensor network and the second part is a group of mobile robots in the network. The sensor network gives the mesh measurement of the environment information, which is represented by a density or concentration function $\rho(x, y)$. The mobile robots may have more powerful communication, computation, and sensing

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capabilities. We assume that each robot knows its own position. Our proposed network structure is readily applicable for power-efficient network protocols [16], robust localization method [17] and variations of collaborative data processing algorithms [18]. Our proposed CVT-based 2-level network can be constructed in the following steps:

1) Sensors are densely deployed to the concerned area, for example, by air-born.
2) Mobile robots are sent to the concerned area and randomly distributed.
3) Robots broadcast their positions and ID's.
4) Each sensor estimates its position by some relative localization method.
5) Each robot communicates with its neighboring robots and access the neighboring sensors.
6) Each robot builds its Voronoi cell and tries to move to the mass central of its Voronoi cell.

Because the concentration function $\rho(x, y)$ may be time-varying, that is in the form of $\rho(x, y, t)$, the mobile robot will keep moving towards its mass central of current Voronoi cell while sensor provides measurements of $\rho(x, y, t)$ to its neighboring robots. A network constructed in this way can accommodate some practical applications, for example, environment surveillance, patrol and moving target tracking. Because of its distributed nature, only local information is needed for the motion control and decision making of each robot, which brings robustness and scalability for the network. Moreover, because the sensor only needs to sense the environment when required by mobile robots, power-efficient network protocol can be easily employed.

III. Path planning for the mobile robot in a dynamic environment by using central Voronoi tessellations

Centroidal Voronoi tessellations has many useful properties which provides a solution to the optimal placement of resources [15]. In this section, we apply CVT to the dynamic environment coverage and active sensing/control.

A. Problem formulation

Let Ω be a convex polytope in $\mathcal{R}^2$, including its interior. A time-varying distribution density function is a map $\rho(x, y, t) : \Omega, t \rightarrow \mathcal{R}_+$ that represents a measurement of environment information. This environment information can be some physical signal measurement or the probability that some events take place over Ω [1]. Assume static sensors are densely deployed within Ω and give mesh measurement of $\rho(x, y, t)$ over time t. A group of mobile robots are then sent to cover Ω with a specific task.

Here, we can have two different application scenarios. First, each mobile robot has more powerful sensing capabilities and $\rho(x, y, t)$ is the measurement information for some concerned physical variables. To achieve a better measurement performance, the robots should be close to where $\rho(x, y, t)$ is high. But they should be distributed and cover the whole area in such a way that they should also be responsible for the information comes from the other parts of the area. This is what "covering" means in the context of this paper. Second, $\rho(x, y, t)$ may give the probability that some events will happen and the robots should serve the events. Then, the robots should also be close to the points where $\rho(x, y, t)$ is high. But there is also a possibility that some events can happen where $\rho(x, y, t)$ is low. To be reactive to the events, it is more reasonable to put "covering" task on the robots. In both cases, robots should be distributed in such a way that they can "cover" the whole area while close to where $\rho(x, y, t)$ is high.

Let $P = (p_1, \cdots, p_n)$ be the location of n robots and let $\|\cdot\|$ the Euclidean distance function. Similar to [1], n robots will partition Ω into a collection of n polytopes $\mathcal{V} = \{V_1, \cdots, V_n\}$, $p_i \in V_i$, $V_i \cap V_j = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{n} \bar{V}_i = \bar{\Omega}$ ($\bar{V}_i = V_i \cup \partial V_i$ and $\bar{\Omega} = \Omega \cup \partial\Omega$). For each time $t = T$, We consider to minimize the following cost function:

$$\mathcal{K}(P, \mathcal{V}) = \sum_{i=1}^{n} \int_{V_i} \rho(q)|q - p_i|^2 dq \text{ for } q \in \Omega. \quad (1)$$

It is clear that to minimize $\mathcal{K}$, the distance $\|q - p_i\|$ should be small when $\rho(q, t)$ is large. It is the density function $\rho(q, t)$ that determines the optimal positions of the robots. A necessary condition for $\mathcal{K}$ to be minimized is that $\{p_i, V_i\}_{i=1}^{k}$ is a Centroidal Voronoi Tessellations of Ω at time $t = T$ [15].

B. Proposed algorithms

Although CVT is applied only in a static environment [1]. We claim that when the environment evolves slowly compared with the convergence rate of the adopted algorithm and the CVT updating period, CVT can still be a valid solution to the optimal coverage problem with active sensing and/or control. In [19], we have successfully applied CVT for the mobile robot motion planning in the feedback control of a diffusion process. In this paper, we extend the static coverage control by CVT to a dynamic environment with active sensing task. The proposed algorithm to compute the locations of robots by centroidal Voronoi tessellations in a dynamic environment will be given below. The proposed algorithm is based on a discrete version of (1) and the density information comes from the measurements of the static, low-cost mesh sensors.

Lloyd's method is a deterministic algorithm for determining the central Voronoi tessellations. For details of Lloyd's method, please refer to [15]. The Lloyd's method will be executed periodically so that the motion of the robots can be adaptive to the evolution of the environment. The dynamics of the environment should be slower than the updating rate of the CVT. Lloyd's method converges fast and requires few iterations, but it has higher computation requirements for each iteration. In many applications, the robot has only limited communication abilities. To avoid the communication collision and reduce the computation load, a distributed asynchronous algorithm to construct the Voronoi diagram based on the local information is clearly more desirable. We assume that the robot can communicate with the sensors and other robots within a radius R_i. R_i is an adjustable parameter. Here, we present a distributed algorithm from [1] with mild modification.

To construct the Voronoi diagram in the Lloyd's algorithm, each robot will do the following:

1) Assign its detection range R_i with a small initial value, detect all its neighboring robots with radius R_i.
2) Construct its own Voronoi cell $V_i$within the radius R_i.
3) For every sensor $q_i \in V_i$, compute $d_i = \max|q_i - p_i|$.
4) If $R_i > 2 \times d_i$, stop. Otherwise set $R_i = 2 \times R_i$, go to step 2.

R_i obtained at the preceding execution will be used as the initial values for the following executions. The initial value of R_i for the first execution can be obtained by the way below:

At the beginning of the procedure,
1: each robot i broadcasts its current position p_i.
2: each sensor finds its closest p_i.
3: each sensor sends its position and measurement
to its closest robot i. If
4: robot i knows which sensor in its VD.
5: robot i finds the farthest sensors j and its position s_j.
6: robot i set $R_i = \|R_i - S_j\|$.

for some time, for example, successive 3 updates, the R_i remains unchanged, then R_i can be decreased, $R_i = R_i - \Delta r$ for some $\Delta r > 0$. This improvement on the algorithm from [1] helps to reduce the computation load.

If the mobile robots are sparsely distributed in the concerned area, an approximation approach that can reduce the communication and computation load while not causing much loss of accuracy in constructing the Voronoi diagram is more desirable. One distributed approximation approach that uses Gabriel graph is given in [20].

The mobile robots are treated as virtual particles and obey the second-order dynamical equation:

$$\ddot{p}_i = F_i$$

where F_i is given by

$$F_i = f_i - k_v \dot{p}_i \tag{2}$$

with f_i the force input to control the motion of the robot and f_i given by a proportional control law:

$$f_i = -k(p_i - \bar{p_i})$$

where $\bar{p_i}$ is the computed mass centroid of the current Voronoi cell.

The second term of (2) on the right-hand side is the viscous friction artificially introduced. k_v is the friction coefficient and $\dot{p}_i$ denotes the velocity of the robot i.

IV. Grouping behavior in coverage control and dynamic targets tracking by using central Voronoi tessellations

In this section, we discuss the grouping behavior of mobile robots in target tracking under the geometric constraints introduced by central Voronoi tessellations. What underlines is the problem of distritbuted task assignment for each robot.

Target tracking is an important application for wireless sensor network. Usually, the sensor node will observe the states of the target and report them to the remote base-station. Power-efficient dynamic target tracking can be achieved by activating only the sensors that are close to the moving target [11]. But the scenarios where multiple mobile targets are present are seldom discussed.

In [14], mobile robots are used to observe the states of multiple targets. An interesting problem is how to assign the task for each robot in a distributed way so that mobile robots are automatically grouped with one group of robots tracking one moving target. In our proposed 2-level mobile actuator/sensor network, we use densely distributed sensors to estimate the signal strength of the moving targets and use the mobile robot(s) to track the moving targets for advanced state measurement. In our scenario, the task of target tracking is executed in a probabilistic way. It means the area where $\rho(x, y)$ is high only means that the probability of the target on that area is high. For example, in acoustic target tracking, the sensor measurement only gives information about how close the target can be, but not exactly what its position is. So, the robot should be distributed in such a way that it "covers" the whole area and should also be responsible to potential target comes out where $\rho(x, y)$ may be low. We call this *coverage and tracking* problem.

We can use CVT to determine the robot position in this *coverage and tracking* problem. In our proposed 2-level mobile actuator/sensor network, the densely distributed sensors give the mesh measurement of $\rho(x, y)$, which indicates the possibility of the target coming out there. The positions of the robots are the mass centers in their central Voronoi tessellations. The distribution of $\rho(x, y)$ produced by the motion of mobile targets will continuously change the desired positions of the mobile robots so that each robot tends to move close to where $\rho(x, y)$ is high, and at the same time, keep away from other robot. If multiple mobile targets are present, $\rho(x, y)$ may have several local maximum points, causing mobile robots automatically gathering together to different groups. Once formed, each group will track different target separately, if the moving targets are apart from each other far enough. What we can see is that without the need of consensus between individual robots, CVT gives a distributed solution for the task assignment on mobile robots for target tracking. By collaborative data processing, robots do not necessary go to the point where the density function $\rho(x, y)$ is maximum. More illustrations of our proposed method will be given in the simulation section.

Similar idea can be used to extend our previous work for pollutant diffusion control [19] to the case where multiple moving pollution sources present. In this case, we can show that mobile robots also exhibit automatic grouping behaviors. The results are presented in Sec. VI-B.

To achieve desired gathering behavior of the mobile robots, the sensor measurement $\rho(x, y)$ can be scaled by some class $\mathcal{K}$ function $\mathcal{SK}(\rho)$. In this way, more flexible robot behaviors can be achieved. The class $\mathcal{K}$ function $\mathcal{SK}(\rho)$ can be a design component which depends on specific applications which deserves further investigations.

V. Hierarchical structure and robot formation

Just as 2-D Voronoi diagram can be extended to the high-order Voronoi diagram, we can also extend the hierarchy architecture of the 2-level mobile actuator/sensor network in such a way that in each Voronoi cell, there is no longer a single robot, but a group of mobile robots. Each robot within a cell may have different capabilities which are necessary to fulfill a common mission. We can apply distributed formation on the group of mobile robots within a cell. What we propose here has real impact on the real-life applications. For example, the idea we employ here can be used in using intelligent mobile robots to simulate the deployments of military units in a battle field where each member of the units will cooperate with each other for a designed mission. We need to point out that we do not need to consider the collision between robots from different groups. Every time the robot group moves, it moves within its cell decided by the Voronoi diagram and the cells do not overlap, so there is no possibility that two robots from two different groups will run into each other.

Formation control of the group of mobile robots within a cell may introduce extra advantages into the network for some practical applications [21]. If the robots follow a virtual leader, the position of the virtual leader can be decided by the mass central of CVT. Robots can even change their formation corresponding to the space they can occupy or to the local information they can obtain.

VI. Simulation Examples

In this section, we show two simulation examples of CVT-based mobile actuator/sensor network for mobile targets tracking and diffusion feedback control of multiple moving pollution sources. For the robot motion control in the simulation, the viscous coefficient is given by $k_v = 1$ and the control input is given by

$$F_i = -3(p_i - \bar{p_i}) - \dot{p}_i.$$

where $\bar{p_i}$ is the mass central for the current Voronoi cell.

A. Acoustic mobile target tracking

Here, CVT-based mobile actuator/sensor network is used for the multiple acoustic mobile targets tracking. The area concerned is given by $\Omega = \{(x, y)|0 \le x \le 1, 0 \le y \le 1\}$. There are two mobile targets moving in Ω: one starts from $ps_1 = (0.23, 0.2)$ and moves parallel to y axis with the velocity $vs_1 = 0.12/s$; the other one starts from $ps_2 = (0.73, 0.8)$ and moves parallel to y axis with the velocity $vs_2 = -0.12/s$. 40×40 sensors are uniformly deployed in Ω to give mesh measurement of the acoustic signals. The strength of the acoustic signal received by the static sensor i is given by [11]:

$$as_i = a \times \|ps_1 - s_i\|^{-\alpha} + a \times \|ps_2 - s_i\|^{-\alpha}, \quad 1 \le i \le 1600. \tag{3}$$

where $a \in \mathcal{R}$ is a system parameter that indicates the strength of the signal at the source; s_i is the sensor position, and α is the attenuation coefficient.

We have 9 mobile robots that implement the task of coverage and tracking. The simulation time is chosen as $t = 5$ and the updating interval to compute CVT is given by $\Delta t = 0.01$ sec. At first, robots are uniformly distributed. Figure 1 shows the initial layout of the robots and the targets. When the robots receive the neighboring senor measurements, they partition Ω by CVT and begin to move to the mass central of their current Voronoi cell. The motion of the targets change the mesh measurement from the sensors and make the movement of the mobile robot adaptive to target motion.

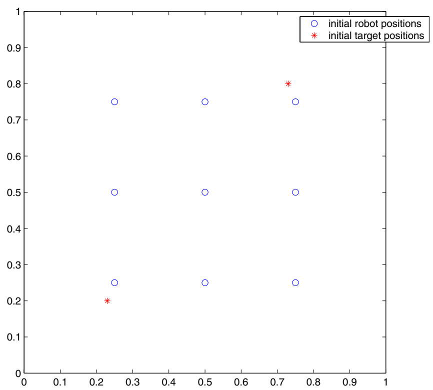
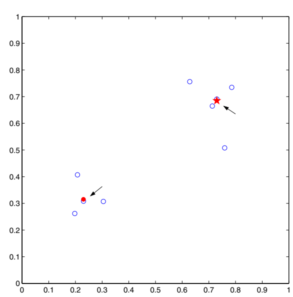
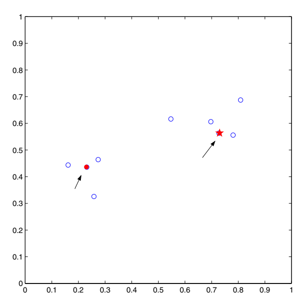


Fig. 1. Initial layout of actuators and targets.

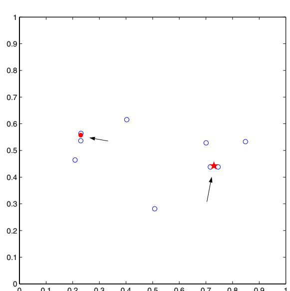
Figure 2 gives the snapshots of the motion of the targets and the robots at $t = 1.0s, 2.0s, 3.0s, 4.0s$, respectively. The arrow shows the positions of the targets. It can be seen that the mobile robots are automatically grouped to track different targets. The targets are closely followed. Depending on the variation of $\rho(x, y)$, some robot may even change their group memberships. The robot behavior actually resembles the behavior of ants, animals and even the human being. Figure 3 shows the evolution of the Voronoi diagram for each robot at the snapshots.



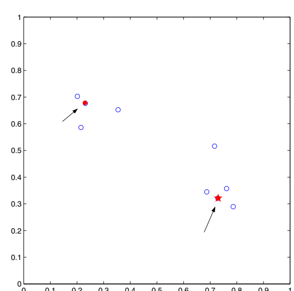
(a) Robots and targets position at $t = 1.0$ sec.



(b) Robots and targets position at $t = 2.0$ sec.



(c) Robots and targets position at $t = 3.0$ sec.



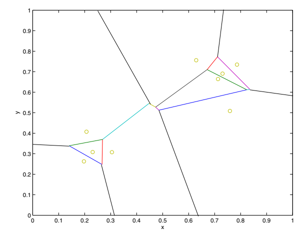
(d) Robots and targets position at $t = 4.0$ sec.

Fig. 2. Snapshots of robot and target motion

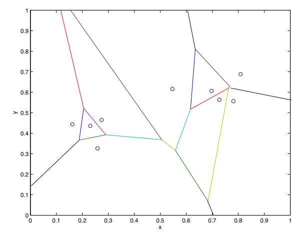
Figure 4 shows the trajectories of the robots and the moving targets. The bold line shows the target trajectory and the arrow shows the movement direction of its near line.

B. *Feedback control of pollution diffusion*

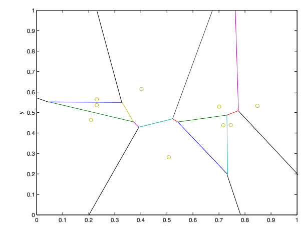
Here we extend our previous work in [19] and try to use CVT for diffusion feedback control of multiple moving pollution sources. The diffusion process is modelled as a parabolic PDE system. The



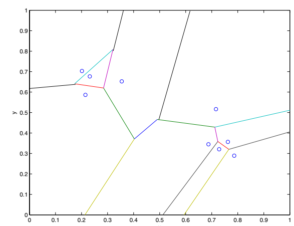
(a) Voronoi diagram at $t = 1.0$ sec.



(b) Voronoi diagram at $t = 2.0$ sec.



(c) Voronoi diagram at $t = 3.0$ sec.



(d) Voronoi diagram at $t = 4.0$ sec.

Fig. 3. Evolution of Voronoi diagram in tracking

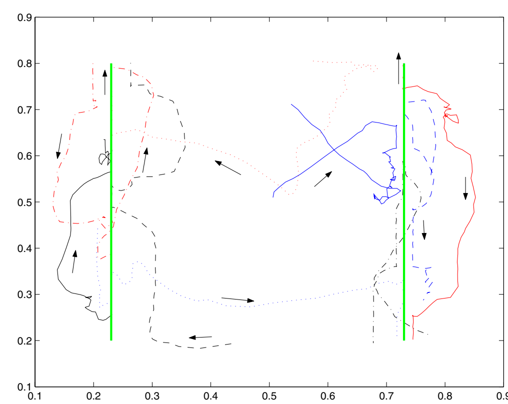


Fig. 4. Trajectories of robots and targets

mobile robots are equipped with controllable dispensers that can release chemicals to neutralize the pollution. The amount released is proportional to the pollution measurements. `Diff-MAS2D`[22] is used as the simulation platform for our implementation. The area concerned is given by $\Omega = \{(x, y)|0 \leq x \leq 1, 0 \leq y \leq 1\}$. For details of the system model and the simulation platform, please refer to [19].

In our scenario, there are two moving pollution sources. The pollution sources are modelled as a point disturbance to the PDE system with the pollution at the source given by

$$f_d(t) = 20e^{-t}.$$

One source starts from $(x = 0.21, y = 0.79)$ and moves parallel to x-axis with the velocity $vp_1 = 0.06/s$; the other one starts from $(x = 0.8, y = 0.21)$ and moves parallel to x axis with the velocity $vp_2 = -0.06/s$. In our simulation, we assume that once deployed, the sensors remain static. There are 29×29 sensors evenly distributed in a square area $(0, 1)^2$ and they form a mesh over the area. There are 4 robots that can release the neutralizing chemicals. The pollution source begins to diffuse at $t = 0$ to the area Ω, 4 robots are deployed with initial positions at $(0.33, 0.33), (0.33, 0.66), (0.66, 0.33), (0.66, 0.66)$, respectively.

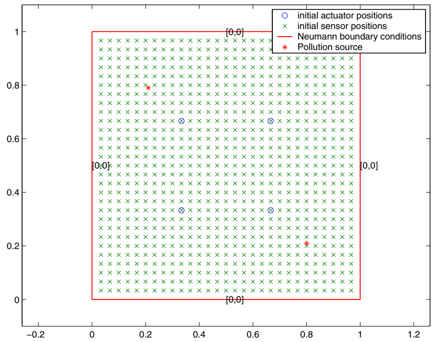


Fig. 5. Evolution of the diffusion process at $t = 8.3$ sec.

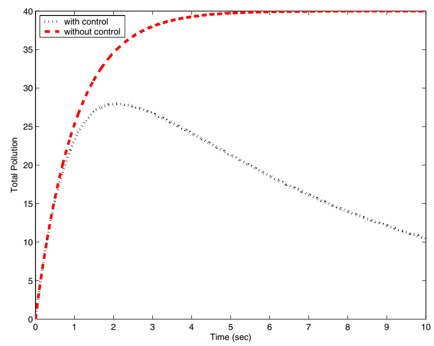


Fig. 6. Evolution of the amount of pollutants (with and without control)

Figure 5 shows the initial positions of the robots, the positions of the sensors and the positions of the pollution sources.

We choose the simulation time as $t = 10$ sec. and the time step is chosen as $\Delta t = 0.002$ sec. The robot recomputes its desired position every 0.2 sec. The system evolves under the effects of diffusion of pollutants and diffusion of neutralizing chemicals released by robots. In Fig. 6, the y-axis is the sum of the sensor measurements. It shows that the amount of pollutants decreases to 36% of its peak value at the end of the simulation. And the the decreasing process is monotonic. The evolution of the amount of pollutants without control is also shown in Fig. 6 for comparison.

Figure 7 shows the trajectories of the robots for $t \leq 10$ sec. It can be seen that the robots move towards the pollution source to suppress the diffusion of the source. And they also move around the source to track the pollution that has already diffused and try to back-chase and neutralize it.

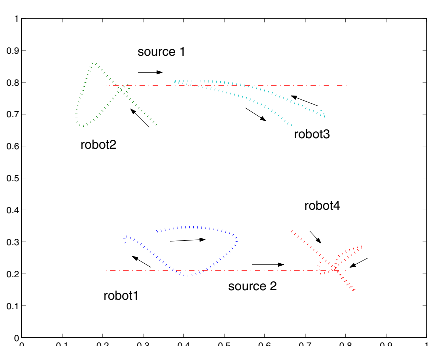


Fig. 7. Robots trajectories

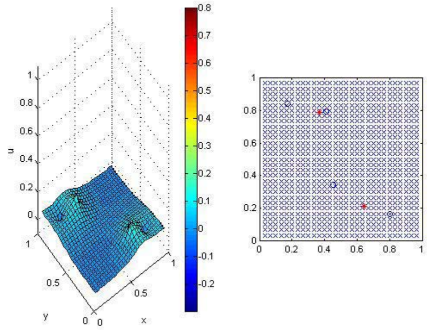


Fig. 8. Evolution of the diffusion process at $t = 2.6$ sec.

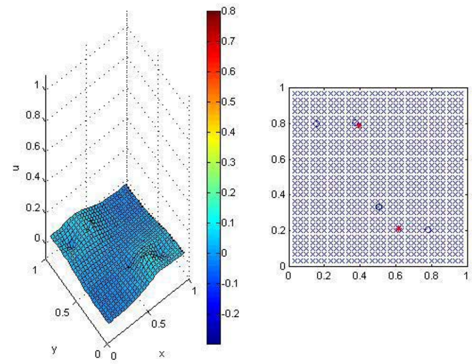


Fig. 9. Evolution of the diffusion process at $t = 3.1$ sec.

Figures 8 to 11 show the evolution of the diffusion process at $t = 2.6, 3.1, 3.5, 8.3$ sec., respectively. The left part of the figure is a 3D plot of $\rho(x, y, t)$ and the right part of the figure shows the current position of the robots. We can see that the mobile robots form two groups with two robot in each group to trace the diffusion source. In each group, one robot is close to the pollution source and try to suppress the peak at the source while the other is "lagged" behind to neutralize the pollution that is already diffused. This is because the diffusion introduces a "delay" effect in the robot motion. When the pollution becomes evenly distributed, so the robots.

Figure 12 shows the evolution of Voronoi diagrams at those time instants.

VII. Conclusion

In this paper, Voronoi diagram and central Voronoi tessellations are used for the the distributed control of mobile robots so that the robots can exhibit automatic grouping behaviors for some specific applications. The robot reacts to the dynamic environment depending only on the local information. Our control strategy is distributed, robust and scalable. In the future, we will consider the spatial sampling and time sampling issues in the network and the systematic design of the class $\mathcal{K}$ function $\mathcal{SK}(\rho)$ to control the grouping or flocking behavior of the mobile robots. We will also consider how to design the formation controller for the subgroup of mobile robots in extended CVT-based mobile actuator/sensor network.

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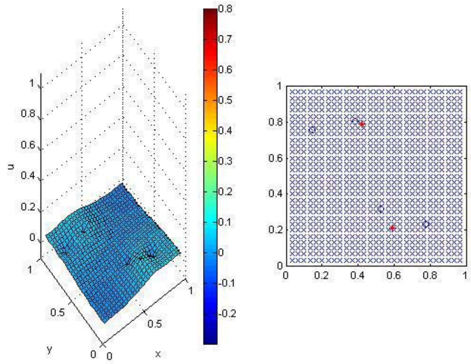


Fig. 10. Evolution of the diffusion process at $t = 3.5$ sec.

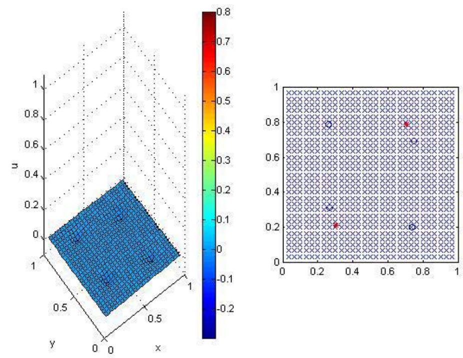
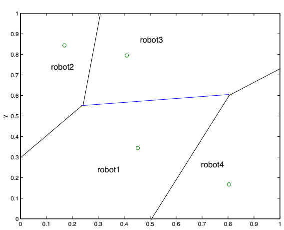
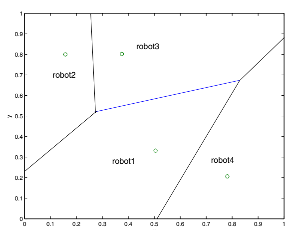


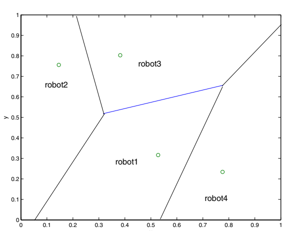
Fig. 11. Evolution of the diffusion process at $t = 8.3$ sec.



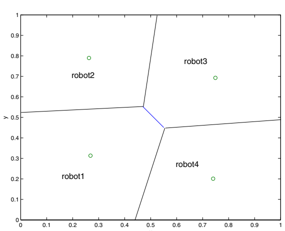
(a) Voronoi diagram at $t = 2.6$ sec.



(b) Voronoi diagram at $t = 3.1$ sec.



(c) Voronoi diagram at $t = 3.5$ sec.



(d) Voronoi diagram at $t = 8.3$ sec.

Fig. 12. Evolution of Voronoi diagrams for feedback control of the diffusion

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