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Iterative Learning Control for Cross-Coupled Contour Motion Systems

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Abstract— **Iterative Learning control (ILC) is a powerful control concept that iteratively improves the behavior of processes that are repetitive in nature and Cross-Coupled Control (CCC) has been developed to effectively reduce the contouring error. This paper proposes an integrated controller which combines ILC and CCC to achieve a better performance. In the presented preliminary simulation study, smaller contour error and tracking error can be achieved and moreover, contouring error is less oscillatory because CCC suppresses the inter-axis mismatch "cross error".**

Keywords: Iterative learning control; cross-coupled control; contouring error; motion control systems.

I. INTRODUCTION

In multiple-axis motion applications, conventional feedback controllers are usually designed independently for each axis to reduce the tracing error. In addition to feedback loop, many feed-forward controllers have been studied to greatly improve the tracking accuracy. For example, zero phase error tracking control (ZPETC) can reduce tracking error effectively.

In many contour control system like the computerized numerical control (CNC) machine, contouring accuracy is very crucial. In addition to the control design in a feedback loop [1], cross-coupled control (CCC) proposed by Koren [2] were developed to further reduce contouring error and substantially improve the contouring accuracy of multi-axis systems. Hsu and Houng [3] intuitively integrated a well-tuned CCC and a ZPETC, which reduced both the contouring error and the tracking error. Syh-Shiuh Yeh and Pau-lo Hsu [4] developed the theory greatly and proposed a contouring error transfer function (CETF) that is equivalent to the sensitivity function in an SISO control system.

In recent years, much attention has been devoted to the design of iterative learning controllers [e.g., 5, 6, 7, 8] that progressively improve the performance in repeatedly attempting to track pre-specified trajectories. It is a recursive online control method that relies on less calculation and requires less knowledge about the system dynamics.

The major contributions of this paper are as follows:

- In contouring motion control systems such as CNC machine tools, the CCC (cross-coupled control) combined with ILC (iterative learning control) can greatly reduce the contour error and tracking error.
- Contour error has less oscillation can be reduced smoothly because CCC suppresses the axis mismatch "cross error".
- ILC learning gain matrix can be designed for a diagonal matrix although CCC system is time-varying.

The rest of this paper is organized as follows. In Sec. 2, some notations and preliminaries are given. In Sec. 3, some preliminaries on ILC, CCC and integrated ILC+CCC system are discussed. A simulation example is also presented in Sec. 3 for illustration. Finally, conclusions are presented in Sec. 4 with several remarks on further investigation.

II. NOTATIONS AND PRELIMINARIES ON ILC-CCC

A. A General Formulation of ILC Problems

Let a trial, denoted by the subscript k, be a single operation of the system and let time during a given trial be denoted by $t \in [0,T]$. The system has input $U_k(t) \in R^p$ and output $Y_k(t) \in R^p$. $Y_d(t) \in R^p$ is a desired output. Also we assume the standard ILC resetting condition: $Y_k(0)= Y_d(0)=0$ for all k. The ILC's goal is to derive an input $U^*(t)$ to make the error $E_k(t)=Y_d(t)-Y_k(t)$ as small as possible. This is accomplished by adjusting the input from the current trial $U_k(t)$ to a new input $U_{k+1}(t)$ for the next trial according to an appropriate algorithm.

The learning control algorithm, proposed by Arimoto, *et al*, had the form

$$U_{k+1}(t) = U_k(t) + \Gamma \dot{E}_k(t) \qquad (1)$$

where $\dot{E}_k(t)$ is the derivative of the tracking error $E_k(t) = Y_d(t) - Y_k(t)$. Γ is the learning gain matrix, t is the time variable and k is the trial index. For a linear system, with a state-space description (A,B,C), if CB is not zero and the induced operator norm $\|I - CB\Gamma\|_i < 1$, it was proved that the time derivative of the output converges to the time derivative of the desired trajectory of the system. However, this learning control suffers from a few drawbacks, such as non-causal operator (i.e. the differentiator) and only velocity convergence guaranteed.

In this paper, we consider a 2nd-order iterative learning control of the form

$$U_{K+1}(t) = P_1 u_k(t) + P_2 u_{k-1}(t) + Q_1 e_k(t) + Q_2 e_{k-1}(t) \cdot \quad (2)$$

It is observed that, for any fixed t, the above updating algorithm is a second-order difference equation in iterative number k. For a linear system, with a state-space description (A,B,C,D), let $p \times p$ matrices P_i, i=1, 2, and $p \times p$ matrices Q_i, i=1, 2, be given so that the following two conditions are satisfied [9]:

$$\begin{aligned} &P_1 + P_2 = I \\ &\|P_1 - Q_1 D\|_\infty + \|P_2 - Q_2 D\|_\infty < 1. \end{aligned} \quad (3)$$

Then, for a given desired output $Y_d(t)$, the iterative control law guarantees that, for each $t \in [0,T]$, $Y_k(t) \rightarrow Y_d(k)$ as $k \rightarrow \infty$. For a linear system with a state-space description (A,B,C), there is no D term, we can modify the output equation

$$\hat{Y}(t) = \dot{Y}(t) = CAX + CBU(t) \quad \cdot \quad (4)$$

Let $\hat{Y}_d(t) = \dot{Y}_d(t)$, $\hat{D} = CB$. It can be seen that what steers the system output $\hat{Y}(t)$ to the desired output $\hat{Y}_d(t)$ also steers the output $Y(t)$ to the desired output $Y_d(t)$.

An extended form can be applied for nonlinear dynamical systems

$$\begin{aligned} &\dot{X}(t) = F(X,t) + B(t)U(t) \\ &Y(t) = G(X,t) + D(t)U(t). \end{aligned} \quad (5)$$

Assume that the functions $F(X,t)$, $G(X,t)$, $B(t)$ and $D(t)$ are periodic in t with a period T, and for each $t \in [0,T]$, the nonlinear functions $F(X,t)$ and $G(X,t)$ satisfy a Lipschitz continuity condition

$$\begin{aligned} &\|F(X_1,t) - F(X_2,t)\| \le f_0 \|X_1 - X_2\|_\infty \\ &\|G(X_1,t) - G(X_2,t)\| \le g_0 \|X_1 - X_2\|_\infty \end{aligned} \quad (6)$$

for any pair (X_1,X_2) in $R^n \times R^n$ while the function $B(.)$ and $D(.)$ are bounded on $[0,T]$.

Suppose we apply an N-th order updating scheme in iteration axis

$$U_{k+1}(t) = P_1 U_k(t) + \ldots\ldots + P_N U_{k-N+1}(t) + Q_1 E_k(t) + \ldots\ldots + Q_N E_{k-N+1}(t) \quad (7)$$

where P_i and Q_i, i=1, 2, …., N, satisfy

$$\sum_{i=1}^{N} P_i = I \quad (8)$$

and the polynomial $P_l(z) = z^N - l_1 z^{N-1} - \ldots\ldots - l_N$ has all its roots inside the unit circle, where $l_i = \sup_{t \in [0,T]} \|P_i - Q_i D(t)\|_\infty$, then $Y_k(t) \rightarrow Y_d(t)$ as $k \rightarrow \infty$.

Similar to (4), if $D(t)$=0, the output equation can be modified and $\hat{D}(t) = G_x(t)B(t)$, where G_x is differentiable in x and globally Lipchitzian in x on $[0,T]$.

B. Cross-Coupled Control for Motion System

In multi-axis motion system, feedback controllers are usually designed independently for each axis and each axial servomechanism tracks input commands to reduce the tracking error. In addition to tracking accuracy, contouring accuracy is more important for any precise multi-axis motion system. Basically, by applying the position error adjustment to each axis, the CCC substantially improves the contouring accuracy. Note that although the CCC has been verified to reduce contouring error [4], it cannot effectively reduce the tracking error. Therefore, all CCC controllers are based on the position feedback loops which usually apply P or PD regulator.

The typical CCC [1, 3, 4] schemes are presented in Fig.1. Note that in CCC, the cross-coupled gain Cx=sinθ、Cy=cosθ and θ is the angle between the movement direction and X-axis. The gain in CCC varies according to different contours. The important relationship for the contouring error with and without CCC is obtained as [3]

$$\varepsilon_c = \frac{1}{1+CK}\varepsilon_o = H\varepsilon_o \quad (9)$$

where C is the designed CCC; H=1/(1+CK) the contouring error transfer function(CEFT);

$$\begin{aligned} K &= \frac{(1+G_1)c_y c_y G_2 + (1+G_2)c_x c_x G_1}{(1+G_1)(1+G_2)} \\ &= C_y^2 \frac{G_2}{1+G_2} + C_x^2 \frac{G_1}{1+G_1} \end{aligned} \quad (10)$$

It can be obviously observed from (9) that CETF can be treated as a sensitivity function of an SISO feedback control system. The design of controller C depends on each axis position feedback loop from (10) as Cx and Cy vary. Although the CCC controller can be designed using suitable algorithms, for example the quantitative feedback theory (QFT), to stabilize the equivalent system and reduce the contouring error, its design procedures are relatively complicated and small enough contouring error is hard to achieve.

C. An Integrated Control System Combining ILC and CCC

Integrated ILC-CCC based on the typical CCC system structure is proposed in Fig. 2, where CCC controller C is treated as a constant gain for simplicity in the discussion below. For ILC systems, the input signals are $ilcx$ and $ilcy$, and the output signals are xo and yo. The desired trajectories are $x_d(t)=x_r(t)$, and $y_d(t)=y_r(t)$. Here, x_r and y_r in Fig.2 can be considered as disturbances that are invariant from trial to trial.

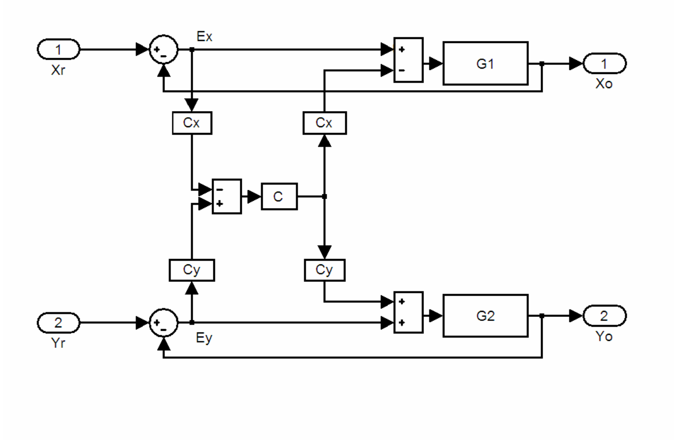

Fig. 1 A typical CCC system scheme

Independent position controller for each axis is selected to be a PD regulator [1]. Then, the plants G_1 and G_2 can be simplified as follows:

$$G_1 = \frac{a_1 s + a_2}{s^2 + b_1 s} \qquad G_2 = \frac{c_1 s + c_2}{s^2 + d_1 s}.$$

Then, we can get the following relationships:

$$\left[1 + (1 + cc_x^2)G_1\right]x_o = G_1 ilcx + cc_x c_y G_1 y_o,$$

$$\left[1 + (1 + cc_y^2)G_2\right]y_o = G_2 ilcy + cc_x c_y G_2 x_o.$$

The corresponding state space representation is given by

$$\dot{X} = AX + BU \qquad Y = CX$$

where $Y = \begin{bmatrix} x_o \\ y_o \end{bmatrix}$ and $U = \begin{bmatrix} ilcx \\ ilcy \end{bmatrix}$

We can obtain $CB = \begin{bmatrix} a_1 & 0 \\ 0 & c1 \end{bmatrix}$. Note that CB is not time-dependent.

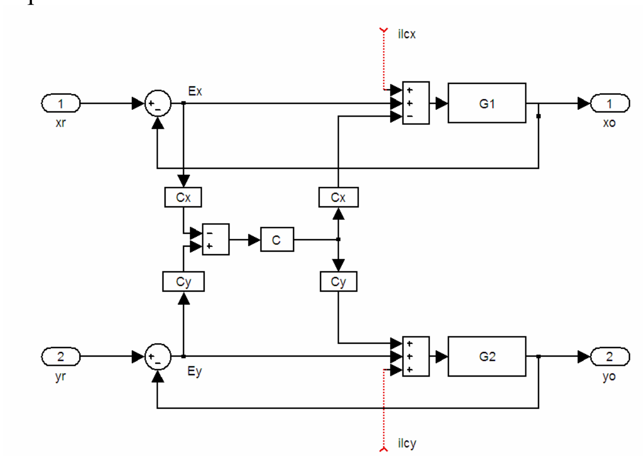

Fig.2 ILC-CCC integrated control system scheme

III. DEMONSTRATIVE SIMULATION

Applying the ILC algorithm with P_2=0 and Q_2=0, the proposed learning update law (2) is essentially a proportional-type ILC. We simply apply

$$Q_1 = D^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix}.$$

For a two axis typical servo system *X, Y* with PD position controllers,

$$P_1(s) = \frac{a_1 s + a_2}{s^2 + b_1 s} = \frac{2s + 11.76}{s^2 + 12s},$$

$$P_2(s) = \frac{c_1 s + c_2}{s^2 + d_1 s} = \frac{1.5s + 3}{s^2 + 5s}.$$

Consider a CCC controller with C=20. The test motion contour is a line with $\pi/4$ angle along *X* axis and the contouring error $\varepsilon_c = E_y \cos\theta - E_x \sin\theta = E_y C_y - E_x C_x$.

When the CCC controller is ineffective and *X* and *Y* have iterative learning operation, we can obtain contour error $E_c(t)$ and *X, Y* axis tracking errors $E_x(t)$, $E_y(t)$ for the *k*-th iteration shown in Fig.3.

From Fig. 3, we can observe that the contour error and *X, Y* axis tracking errors are relatively large when CCC is not applied, but decrease rapidly due to iterative learning control. An important result is that the contour error is not small enough although for each axis the tracking error decreases rapidly with iterative learning. This indicates that decreasing tracking error to some extent can not make the contour error small enough. Another result we can see from Fig. 3 is that contour error may oscillate due to the two axis dynamic performance mismatch under the condition that there is no CCC controller.

From Fig. 4, we have ILC system with an effective CCC applied. Compared with ILC without CCC, smaller contour error and tracking error can be clearly observed although it seems that similar decreasing trend is achieved due to ILC. At the meantime, contour error curve has no oscillation because CCC suppresses the system "*cross coupling error*" due to axis dynamic mismatch.

IV. CONCLUSIONS

In this paper, we have presented a preliminary study of an integrated contour motion control system combining the iterative learning controller (ILC) and cross-coupled controller (CCC). CCC may decrease contour error and tracking error significantly. Feed forward type ILC makes the tracking errors tend to zero and suppresses the system "cross error" due to the dynamic mismatch between axes.

In our further research efforts, the high order ILC (in time and in iteration) will be designed for further performance improvement. Moreover, some cross coupled influences between the plants will be further investigated.

We remark that iterative learning controller can be used to improve the performance in cross-coupled multi-axis servo control.

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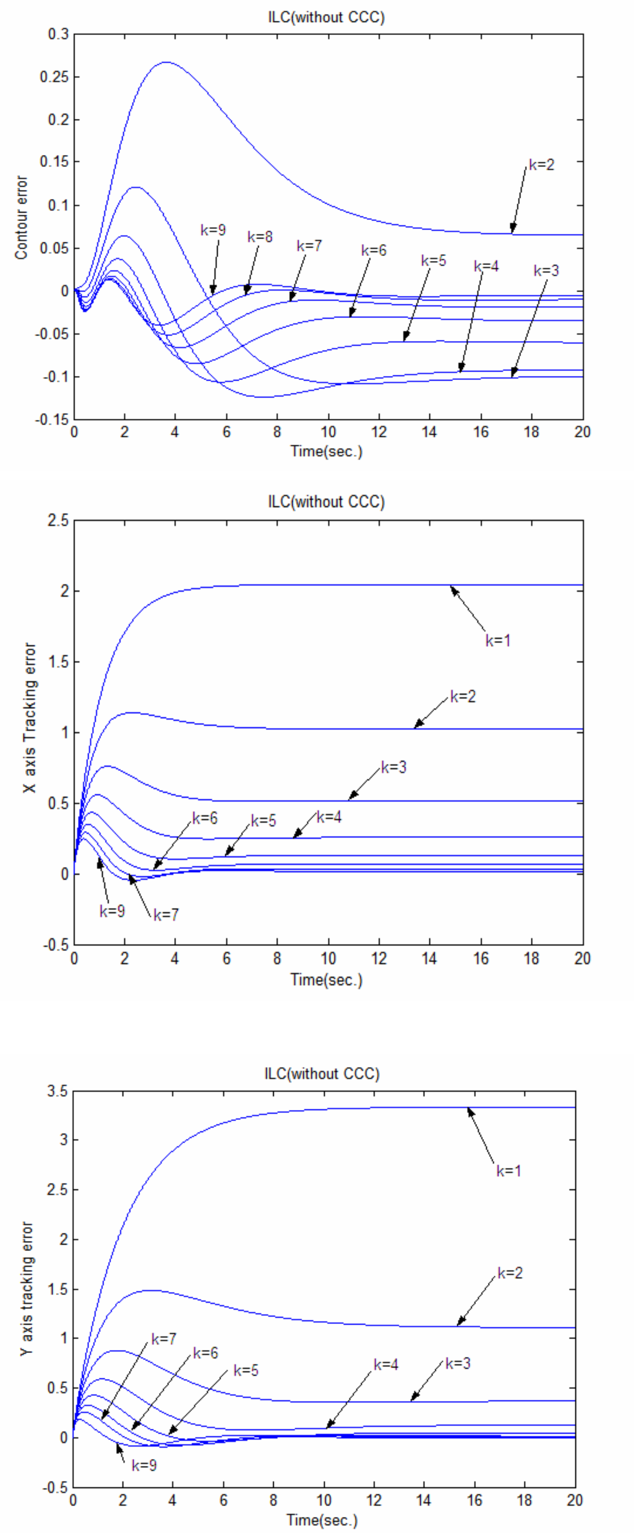

Fig. 3. ILC without CCC controller

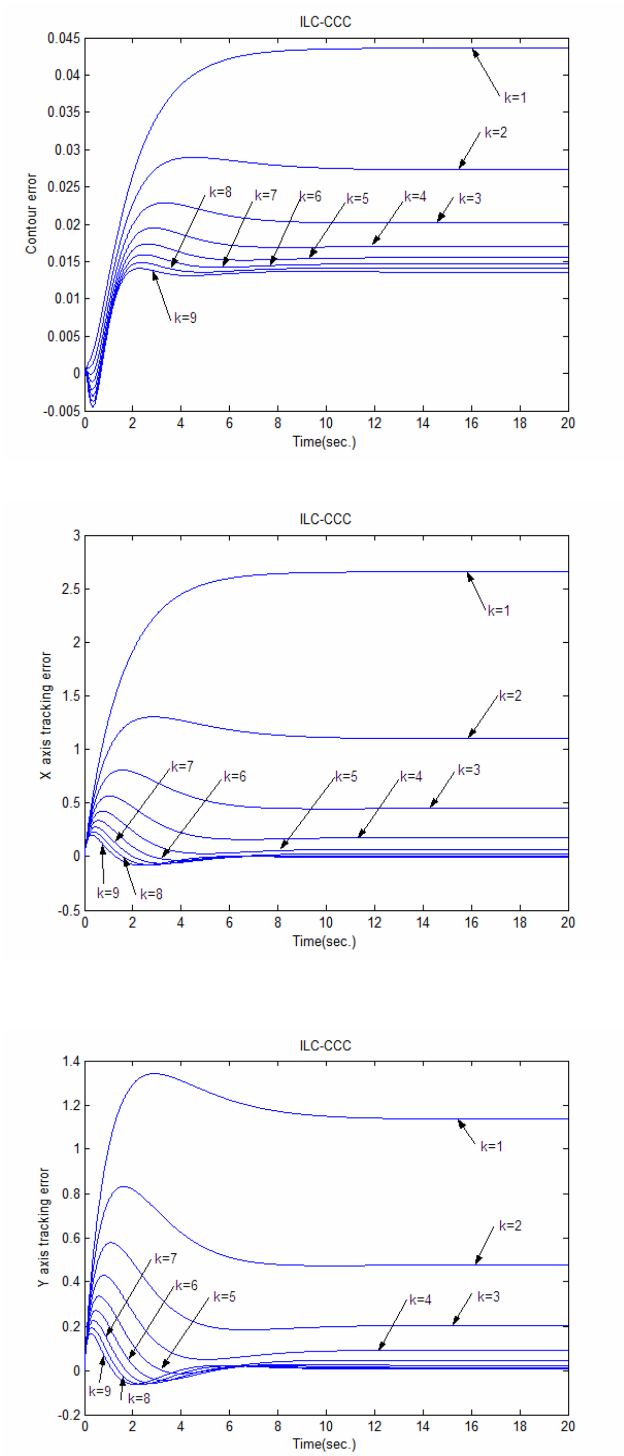

Fig. 4. ILC with CCC controller