Relay Feedback Tuning of Robust PID Controllers With Iso-Damping Property

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***Abstract*—A new tuning method for proportional-integral-derivative (PID) controller design is proposed for a class of unknown, stable, and minimum phase plants. We are able to design a PID controller to ensure that the phase Bode plot is flat, i.e., the phase derivative w.r.t. the frequency is zero, at a given frequency called the "tangent frequency" so that the closed-loop system is robust to gain variations and the step responses exhibit an iso-damping property. At the "tangent frequency," the Nyquist curve tangentially touches the sensitivity circle. Several relay feedback tests are used to identify the plant gain and phase at the tangent frequency in an iterative way. The identified plant gain and phase at the desired tangent frequency are used to estimate the derivatives of amplitude and phase of the plant with respect to frequency at the same frequency point by Bode's integral relationship. Then, these derivatives are used to design a PID controller for slope adjustment of the Nyquist plot to achieve the robustness of the system to gain variations. No plant model is assumed during the PID controller design. Only several relay tests are needed. Simulation examples illustrate the effectiveness and the simplicity of the proposed method for robust PID controller design with an iso-damping property.**

***Index Terms*—Bode's integral, flat phase condition, iso-damping property, proportional-integral-derivative (PID) controller, PID tuning, relay feedback test.**

I. Introduction

According to a survey [1] of the state of process control systems in 1989 conducted by the Japan Electric Measuring Instrument Manufacturer's Association, more than 90° of the control loops were of the proportional-integral-derivative (PID) type. It was also indicated [2] that a typical paper mill in Canada has more than 2,000 control loops and that 97% use PI control. Therefore, the industrialist had concentrated on PI/PID controllers and had already developed *one-button type* relay auto-tuning techniques for fast, reliable PI/PID control yet with satisfactory performance [3]–[7]. Although many different methods have been proposed for tuning PID controllers, the Ziegler–Nichols method [8] is still extensively used for determining the parameters of PID controllers. The design is based on the measurement of the critical gain and critical frequency of the plant and using simple formulae to compute the controller parameters. In 1984, Åström and Hägglund [9] proposed an automatic tuning method based on a simple relay feedback test which uses the describing function analysis to give the critical gain and the critical frequency of the system. This information can be used to compute a PID controller with desired gain and phase margins. In relay feedback tests, it is a common practice to use a relay with hysteresis [9] for noise immunity. Another commonly used technique is to introduce an artificial time delay within the relay closed-loop system, e.g., [10], to change the oscillation frequency in relay feedback tests.

After identifying a point on the Nyquist curve of the plant, the so-called modified Ziegler–Nichols method [4], [11] can be used to move this point to another position in the complex plane. Two equations for phase and amplitude assignment can be obtained to retrieve the parameters of a PI controller. For a PID controller, however, an additional equation should be introduced. In the modified Ziegler–Nichols method, α, the ratio between the integral time T_i and the derivative time T_d, is chosen to be constant, i.e., $T_i = \alpha T_d$, in order to obtain a unique solution.

The control performance is heavily influenced by the choice of α as observed in [10]. Recently, the role of α has drawn much attention, e.g., [12]–[14]. For the Ziegler–Nichols PID tuning method, α is generally assigned as a magic number four [4]. Wallén, Åström, and Hägglund proposed that the tradeoff between the practical implementation and the system performance is the major reason for choosing the ratio between T_i and T_d as four [12].

The main contribution of this paper is the use of a new tuning rule which gives a new relationship between T_i and T_d instead of the equation $T_i = 4T_d$ proposed in the modified Ziegler–Nichols method [4], [11]. We propose to add an extra condition that the phase Bode plot at a specified frequency w_c at the point where sensitivity circle touches Nyquist curve is locally flat which implies that the system will be more robust to gain variations. This additional condition can be expressed as $\mathrm{d}\angle G(s)/\mathrm{d}s|_{s=jw_c} = 0$, which can be equivalently expressed as

$$\angle \left.\frac{\mathrm{d}G(s)}{\mathrm{d}s}\right|_{s=jw_c} = \angle G(s)|_{s=jw_c} \tag{1}$$

where w_c is the frequency at the point of tangency and $G(s) = K(s)P(s)$ is the transfer function of the open loop system including the controller $K(s)$ and the plant $P(s)$. The above equivalence in (1) is mathematically explained in detail

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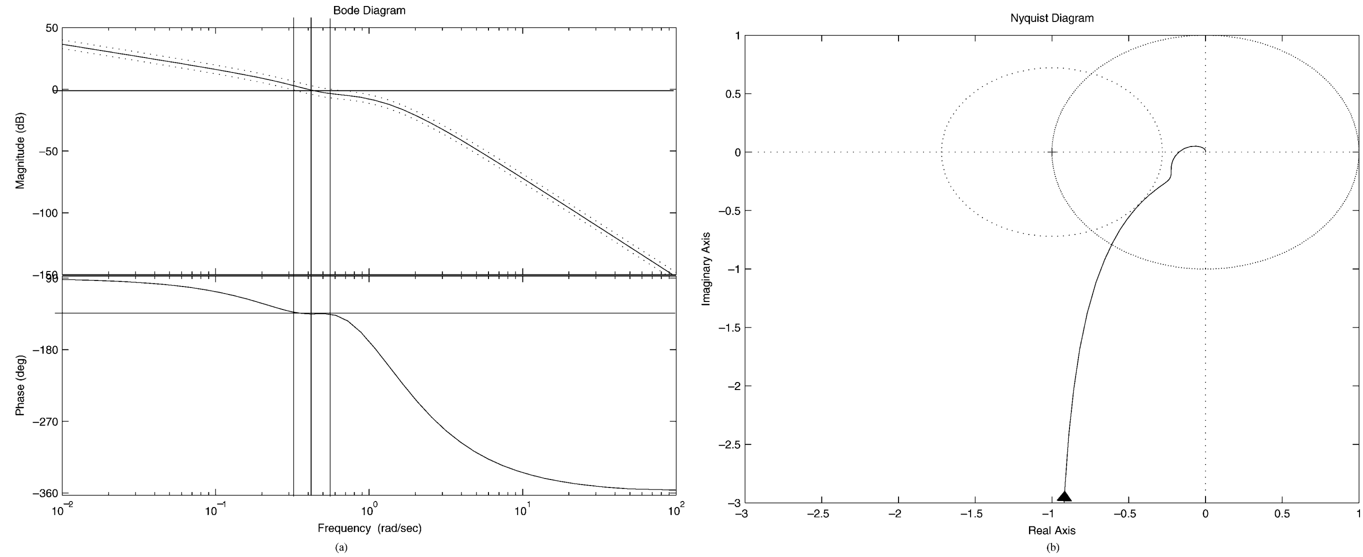

Fig. 1. Illustration of the basic idea for isodamping robust PID tuning.

in the Appendix. In this paper, we consider the PID controller of the following form:

$$K(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right). \tag{2}$$

This "flat phase" idea proposed earlier is illustrated in Fig. 1(a) where the Bode diagram of the open loop system is shown with its phase being tuned locally flat around w_c. We can expect that, if the gain increases or decreases a certain percentage, the phase margin will remain unchanged. Therefore, in this case, the step responses under various gains changing around the nominal gain will exhibit an iso-damping property, i.e., the overshoots of step responses will be almost the same. This can also be explained by Fig. 1(b) where the sensitivity circle touches the Nyquist curve of the open loop system at the flat phase point. Clearly, since gain variations are unavoidable in the real world due to possible sensor distortion, environment change and etc., the iso-damping is a desirable property which ensures that no harmful excessive overshoot is resulted due to gain variations.

Assume that the phase of the open loop system at w_c is

$$\angle G(s)|_{s=jw_c} = \Phi_m - \pi. \tag{3}$$

So, the definition of Φ_m is the phase angle of $G(s) = K(s)P(s)$ at the frequency w_c. Then, the corresponding gain can be expressed by

$$|G(jw_c)| = \cos(\Phi_m). \tag{4}$$

With these two conditions, (3) and (4), and the new condition (1), all the three parameters of PID controller can be calculated.

As in the Ziegler–Nichols method, T_i and T_d are used to tune the phase condition (3) and K_p is determined by the gain condition (4). However, the condition (1) gives a relationship between T_i and T_d instead of $T_i = \alpha T_d$.

Note that in this new tuning method, w_c is not necessarily the gain crossover frequency although close. Precisely, w_c is the frequency at which the Nyquist curve tangentially touches the sensitivity circle. Similarly, Φ_m, the tangent phase, is not necessarily the phase margin usually used in previous PID tuning methods. According to [4], the phase margin is always selected from 30° to 60°. Due to the flat phase condition (1), the derivative of the phase near w_c will be relatively small. Therefore, if Φ_m is selected to be around 30°, such as 35°, the phase margin will be generally within the desired interval.

II. Slope Adjustment of the Phase Bode Plot

In this section, we will show how T_i and T_d are related under the new condition (1).

Substitute s by jw so that the closed loop system can be written as $G(jw) = K(jw)P(jw)$, where

$$K(jw) = K_p \left(1 + \frac{1}{jwT_i} + jwT_d\right) \tag{5}$$

is the PID controller obtained from (2). The phase of the closed loop system is given by

$$\angle G(jw) = \angle K(jw) + \angle P(jw). \tag{6}$$

The derivative of the closed loop system $G(jw)$ with respect to w can be written as follows:

$$\frac{\mathrm{d}G(jw)}{\mathrm{d}w} = P(jw)\frac{\mathrm{d}K(jw)}{\mathrm{d}w} + K(jw)\frac{\mathrm{d}P(jw)}{\mathrm{d}w}. \tag{7}$$

From (1), the phase of the derivative of the open loop system can not obviously be obtained directly from (7). So, we need to simplify (7).

The derivative of the controller with respect to w is

$$\frac{\mathrm{d}K(jw)}{\mathrm{d}w} = jK_p\left(T_d + \frac{1}{w^2 T_i}\right). \tag{8}$$

To calculate $\mathrm{d}P(jw)/\mathrm{d}w$, since we have

$$\ln P(jw) = \ln|P(jw)| + j\angle P(jw) \tag{9}$$

differentiating (9) with respect to w gives

$$\frac{\mathrm{d}\ln P(jw)}{\mathrm{d}w} = \frac{1}{P(jw)}\frac{\mathrm{d}P(jw)}{\mathrm{d}w} = \frac{\mathrm{d}\ln|P(jw)|}{\mathrm{d}w} + j\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}. \tag{10}$$

Straightforwardly, we arrive at

$$\frac{\mathrm{d}P(jw)}{\mathrm{d}w} = P(jw)\left[\frac{\mathrm{d}\ln|P(jw)|}{\mathrm{d}w} + j\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right]. \tag{11}$$

Substituting (5), (8), and (11) into (7) gives

$$\frac{\mathrm{d}G(jw)}{\mathrm{d}w} = K_p P(jw)\left[j\left(T_d + \frac{1}{w^2 T_i}\right) + \left(1 + j\left(T_d w - \frac{1}{wT_i}\right)\right)\left(\frac{\mathrm{d}\ln|P(jw)|}{\mathrm{d}w} + j\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right)\right]. \tag{12}$$

Hence, the slope of the Nyquist curve at any specific frequency w_0 is given by

$$\angle\left.\frac{\mathrm{d}G(jw)}{\mathrm{d}w}\right|_{w_0} = \angle P(jw_0) + \tan^{-1}\left[\frac{(T_dT_iw_0^2+1)+(T_dT_iw_0^2-1)s_a(w_0)+s_p(w_0)T_iw_0}{s_a(w_0)T_iw_0-(T_dT_iw_0^2-1)s_p(w_0)}\right] \tag{13}$$

where, following the notations introduced in [15], [16], $s_a(w_0)$ and $s_p(w_0)$ are used throughout this paper defined as follows:

$$s_a(w_0) = w_0 \left.\frac{\mathrm{d}\ln|P(jw)|}{\mathrm{d}w}\right|_{w_0} \tag{14}$$

$$s_p(w_0) = w_0 \left.\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right|_{w_0}. \tag{15}$$

Here, our task is to adjust the slope of the Nyquist curve to match the condition shown in (1). By combining (1), (6), and (13), one obtains

$$\angle K(jw)|_{w_0} = \tan^{-1}\left[\frac{(T_dT_iw_0^2+1)+(T_dT_iw_0^2-1)s_a(w_0)+s_p(w_0)T_iw_0}{s_a(w_0)T_iw_0-(T_dT_iw_0^2-1)s_p(w_0)}\right]. \tag{16}$$

After a straightforward calculation, one obtains the relationship between T_i and T_d as follows:

$$T_d = \frac{-T_iw_0 + 2s_p(w_0) + \sqrt{\Delta}}{2s_p(w_0)w_0^2T_i} \tag{17}$$

where $\Delta = T_i^2w_0^2 - 8s_p(w_0)T_iw_0 - 4T_i^2w_0^2s_p^2(w_0)$. Note that due to the nature of the quadratic equation, an alternative relationship, is that $T_d = (-T_iw_0 + 2s_p(w_0) - \sqrt{\Delta})/2s_p(w_0)w_0^2T_i$. We should discard one to ensure that the T_d gain is a real positive number to avoid the right half plane zeros in K. In what follows, (17) is used. Additional, Δ could be negative if w_0 is not specified properly.

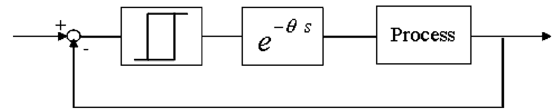

Fig. 2. Relay plus artificial time delay (θ) feedback system.

The approximation of s_p for stable and minimum phase plant can be given as follows [17]:

$$s_p(w_0) = w_0 \left.\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right|_{w_0} \approx \angle P(jw_0) + \frac{2}{\pi}\left[\ln|K_g| - \ln|P(jw_0)|\right] \tag{18}$$

where $|K_g| = P(0)$ is the static gain of the plant, $\angle P(jw_0)$ is the phase and $|P(jw_0)|$ is the gain of the plant at the specific frequency w_0.

It is obvious that T_i and T_d are related by s_p alone. For this new tuning method, s_p includes all the information that we need of the unknown plant. In what follows, we show that the s_p estimate formula can be extended to plants with integrators and/or time delay.

Consider the plant with m integrators

$$P(s) = \frac{\tilde{P}(s)}{s^m}, \qquad m = 1, 2, 3, \ldots. \tag{19}$$

Clearly, one can not get the static gain of such systems to compute s_p directly. But from (15)

$$s_p(w_0) = w_0 \left.\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right|_{w_0} = w_0 \left.\frac{\mathrm{d}(\angle\tilde{P}(jw) - \frac{m\pi}{2})}{\mathrm{d}w}\right|_{w_0} = w_0 \left.\frac{\mathrm{d}\angle\tilde{P}(jw)}{\mathrm{d}w}\right|_{w_0} \tag{20}$$

for the systems with integrators, s_p should be estimated according to the systems without any integrator.

For the plant with a time delay τ

$$\tilde{P}(s) = \bar{P}(s)e^{-\tau s} \tag{21}$$

in the same way

$$s_p(w_0) = w_0 \left.\frac{\mathrm{d}\angle\tilde{P}(jw)}{\mathrm{d}w}\right|_{w_0} = w_0 \left.\frac{\mathrm{d}\angle\bar{P}(jw)}{\mathrm{d}w}\right|_{w_0} - \tau w_0. \tag{22}$$

Consequently, substituting (18) we obtain

$$s_p(w_0) \approx \angle\bar{P}(jw_0) + \frac{2}{\pi}[\ln|K_g| - \ln|\bar{P}(jw_0)|] - \tau w_0 \approx \angle\tilde{P}(jw_0) + \frac{2}{\pi}[\ln|K_g| - \ln|\tilde{P}(jw_0)|]. \tag{23}$$

Obviously, the time delay will not contribute to the estimation of s_p.

So, in general, for the plant with both integrators and a time delay

$$P(s) = \frac{\tilde{P}(s)}{s^m} = \frac{\bar{P}(s)e^{-\tau s}}{s^m}, \qquad m = 1, 2, 3, \ldots \tag{24}$$

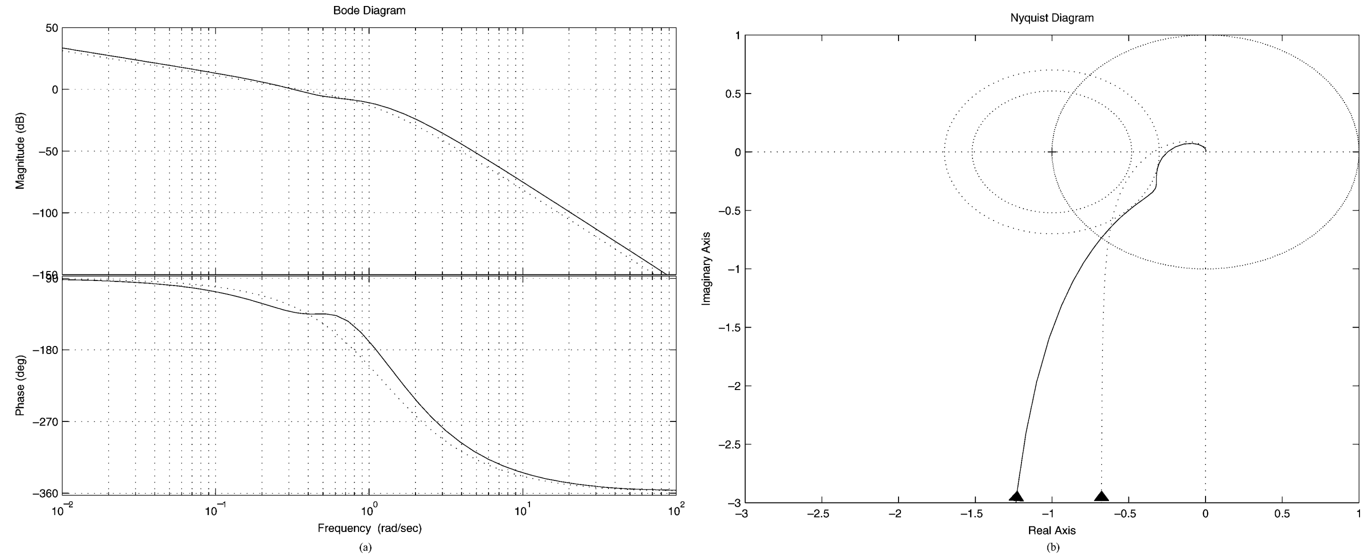

Fig. 3. Frequency responses of $K_{1p}(s)P_2(s)$ and $K_1(s)P_2(s)$ (dashed line: the modified Ziegler–Nichols, solid line: the proposed). (a) Comparison of Bode plots. (b) Comparison of Nyquist plots.

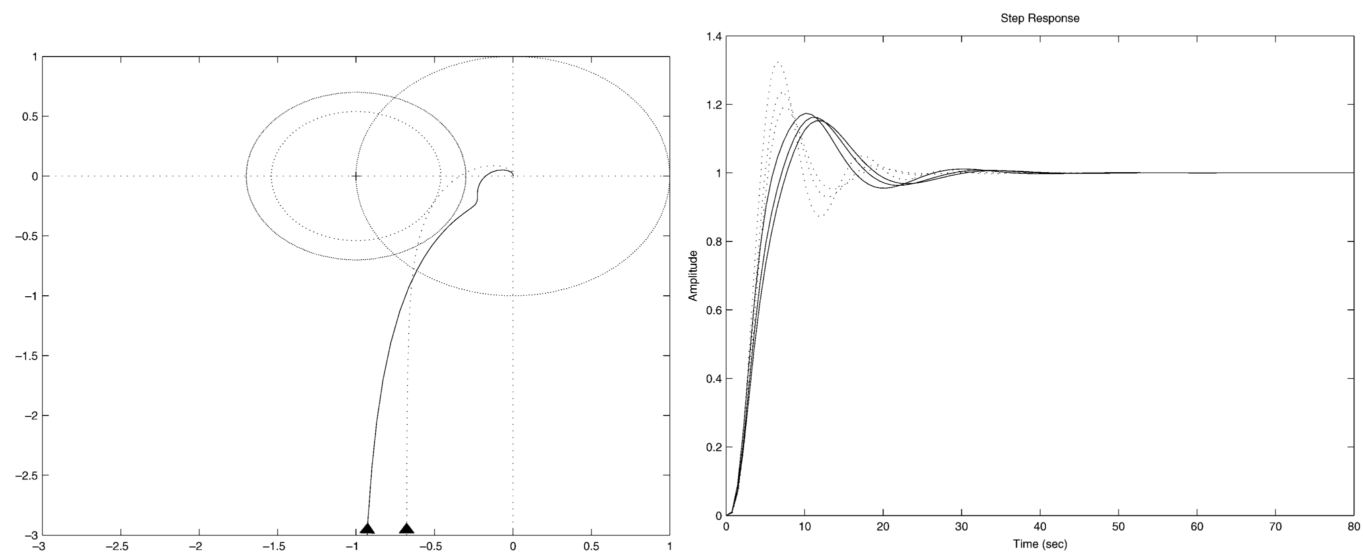

Fig. 4. Comparisons of frequency responses and step responses of $0.7K_{1p}(s)P_2(s)$ and $K_1(s)P_2(s)$ (dashed line: the modified Ziegler–Nichols, solid line: the proposed. For both schemes, gain variations 1, 1.1, 1.3 are considered in step responses). (a) Comparison of Nyquist plots. (b) Comparison of step responses.

according to (20) and (23)

$$s_p(w_0) = w_0 \left.\frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}\right|_{w_0} = w_0 \left.\frac{\mathrm{d}\angle \tilde{P}(jw)}{\mathrm{d}w}\right|_{w_0} \approx \angle \tilde{P}(jw_0) + \frac{2}{\pi}[\ln|K_g| - \ln|\tilde{P}(jw_0)|]. \quad (25)$$

III. New PID Controller Design Formulae

Suppose that we have known s_p at w_c. How to experimentally measure $s_p(w_c)$ will be discussed in the next section based on the measurement of $\angle P(jw_c)$ and $|P(jw_c)|$.

To write down explicitly the formulae for K_p, T_i, and T_d, let us summarize what are known at this point. We are given

1) w_c, the desired tangent frequency;
2) Φ_m, the desired tangent phase;
3) measurement of $\angle P(jw_c)$ and $|P(jw_c)|$
4) the estimation of $s_p(w_c)$.

Furthermore, using (3) and (4), the PID controller parameters can be set as follows:

$$K_p = \frac{\cos(\Phi_m)}{\left|P(jw_c)\sqrt{1+\tan^2(\Phi_m - \angle P(jw_c))}\right|} \quad (26)$$

$$T_i = \frac{-2}{w_c[s_p(w_c) + \hat{\Phi}) + \tan^2(\hat{\Phi})s_p(w_c)]} \quad (27)$$

where $\hat{\Phi} = \Phi_m - \angle P(jw_c)$. Finally, T_d can be computed from (17).

Remark III.1: The selection of w_c heavily depends on the system dynamics. For most of plants, there exists an interval for the selection of w_c to achieve flat phase condition. If no better idea about w_c, the desired cutoff frequency can be used as the initial value. For Φ_m, a good choice is within 30° to 35°.

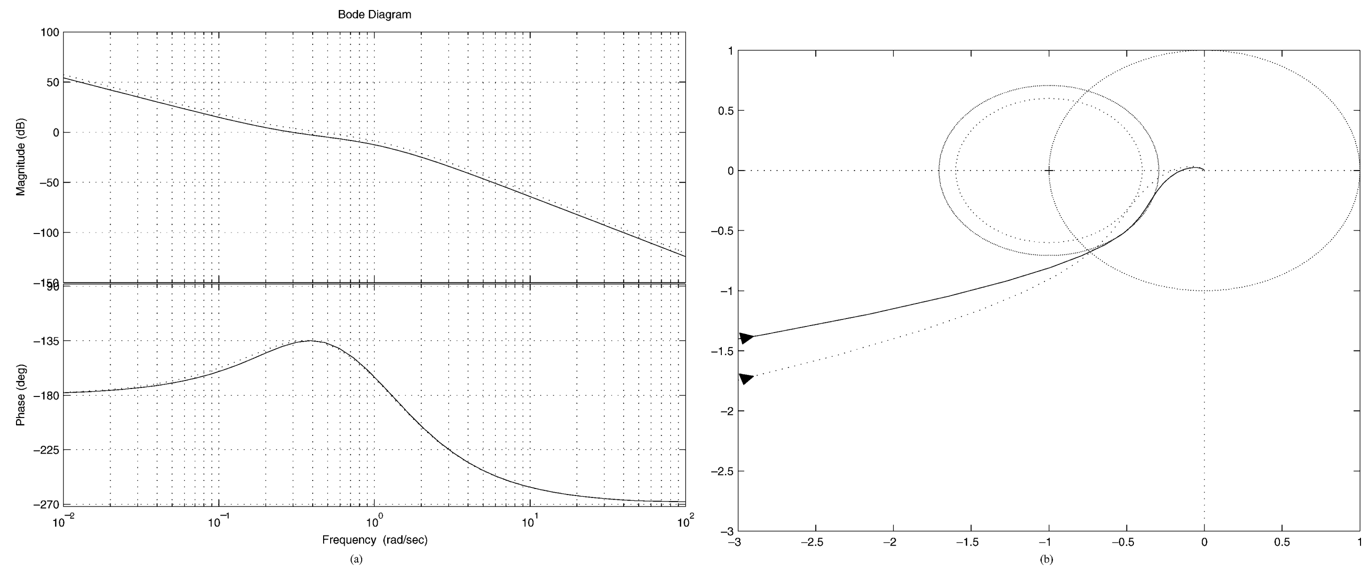

Fig. 5. Comparisons of frequency responses of $K_{2p}(s)P_5(s)$ and $K_2(s)P_5(s)$ (dashed line: the modified Ziegler–Nichols, solid line: the proposed). (a) Comparison of Bode plots. (b) Comparison of Nyquist plots.

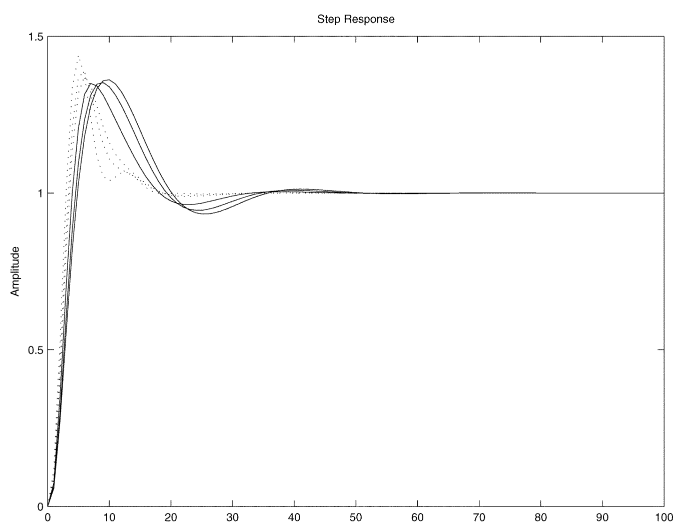

Fig. 6. Comparison of step responses of $K_{2p}(s)P_5(s)$ and $K_2(s)P_5(s)$ (solid line: the proposed modified controller with gain variations 1, 0.9, 0.8; dotted line: the modified Ziegler–Nichols controller with gain variations 1, 0.9, 0.8).

IV. Measuring arg$P(jw_c)$, $|P(jw_c)|$ and $s_p(w_c)$ via Relay Feedback Tests

Following the discussion in the previous section, the parameters of a PID controller can be calculated straightforwardly if we know $\angle P(jw_c)$, $|P(jw_c)|$, and $s_p(w_c)$.

As indicated in (18), $s_p(w_c)$ can be obtained from the knowledge of the static gain $|P(0)|$, $\angle P(jw_c)$, and $|P(jw_c)|$. The static gain $|P(0)|$ or K_g is very easy to measure and it is assumed to be known. The relay feedback test, shown in Fig. 2, can be used to "measure" $\angle P(jw_c)$ and $|P(jw_c)|$. In the relay feedback experiments, a relay is connected in closed-loop with the unknown plant as shown in Fig. 2 which is usually used to identify one point on the Nyquist diagram of the plant. To change the oscillation frequency due to relay feedback, an artificial time delay is introduced in the loop. The artificial time delay θ is the tuning knob here to change the oscillation frequency. Our problem here is how to get the right value of θ which corresponds to the tangent frequency w_c. To solve this problem, an iterative method can be used as summarized in the following:

1. Start with the desired tangent frequency w_c.
2. Select two different values (θ_{-1} and θ_0) for the time delay parameter properly and do the relay feedback test twice. Then, two points on the Nyquist curve of the plant can be obtained. The frequencies of these points can be represented as w_{-1} and w_0 which correspond to θ_{-1} and θ_0, respectively. The iteration begins with these initial values (θ_{-1}, w_{-1}) and (θ_0, w_0).
3. With the values obtained in the previous iterations, the artificial time delay parameter θ can be updated using a simple interpolation/extrapolation scheme as follows:

$$\theta_n = \frac{w_c - w_{n-1}}{w_{n-1} - w_{n-2}}(\theta_{n-1} - \theta_{n-2}) + \theta_{n-1}$$

where n represents the current iteration number. With the new θ_n, after the relay test, the corresponding frequency w_n can be recorded.
4. Compare w_n with w_c. If $|w_n - w^*| < \delta$, quit iteration. Otherwise, go to Step 3. Here, δ is a small positive number.

The iterative method proposed above is feasible because in general the relationship between the delay time θ and the oscillation frequency w is one-to-one.

After the iteration, the final oscillation frequency is quite close to the desired one w_c so that the oscillation frequency is considered as w_c. Hence, the amplitude and the phase of the

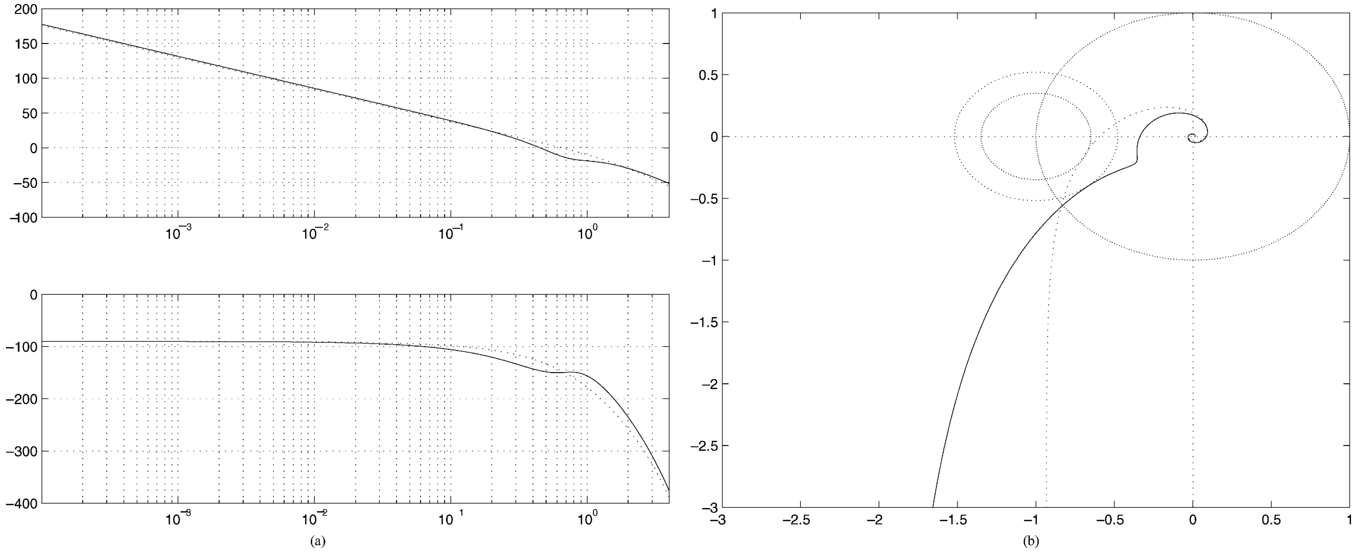

Fig. 7. Comparisons of frequency responses of $K_{3p}(s)P_6(s)$ and $K_3(s)P_6(s)$ (dashed line: the modified Ziegler–Nichols, solid line: the proposed). (a) Comparison of Bode plots. (b) Comparison of Nyquist plots.

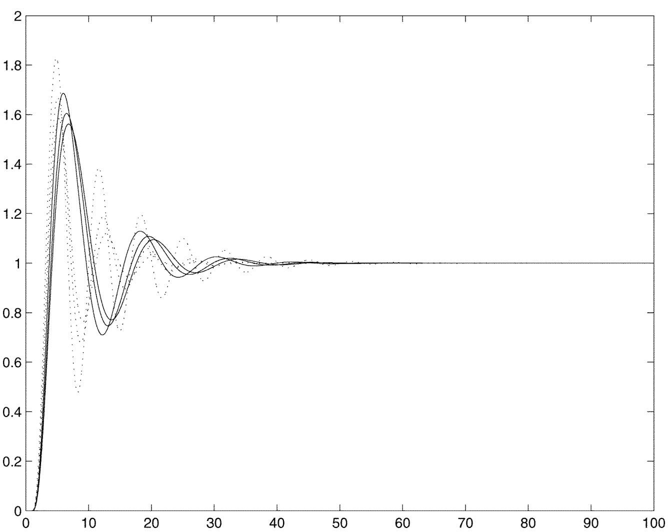

Fig. 8. Comparison of step responses of $K_{3p}(s)P_6(s)$ and $K_3(s)P_6(s)$ (solid line: the proposed modified controller with gain variations 1, 1.5, 1.7; dotted line: the modified Ziegler–Nichols controller with gain variations 1, 1.5, 1.7).

plant at the specified frequency can be obtained. Using (18), one can calculate the approximation of s_p.

V. Illustrative Examples

The new PID design method presented above will be illustrated via some simulation examples. In the simulation, the following classes of plants, studied in [12], will be used.

$$P_n(s) = \frac{1}{(s+1)^{(n+3)}}, \qquad n = 1, 2, 3, 4 \tag{28}$$

$$P_5(s) = \frac{1}{s(s+1)^3} \tag{29}$$

$$P_6(s) = \frac{1}{(s+1)^3} e^{-s} \tag{30}$$

$$P_7(s) = \frac{1}{s(s+1)^3} e^{-s}. \tag{31}$$

A. High-Order Plant $P_2(s)$

Consider plant $P_2(s)$ in (28). This plant was also used in [15]. The specifications are set as $w_c = 0.4$ rad./s and $\Phi_m = 45°$. The PID controller designed by using the proposed tuning formulae is

$$K_{1p}(s) = 0.921\left(1 + \frac{1}{1.961s} + 1.969s\right). \tag{32}$$

The PID controller designed by the modified Ziegler–Nichols method is

$$K_1(s) = 1.131\left(1 + \frac{1}{3.124s} + 0.781s\right). \tag{33}$$

The Bode and the Nyquist plots are compared in Fig. 3. From the Bode plots, it is seen that the phase curve near the frequency $w_c = 0.4$ rad./s is flat. The phase margin roughly equals 45°. That means the controller moves the point $P(0.4j)$ of the Nyquist curve to $K(0.4j)P(0.4j)$ on the unit circle with a phase of 135° and at the same time makes the Nyquist curve satisfy (1).

However, in Fig. 3(b), the Nyquist plot of the open loop system is not tangential to the sensitivity circle at the flat phase but to another point on the Nyquist curve. Define $[w_l, w_h]$ the frequency interval corresponding to the flat phase. So, the gain crossover frequency w_c can be moved within $[w_l, w_h]$ by adjusting K_p by $K'_p = \beta K_p$ where $\beta \in [w_l/w_c, w_h/w_c]$. For this example, if K_p is changed to $K'_p = 0.7K_p = 0.652$, the flat phase segment will tangentially touch the sensitivity circle. The Nyquist plot of the open loop system with the modified proposed PID controller, i.e., $0.7C_{1p}(s)$, is shown in Fig. 4(a) and the step responses of the closed loop system are compared in Fig. 4(b). Comparing the closed-loop system with the modified proposed PID controller to that with the modified Ziegler–Nichols controller, the overshoots of the step responses from the proposed scheme remain almost invariant under gain variations. However, the overshoots using the modified Ziegler–Nichols controller change remarkably.

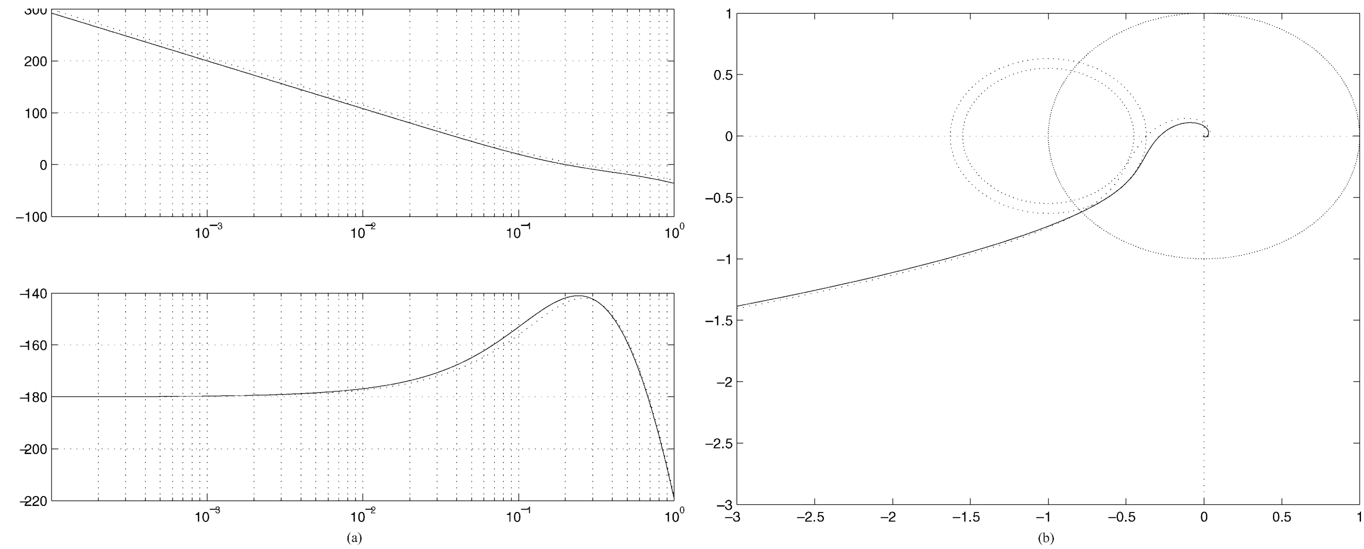

Fig. 9. Comparisons of frequency responses of $K_{4p}(s)P_7(s)$ and $K_4(s)P_7(s)$ (Dashed line: The modified Ziegler–Nichols, Solid line: The proposed). (a) Comparison of Bode plots. (b) Comparison of Nyquist plots.

B. Plant With an Integrator $P_5(s)$

For the plant $P_5(s)$, the proposed controller is

$$K_{2p}(s) = 0.33\left(1 + \frac{1}{6.53s} + 1.89s\right)$$

with respect to $\beta = 1$, $w_c = 0.4$ rad/s and $\Phi_m = 45°$. The controller designed by the modified Ziegler–Nichols method is

$$K_2(s) = 0.528\left(1 + \frac{1}{7.195s} + 1.799s\right).$$

The Bode plot of this situation, shown in Fig. 5(a), is quite different with that of plant $P_2(s)$. The flat phase occurs at the peak of the phase Bode plot. The Nyquist diagrams are compared in Fig. 5(b). The step responses are compared in Fig. 6 where the proposed controller does not exhibit an obviously better performance than the modified Ziegler–Nichols controller for the iso-damping property because of the effect of the integrator.

C. Plant With a Time Delay $P_6(s)$

For the plant $P_6(s)$ the proposed controller is

$$K_{3p}(s) = 1.024\left(1 + \frac{1}{1.241s} + 1.539s\right)$$

with respect to $\beta = 0.7$, $w_c = 0.6$ rad/s, and $\Phi_m = 30°$. The controller designed by the modified Ziegler—Nichols method is

$$K_3(s) = 1.674\left(1 + \frac{1}{2.57s} + 0.643s\right).$$

The Bode plots and Nyquist plots are compared in Fig. 7. The step responses are compared in Fig. 8 where the iso-damping property can be clearly observed.

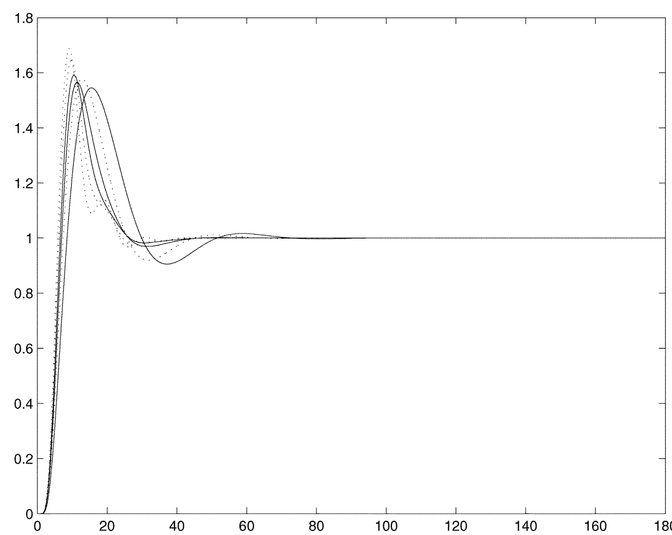

Fig. 10. Comparison of step responses of $K_{4p}(s)P_7(s)$ and $K_4(s)P_7(s)$ (Solid line: The proposed modified controller with gain variations 1, 1.5, 1.7; Dotted line: The modified Ziegler–Nichols controller with gain variations 1, 1.5, 1.7).

D. Plant With an Integrator and a Time Delay $P_7(s)$

For the plant $P_7(s)$, the proposed controller is

$$K_{4p} = 0.212\left(1 + \frac{1}{9.52s} + 2.061s\right)$$

with respect to $\beta = 1$, $w_c = 0.25$ rad/s, and $\Phi_m = 39°$. The controller designed by the modified Ziegler–Nichols method is

$$K_4 = 0.273\left(1 + \frac{1}{2.161s} + 8.644s\right).$$

The Bode plots and Nyquist plots are compared in Fig. 9. The step responses are compared in Fig. 10 where the iso-damping property can be clearly observed.

VI. Conclusion

A new PID tuning method is proposed for a class of unknown, stable and minimum phase plants. Given the tangent frequency w_c, the tangent phase Φ_m and with an additional condition that the phase Bode plot at w_c is locally flat, we can design the PID controller to ensure that the closed loop system is robust to gain variations and to ensure that the step responses exhibit an iso-damping property. No plant model is assumed during the PID controller design. Only several relay tests are needed. Simulation examples illustrate the effectiveness and the simplicity of the proposed method for robust PID controller design with an iso-damping property for different types of plants.

Our further research efforts include

1) determining the width and the position of the flat phase so as to achieve the performance of the proposed controller and simplify the design procedure;
2) testing on more types of plants;
3) exploring nonminimum phase, open loop unstable systems.

Appendix
Derivation of (1)

Assume that $G(jw) = x(w) + jy(w)$. Then,

$$\angle G(jw) = \tan^{-1}\left(\frac{y(w)}{x(w)}\right).$$

The derivative of $G(jw)$ with respect to w is that

$$\begin{aligned}\frac{\mathrm{d}\angle G(jw)}{\mathrm{d}w} &= \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\mathrm{d}\left(\frac{y}{x}\right)}{\mathrm{d}w}\\ &= \frac{x^2}{x^2+y^2}\left(\frac{\frac{\mathrm{d}y}{\mathrm{d}w}}{x} - \frac{\frac{y\mathrm{d}x}{\mathrm{d}w}}{x^2}\right)\\ &= \frac{1}{x^2+y^2}\left(x\frac{\mathrm{d}y}{\mathrm{d}w} - y\frac{\mathrm{d}x}{\mathrm{d}w}\right) = 0.\end{aligned}$$

Furthermore, one has $x(\mathrm{d}y/\mathrm{d}w) - y(\mathrm{d}x/\mathrm{d}w) = 0$, which means that $y/x = (\mathrm{d}y/\mathrm{d}w)/(\mathrm{d}x/\mathrm{d}w)$. Since $\tan\angle(\mathrm{d}G(jw)/\mathrm{d}w) = (\mathrm{d}y/\mathrm{d}w)/(\mathrm{d}x/\mathrm{d}w)$ and $\tan\angle G(jw) = y/x$, therefore, $\angle(\mathrm{d}G(jw)/\mathrm{d}w) = \angle G(jw)$.

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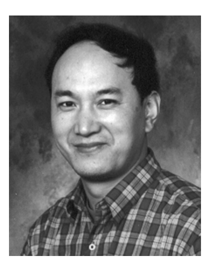

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