Intermittent Iterative Learning Control

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***Abstract*— In this paper we present a mathematical formulation of the problem of robust iterative learning control (ILC) design when the system is subject to data dropout. It is assumed that an ILC scheme is implemented via a networked control system (NCS) and that during the data transfer from the remote plant to the ILC controller data dropout occurs, resulting in what we call intermittent measurement. Using the Kalman filtering approach, we show that it is possible to design a learning gain such that the system eventually converges to a desired trajectory as long as there is not complete data dropout.**

***Index Terms*— Iterative learning control, Intermittent measurement, Networked control system, Kalman filtering**

I. Introduction

Robustness has been studied in ILC from a number of different perspectives, depending on the underlying assumptions made about the system to be controlled. For example, in stochastic ILC, learning gain matrices are designed using Kalman filters to primarily take into account measurement noise [1], [2], [3], [4]. In H_∞ ILC, external disturbances and model uncertainty are considered on the time axis [5]. Robustness with respect to initial resetting has been one of the most important research topics in ILC, given that by assumption ILC requires an initial equivalent condition at every iteration [6], [7], [8], [9], [10]. Frequency-domain analysis and/or synthesis based on frequency-based filtering considers noise and disturbances in the frequency domain [11], [12], [13], [14]. In interval ILC, parametric interval uncertainties in the plant model have been addressed with a particular focus on monotonically-convergent ILC design [15], [16].

In this paper we discuss another type of uncertainty that can arise during the implementation of an ILC system – data dropout during transmission of measurements or control signals between a remote plant and the ILC controller – and we provide an analytical ILC learning gain design method to ensure convergence in the face of such uncertainty. The paper is organized as follows: In Section II, we explain the idea of intermittent ILC in detail, including the motivation for this study. In Section III, we provide a synthesis method and an analysis for convergence. Using a stochastic Kalman filtering approach, we show the surprising result that it is possible to design a learning gain such that the system eventually converges to a desired trajectory as long as all the data is not lost. Conclusions are given in Section IV.

II. Intermittent ILC

A. Networked Control Systems

Recently networked control systems (NCS) have become very popular, due to the benefit of reducing the complexity of directly wiring between computers, saving the costs of maintenance of controllers at remote plants, etc. Driven by these benefits, real-time industrial networks such as DeviceNet, Profibus, FireWire, etc., have emerged as new technologies for distributed control applications [17]. The key feature of these industrial networks is to connect sensors, actuators, and controllers as network-wired nodes. This feature has enabled reducing the system wiring, increasing system agility, making it easier to diagnose the system, and increasing system reliability. Applications of computer networks include military unmanned vehicles, manufacturing plants, telerobotics, telemedicine, and various kinds of information and data signal exchanges between spatially-distributed system components. Generally, the distributed system consists of a supervisory controller, a remote plant, main controller, actuators, sensors, and a network that connects all these components (see Fig. 1).

Unfortunately, despite the benefits, in NCS applications there are also some drawbacks. First, there is the problem of data congestion, which is caused by lack of a universal clock between the main controller and the remote plant, hardware-inherent data delays, and communication constraints such as channel capacity. Furthermore, in the case of MIMO systems or multiple networked plants, data congestions and other factors, such as hard-nonlinearities, can cause time asynchrony among the multiple remote plants or subsystems. Another drawback is data dropout. In NCS research there have been numerous efforts to compensate for data congestion [18], [19], [20], [17], [21], [22], [23], [24], [25], and to improve the performance of systems that experience data dropout [26], [27], [28], [29], [30], [31], [32], [33], [34]. With regard to data dropout, it has been shown that there is a critical data dropout rate [32], [33], [34] above which the networked-system becomes unstable and therefore the desired performance is no longer achieved.

In this paper we suggest that for iterative learning control systems operated over a networked control system it is possible to address and alleviate the data dropout problem. Our main theoretical result is that if the control gain is updated by an ILC scheme and the remote plant operates in a repetitive way, the resulting NCS will be stable as long as there is a part of the data measurement, whatever amount it is, that gets through.

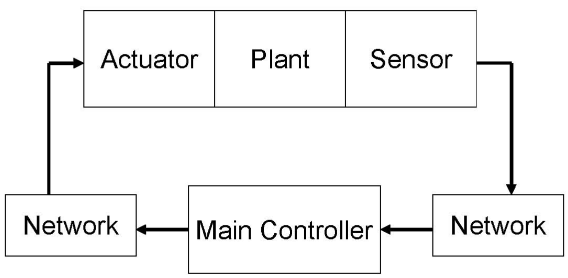

Fig. 1. Direct closed-loop networked control system configuration.

B. What is Intermittent ILC?

The key idea of ILC is to use information from past repetitions to compute the current repetition's control force. Throughout this paper, let us consider the following discrete-time, 2-dimensional system:

$$x_k(t+1) = Ax_k(t) + Bu_k(t) \tag{1}$$
$$y_k(t) = Cx_k(t) \tag{2}$$

where $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}$, and $C \in \mathbf{R}^{l\times n}$. The system is operated repeatedly in the iteration domain and at the k-th iteration, the system is described by the discrete state equations (1) and (2). Basic assumptions of this 2-dimensional system are (i) every trial ends in a fixed time of duration; (ii) a desired trajectory $y_d(t)$ is iteration invariant; and (iii) repetition of the initial resetting condition ($x_k(0) = x_0$ for all k) is satisfied. For more detailed explanation about this 2-dimensional system, refer to [35].

We can consider two different types of data dropouts when the ILC main controller is connected to the remote plant via a network. The first one occurs when the control input $u_k(t)$ is updated. For instance, in a typical ILC update law, the control input $u_{k+1}(t)$ is calculated by $u_{k+1}(t) = u_k(t)+\gamma e_k(t+1)$, where $e_k(t+1) = y_d(t+1)-y_k(t+1)$, y_d is the desired output, and y_k is the actual measured output. In this update, the stored control input information $u_k(t)$ or the error term $\gamma e_k(t+1)$ may be missed during the data transfer in the network. From Fig. 1, the remote ILC controller is located in the box named "Main controller" and the controlled-remote plant is located in the box named "Plant." During the control signal translation through the network, the signal could be dropped, which means that in the ILC control signal calculated by $u_{k+1} = u_k(t)+\gamma e_k(t+1)$, the past control signal $u_k(t)$ and/or error term $\gamma e_k(t+1)$ may be missed. The second data dropout is due to the measurement data loss during the signal transfer from the sensor to the main controller. In this case, mathematically, equation (2) can be changed as

$$y_k(t) = \eta(t)Cx_k(t) \tag{3}$$

where $\eta(t)$ could be zero or one (i.e., $\eta(t) \in \{0, 1\}$). But, this zero or one could be random due to the stochastic characteristic of the data loss. Thus, when we say *intermittent ILC*, we have to consider the above two different data dropout situations. Here we consider only the second case (intermittent measurement), as also done in most existing NCS works [26], [27], [28], [29], [30], [31], [32], [33], [34]. Note that if we use a higher-order ILC updating scheme, we may have to consider intermittent data dropouts from all past control signals and/or all past output measurements, but this is beyond the scope of this paper.

III. Optimal learning gain matrix design of intermittent ILC system

For the intermittent ILC design, we use an existing Kalman filtering ILC approach. Our main theoretical contribution is to add an intermittent measurement signal to the ILC update and then analyze the overall convergence property of the designed intermittent ILC system. The result of this section is influenced by [3]. Specifically, we will design ILC learning gain $K_k(t)$ with stochastic noises and intermittent measurements.[1]

A. Design of an optimal learning gain

Throughout this paper, we use the following update rule:

$$u_{k+1}(t) = u_k(t) + K_k(t)\eta e_k(t+1) \tag{4}$$

where $\eta \in \{0, 1\}$. This equation shows that for the current control signal calculation at the $(k+1)$-th iteration, we use the k-th control signal $u_k(t)$ and error term $e_k(t+1)$. Thus,

[1] In [3], there is no mathematical derivation considering intermittent measurements. Thus, the contribution of this section over [3] is to introduce intermittent measurements into the stochastic ILC framework. Also we note again, the contribution of this paper over existing intermittent Kalman filtering [26], [27], [28], [29], [30], [31], [32], [33], [34] is to show that there is no critical data dropout rate in the point of stability when we enhance the intermittent Kalman filtering results with an ILC update scheme.

it is a basic assumption of this paper that we know, at the $(k+1)$-th iteration, whether the output signal $y_k(t+1)$, which is the past information, was missed or not. That is, if $y_k(t+1)$ was delivered, then the current control signal is updated as

$$u_{k+1}(t) = u_k(t) + K_k(t) e_k(t+1), \tag{5}$$

otherwise, the current control signal is updated as

$$u_{k+1}(t) = u_k(t). \tag{6}$$

Following standard ILC practice, we denote $u_d(t), x_d(t)$, and $y_d(t)$ as the desired input, state, and output signals, respectively. Introducing $\delta u_{k+1}(t) = u_d(t) - u_{k+1}(t)$ and $\delta x_k(t) = x_d(t) - x_k(t)$, and following [3], we obtain the auxiliary system:

$$\begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix} = \begin{bmatrix} I - K_k(t)\eta CB & -K_k(t)\eta CA \\ B & A \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\eta C & K_k(t) \\ -I & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix} \tag{7}$$

Now, we introduce $\eta = \overline{\eta} + \widetilde{\eta}$, where $\overline{\eta}$ is the mean of η and $\widetilde{\eta}$ is a zero-mean stochastic sequence. Following [33], we can calculate the variance of $\widetilde{\eta}$ as $\sigma_{\tilde{\eta}}^2 = (1-\overline{\eta})\overline{\eta}$. Now, inserting $\eta = \overline{\eta} + \widetilde{\eta}$ into (7) yields (8) the following equation.

$$\begin{aligned} \begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix} &= \begin{bmatrix} I - K_k(t)\overline{\eta} CB & -K_k(t)\overline{\eta} CA \\ B & A \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\overline{\eta} C & K_k(t) \\ -I & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix} \\ &+ \begin{bmatrix} -K_k(t)\widetilde{\eta} CB & -K_k(t)\widetilde{\eta} CA \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\widetilde{\eta} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix}. \end{aligned} \tag{8}$$

For convenience, defining

$$\Phi := \begin{bmatrix} I - K_k(t)\overline{\eta} CB & -K_k(t)\overline{\eta} CA \\ B & A \end{bmatrix}$$

$$\Psi := \begin{bmatrix} K_k(t)\overline{\eta} C & K_k(t) \\ -I & 0 \end{bmatrix}$$

$$\Omega := \begin{bmatrix} -K_k(t) CB & -K_k(t) CA \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon := \begin{bmatrix} K_k(t) C & 0 \\ 0 & 0 \end{bmatrix}; \; X^+ := \begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix}$$

$$X := \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix}; \; W := \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix}$$

we rewrite (8) such as

$$X^+ = \Phi X + \widetilde{\eta} \Omega X + \Psi W + \widetilde{\eta} \Upsilon W. \tag{9}$$

Now, taking expectations on both sides of (9), and assuming no correlation among state, random sequence, and random noises, we have

$$\begin{aligned} E[X^+ X^{+^T}] &= E[(\Phi X + \widetilde{\eta}\Omega X + \Psi W + \widetilde{\eta}\Upsilon W) \\ &\quad \times (\Phi X + \widetilde{\eta}\Omega X + \Psi W + \widetilde{\eta}\Upsilon W)^T] \\ &= \Phi E[XX^T]\Phi^T + \sigma_{\tilde{\eta}}^2 \Omega E[XX^T]\Omega^T \\ &\quad + \Psi E[WW^T]\Psi^T + \sigma_{\tilde{\eta}}^2 \Upsilon E[WW^T]\Upsilon^T. \end{aligned} \tag{10}$$

Assuming that we have knowledge of the noise statistics and denoting them as

$$P^+ := E[X^+ X^{+^T}]; \; P := E[XX^T]; \; Q := E[WW^T],$$

we have

$$P^+ = \Phi P \Phi^T + \sigma_{\tilde{\eta}}^2 \Omega P \Omega^T + \Psi Q \Psi^T + \sigma_{\tilde{\eta}}^2 \Upsilon Q \Upsilon^T. \tag{11}$$

To find an optimal learning gain matrix $K_k(t)$, following [3], we use the trace of P^+, a typical procedure in Kalman filtering. In what follows, for simplicity, we omit subscripts the k and $\tilde{\eta}$, and the time index t. Now, we compute the trace of both sides of (11) as:

$$\begin{aligned} \mathrm{trace}(P^+) &= \mathrm{trace}[\Phi P \Phi^T + \sigma_{\tilde{\eta}}^2 \Omega P \Omega^T + \Psi Q \Psi^T \\ &\quad + \sigma_{\tilde{\eta}}^2 \Upsilon Q \Upsilon^T] \end{aligned}$$

Partitioning P and Q according to:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}; \; Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

we have:

$$\begin{aligned} &\mathrm{trace}(P^+) = \\ &\mathrm{trace}\left\{\sigma^2 KC(BP_{11} + AP_{21})B^T C^T K^T \right. \\ &\quad + \sigma^2 KC(BP_{12} + AP_{22})A^T C^T K^T \\ &\quad + [(I - K\overline{\eta}CB)P_{11} - K\overline{\eta}CAP_{21}]\,[I - K\overline{\eta}CB]^T \\ &\quad + [(I - K\overline{\eta}CB)P_{12} - K\overline{\eta}CAP_{22}]\,[-K\overline{\eta}CA]^T \\ &\quad + (BP_{11} + AP_{21})B^T + (BP_{12} + AP_{22})A^T \\ &\quad + (K\overline{\eta}CQ_{11} + KQ_{21})(K\overline{\eta}C)^T \\ &\quad + (K\overline{\eta}CQ_{12} + KQ_{22})K^T + Q_{11} \\ &\quad \left. + \sigma^2 KCQ_{11}(KC)^T \right\}. \end{aligned} \tag{12}$$

Next, substituting the following into the above equation

$$V_1 := (CB, \; CA), \; V_2 := (B, \; A), \; V_3 = (I, \; 0),$$

we can simplify the right-hand side of (12) to be:

$$\begin{aligned} &\mathrm{trace}\left\{\sigma^2 KV_1PV_1^T K^T + \overline{\eta}^2 KV_1PV_1^T K^T + V_2PV_2^T \right. \\ &\quad + V_3PV_3^T - \overline{\eta}V_3PV_1^T K^T - \overline{\eta}KV_1PV_3^T \end{aligned}$$

$$+ Q_{11} + K(\overline{\eta}^2 CQ_{11}C^T + \sigma^2 CQ_{11}C^T + \overline{\eta}CQ_{12} + Q_{21}\overline{\eta}C^T + Q_{22})K^T\} \tag{13}$$

Therefore, we have

$$\frac{\partial \mathrm{trace}(P^+)}{\partial K} = 2\sigma^2 KV_1PV_1^T + 2\overline{\eta}KV_1PV_1^T - 2\overline{\eta}V_3PV_1^T + 2K(\overline{\eta}^2 CQ_{11}C^T + \sigma^2 CQ_{11}C^T + \overline{\eta}CQ_{12} + Q_{21}\overline{\eta}C^T + Q_{22}). \tag{14}$$

Here, assuming no correlation between $w_k(t)$ and $v_k(t)$, we consider $Q_{12} = Q_{21} = 0$. Now, defining

$$\Pi := (\sigma^2 + \overline{\eta}^2)V_1PV_1^T + (\overline{\eta}^2 + \sigma^2)CQ_{11}C^T + Q_{22},$$

when $\frac{\partial \mathrm{trace}(P^+)}{\partial K}$ is set equal to zero, we finally calculate the optimal learning gain as follows:

$$K_k(t) = \overline{\eta}V_3PV_1^T\Pi^{-1}. \tag{15}$$

B. Analysis of convergence

In this subsection, we analyze the convergence of the intermittent ILC system updated by the learning gain given by (15). For this purpose, we need to find a recursive update formula for the covariance matrix $P = E[XX^T]$. For each t and k, let us denote:

$$\begin{aligned} P_{11,k}(t) &= E[\delta u_k(t)\delta u_k(t)^T] \\ P_{12,k}(t) &= E[\delta u_k(t)\delta x_k(t)^T] \\ P_{21,k}(t) &= E[\delta x_k(t)\delta u_k(t)^T] \\ P_{22,k}(t) &= E[\delta x_k(t)\delta x_k(t)^T] \end{aligned}$$

In the sequel we will drop the notational dependence on k and/or t when it is clear from the context.

First, it is easy to show that $P_{12,k}(t) = P_{21,k}(t) = 0$ using the same method given in [3]. Next we consider the diagonal terms $P_{11,k}(t)$ and $P_{22,k}(t)$. Beginning with P_{11}:

$$E[\delta u_{k+1}(t)\delta u_{k+1}(t)^T] = \sigma^2 KV_1PV_1^TK^T + \overline{\eta}^2 KV_1PV_1^TK^T + V_3PV_3^T - \overline{\eta}V_3PV_1^TK^T - \overline{\eta}KV_1PV_3^T + K(\overline{\eta}^2 CQ_{11}C^T + \sigma^2 CQ_{11}C^T + Q_{22})K^T. \tag{16}$$

Now, reformulating the right-hand side of (16) and inserting (15) into this yields

$$E[\delta u_{k+1}(t)\delta u_{k+1}(t)^T] = V_3PV_3^T - \overline{\eta}^2 V_3PV_1^T\Pi^{-1}V_1PV_3^T. \tag{17}$$

Thus we have

$$P_{11,k+1} = (I - \overline{\eta}K_k(t)CB)P_{11,k}, \tag{18}$$

which is very similar to equation (16) of [3]. Here, denoting $\Lambda := CQ_{11}C^T + (\sigma^2 + \overline{\eta}^2)^{-1}Q_{22}$ and $S := CAP_{22}(CA)^T + \Lambda$, we have

$$\begin{aligned} I - \overline{\eta}K_k(t)CB &= I - \overline{\eta}^2 P_{11}(CB)^T(\sigma^2 + \overline{\eta}^2)^{-1} \\ &\quad \times \left[CBP_{11}(CB)^T + S\right]^{-1} CB. \end{aligned} \tag{19}$$

Now, since $\overline{\eta}^2/(\sigma^2 + \overline{\eta}^2) = \overline{\eta}$ and, for convenience, substituting CB by N, then, (19) becomes:

$$I - \overline{\eta}K_k(t)CB = I - \overline{\eta}P_{11}N^T\left[NP_{11}N^T + S\right]^{-1} N \tag{20}$$

Next, using $\left[NP_{11}N^T + S\right]^{-1} = S^{-1} - S^{-1}N(N^TS^{-1}N + P_{11}^{-1})^{-1}N^TS^{-1}$, we obtain

$$\begin{aligned} I - \overline{\eta}K_k(t)CB &= I - \overline{\eta}P_{11}N^TS^{-1}N + \overline{\eta}P_{11}N^TS^{-1}N \\ &\quad \times (N^TS^{-1}N + P_{11}^{-1})^{-1}N^TS^{-1}N. \end{aligned} \tag{21}$$

Also for convenience, denoting $Y := N^TS^{-1}N$ and $W := P_{11}Y$, we finally obtain

$$\begin{aligned} &I - \overline{\eta}K_k(t)CB \\ &= I - \overline{\eta}P_{11}Y + \overline{\eta}P_{11}Y(Y + P_{11}^{-1})^{-1}Y \\ &= I - \overline{\eta}P_{11}Y + \overline{\eta}P_{11}Y(I + (P_{11}Y)^{-1})^{-1} \\ &= I - \overline{\eta}W + \overline{\eta}W(I + W^{-1})^{-1} \\ &= (I - \overline{\eta}W)(I + W^{-1})(I + W^{-1})^{-1} \\ &\quad + \overline{\eta}W(I + W^{-1})^{-1} \\ &= ((1 - \overline{\eta})I + W^{-1})(I + W^{-1})^{-1}. \end{aligned} \tag{22}$$

Now it is observed that S is a positive definite matrix. Hence, if CB is full rank, then Y is positive definite (see page 399 of [36]). Also, it is assumed that the covariance matrix P_{11} is positive definite. Generally, the multiplication of two symmetric matrices is not symmetric. So, we cannot claim that W is symmetric. However, for SISO systems, W is a scalar. Then, we can conclude that $((1-\overline{\eta})I + W^{-1})(I + W^{-1})^{-1} < 1$; so $I - \overline{\eta}K_k(t)CB < 1$. Thus, $P_{11,k} \to 0$ in the manner of $P_{11,k+1} < P_{11,k}$ as $k \to \infty$. For the MIMO case, we prove $\rho\left[((1-\overline{\eta})I + W^{-1})(I + W^{-1})^{-1}\right] < 1$ in the following lemma.

Lemma 3.1: The spectral radius of $((1-\overline{\eta})I + W^{-1})(I + W^{-1})^{-1}$ is less than 1.

Proof: Clearly, W^{-1} has positive real eigenvalues since it is multiplied by two positive definite matrices (page 465 of [36]). Hence $(1 - \overline{\eta})I + W^{-1}$ and $I + W^{-1}$ are nonsingular matrices. Let us say that X is an eigenvector matrix of $(1 - \overline{\eta})I + W^{-1}$ and $\Sigma = \mathrm{diag}(\sigma_i)$ is diagonal eigenvalue matrix. Then, we can write

$$\begin{aligned} X[(1 - \overline{\eta})I + W^{-1}]X^{-1} &= \Sigma \\ \Leftrightarrow -\overline{\eta}XX^{-1} + X(I + W^{-1})X^{-1} &= \Sigma \\ \Leftrightarrow X(I + W^{-1})X^{-1} &= \mathrm{diag}(\overline{\eta} + \sigma_i). \end{aligned} \tag{23}$$

Therefore, we have

$$(1-\overline{\eta})I + W^{-1} = X^{-1}\mathrm{diag}(\sigma_i)X \tag{24}$$

and

$$\begin{aligned}(I+W^{-1}) &= X^{-1}\mathrm{diag}(\overline{\eta}+\sigma_i)X \Rightarrow (I+W^{-1})^{-1}\\ &= X^{-1}\mathrm{diag}(\overline{\eta}+\sigma_i)^{-1}X.\end{aligned} \tag{25}$$

Now, substituting (24) and (25) into (22), we have

$$\begin{aligned}&I-\overline{\eta}K_k(t)CB =\\ &X^{-1}\mathrm{diag}(\sigma_i)XX^{-1}\mathrm{diag}(\overline{\eta}+\sigma_i)^{-1}X\\ =\;& X^{-1}\mathrm{diag}\left(\sigma_i/(\overline{\eta}+\sigma_i)\right)X.\end{aligned} \tag{26}$$

Now, since $\sigma_i > 0$, the spectral radius is less than 1. ■

These results are summarized in the following theorem:

Theorem 3.1: Under the intermittent measurement environment, η, with mean $\overline{\eta}$, the ILC learning gain determined by (15) guarantees $P_{11,k} \to 0$ as $k \to \infty$.

Proof: By Lemma 3.1, the proof is direct. ■

Remark 3.1: From the definition of $\overline{\eta}$, we know that when $\overline{\eta} = 1$, there is no intermittent measurement, while when $\overline{\eta} = 0$, all measurements are lost. From (26), when $\overline{\eta} = 1$, the spectral radius of $I-\overline{\eta}K_k(t)CB$ is smallest. So, we can conclude that without intermittent measurement, the best convergence is achieved. However, from (26), as far as $\overline{\eta} \neq 0$, still the spectral radius of $I-\overline{\eta}K_k(t)CB$ is less than 1. So, in ILC, the convergence is always guaranteed even if most of the measurements are lost (in other words, as far as there is a part of the data measurement available, whatever amount it is).

Next we consider the convergence of $P_{22,k}(t)$. For this we have the following theorem.

Theorem 3.2: If there is no initial resetting error at every iteration, then

$$P_{22,k} \to \sum_{i=0}^{t-1} A^{t-1-i}Q_{11}\sum_{i=0}^{t-1}(A^T)^{t-1-i}. \tag{27}$$

Proof: From (7), and after some manipulations, we obtain:

$$\delta x_k(t) = A^t\delta x_k(0) + \sum_{i=0}^{t-1} A^{t-1-i}[B\delta u_k(i) - w_k(i)] \tag{28}$$

Since there is no initial resetting error, $\delta x_k(0) = 0$, so:

$$\begin{aligned}&E[\delta x_k(t)\delta x_k(t)^T] =\\ &\sum_{i=0}^{t-1} A^{t-1-i}E\left[(B\delta u_k(i) - w_k(i))\right.\\ &\left.\times(\delta u_k(i)^T B^T - w_k(i)^T)\right]\\ &\times\sum_{i=0}^{t-1}(A^T)^{t-1-i}\\ =\;& \sum_{i=0}^{t-1} A^{t-1-i}\left[BE(\delta u_k(i)\delta u_k(i)^T)B^T\right.\\ &\left.+E(w_k(t)w_k(t)^T)\right]\sum_{i=0}^{t-1}(A^T)^{t-1-i}\end{aligned} \tag{29}$$

Therefore, from Theorem 3.1, since $E(\delta u_k(i)\delta u_k(i)^T) = 0$ as $k \to \infty$, the proof is completed. ■

Remark 3.2: In Theorem 3.2, it is shown that $P_{22,k}$ converges to a fixed value as the number of iterations increases. It is observed that the final converged value of P_{22} depends on A and Q_{11}. If there is no noise, then $P_{22} \to 0$.

When $Q_{11} \neq 0$, we can write (27) as:

$$\mathrm{vec}(P_{22}) = \sum_{i=0}^{t-1}(A^i \otimes A^i)\mathrm{vec}(Q_{11}) \tag{30}$$

where $\otimes$ is the Kronecker product, $\mathrm{vec}(P_{22}) = [(P_{22})_1^T, (P_{22})_2^T, \cdots, (P_{22})_n^T]^T$, and $\mathrm{vec}(Q_{11}) = [(Q_{11})_1^T, (Q_{11})_2^T, \cdots, (Q_{11})_n^T]^T$, where $(P_{22})_j$ is the j-th column vector of matrix P_{22} and $(Q_{11})_j$ is the j-th column vector of matrix Q_{11}. Furthermore, using the property $A^2 \otimes A^2 = (A \otimes A)(A \otimes A)$, we can see that the boundedness of P_{22}, on the time domain, depends on eigenvalues of $A \otimes A$ (not the eigenvalues of A).

Remark 3.3: In (15), $K_k(t)$ depends on P, which is the expectation of XX^T. Actually, $K_k(t)$ is calculated based on past information at every iteration. Similar to [3], we can develop an algorithm for updating $K_k(t)$. To avoid confusion and clarify this paper, we provide an algorithm in this remark. Note that from (11), we have

$$P_{22,k}(t+1) = BP_{11,k}(t)B^T + AP_{22,k}(t)A^T + Q_{11}. \tag{31}$$

Let us assume that $P_{11,k}(t)$, when $k = 0$, is available and $P_{22,k}(t)$, when $t = 0$, is also available. Then, the following algorithm can be developed:

- From (31), when $k = 0$, we calculate $P_{22,k}(t+1)$.
- Use (15) for updating $K_k(t)$.
- Calculate $u_{k+1}(t)$ using (4).
- Use (18) to update $P_{11,k}(t)$.
- Repeat whole process (i.e., $k = k+1$).

IV. Concluding remarks

In this paper, we provided a synthesis method for ILC systems subject to intermittent measurements. Although the algorithm requires using full information about the A matrix, as some measurements are available the convergence of the ILC scheme is guaranteed. Compared with existing intermittent estimation theories [26], [27], [28], [29], [30], [31], [32], [33], [34], which have critical data dropout rates above which the system is not stable, the intermittent Kalman filtering scheme enhanced by ILC has no critical data dropout rate. Intuitively, this result can be understood in the following way: even if there may be significant data dropout in the output measurements, if we update the control signals on the repetitive iteration domain, the effect of data loss can be compensated eventually, because the system, when viewed from the repetition-domain, is simple a contractive sequence. In future work, we consider the design of the learning gain matrix without using full information of the A matrix and will also consider the problem of control signal dropout in a delayed NCS system. We are also developing an equivalent theory for interval plants as well as experimental validation using the Utah State University "Smart Wheel." [37].

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