LMI Approach to Iterative Learning Control Design

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***Abstract*— This paper uses linear matrix inequalities to design iterative learning controller gains. Comparisons are made between Arimoto-style gains, causal gains, and non-causal gains, using the supervector approach. The results show that linear time-varying gains have better performance than linear time invariant gains and non-causal terms make the system more stable in the sense of monotonic convergence.**

I. Introduction

Let the SISO discrete-time system $Y_k(z) = H(z)U_k(z)$ have the transfer function

$$H(z) \;=\; h_1 z^{-1} + h_2 z^{-(2)} + \cdots$$

where the system is assumed with no loss of generality to have relative degree one, z^{-1} is the standard delay operator with respect to time t, k denotes the iteration index, and the parameters h_i are the standard Markov parameters of the system $H(z)$. Per the normal ILC methodology, let the trial length be N and lift the time-domain signals to form the so-called supervectors:

$$\begin{aligned} U_k &= (u_k(0), u_k(1), \cdots, u_k(N-1)) \\ Y_k &= (y_k(1), y_k(2), \cdots, y_k(N)) \\ Y_d &= (y_d(1), y_d(2), \cdots, y_d(N)) \end{aligned}$$

from which we can write $Y_k = HU_k$, where H is a lower-triangular Toeplitz matrix of rank n whose elements are the Markov parameters of the plant $H(z)$, given by:

$$H = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_N & h_{N-1} & \cdots & h_1 \end{bmatrix}$$

This is called the supervector representation of the ILC problem [1], [2], [3] An advantage of using the super-vector notation is that the two-dimensional problem of ILC is changed into a one-dimensional multi-input multi-output (MIMO) problem.

Stability conditions for super-vector ILC have been well-analyzed in the literature. However, though many researcher have considered the design of learning gains, the question of finding the best structure of the learning gain matrix has not been fully answered. The difficulties of finding the best structure are that (i) the system is a high-order MIMO plant, so it is not easy to analyze the super-vector ILC system analytically, and, contradictory to (i), (ii) there exists an easy solution, that is, the inverse of the Markov matrix is the best optimal solution. However, this easy solution can be difficult to implement when the plant is ill-conditioned. To overcome these difficulties, we turn to some recently-developed tools in control theory, specifically, linear matrix inequalities.

Recently, with the advent of powerful convex optimization techniques, the linear matrix inequality (LMI) method has been applied widely in control systems applications [4], [5]. A general linear matrix inequality is given as[1]:

$$F(x) \equiv F_0 + \sum_{i=1}^{m} x_i F_i \geq 0, \qquad (1)$$

where $x \in \Re^m$ is the decision variable, $F_i = F_i^T, i = 1, \cdots, m$ are given symmetric matrices, and the constraint ≥ 0 means positive semidefinite (i.e., nonnegative eigenvalues). With the constraint of (1), the LMI method finds an optimal x in the effort of minimizing or maximizing a convex object function $J(x)$. In 2-D dynamic systems similar to the ILC problem, LMI-based stability analysis was performed in [6]. We also note the work of [7], which also looked at LMI-based ILC design both from a parametric uncertainty and a frequency-domain uncertainty perspective. A distinction between that work and our own is that we have additionally considered monotonic convergence.

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[1]We define a matrix inequality as follows: if $A - B$ is positive definite, then $A > B$, and if $A - B$ is semi-positive definite, then $A \geq B$.

In this paper we show that the LMI method can be used to design the optimal ILC learning gain matrix. The paper is organized as follows. The super-vector ILC structure is analyzed in Section II Based on the analysis of Section II, LMI algorithms are designed in Section III and LMI test results are shown in Section IV. Conclusions are given in Section V.

II. Super-vector ILC Analysis

Consider the ILC update equation

$$U_{k+1} = U_k + \Gamma E_k$$

and the following definitions:

Definition 2.1: In the ILC learning gain matrix, a "band" is defined as sets of diagonals line as shown in Fig. 1. There are three different bands that we consider. "Arimoto-type ILC gains or band" are considered a band of size one corresponding to the center diagonal of Γ, "causal ILC gains or bands" consist of diagonals that are below the center diagonal, and "non causal ILC gains or bands" consist of diagonals that are above the center diagonal. Causal and non-causal band sizes can vary according to the number of diagonals included. If the first causal and first non-causal diagonal are used, we call the band size 2. If the second causal and non-causal diagonals are used, the band size is 3. In this way we define the band size by the number causal and non-causal diagonals in the ILC gain matrix.

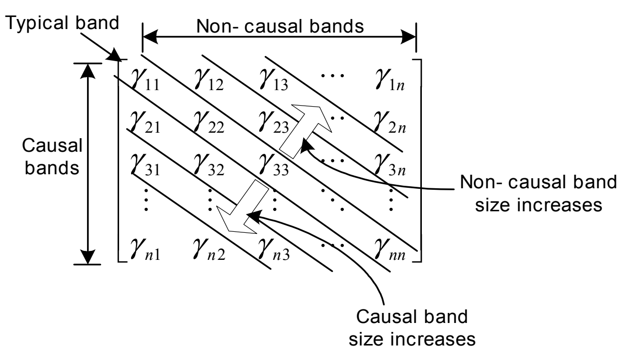

Fig. 1. Band and band size in learning gain matrix

Definition 2.2: We call Γ a linear time-invariant (LTI) ILC gain matrix if all the learning gain components in each diagonal are fixed as the same value. We call Γ a linear time-varying (LTV) ILC gain matrix if the learning gain components in each diagonal are different from each other.

Definition 2.3: We define two stability concepts with respect to iteration: *asymptotic* stability and *monotonic* convergence. The system is asymptotically stable if every finite initial state excites a bounded response, and the error ultimately approaches 0 as $k \to \infty$. It is monotonically convergent if $\|e_{k+1}\| < \|e_k\|$, and ultimately approaches 0 as $k \to \infty$.

For asymptotic stability conditions, two concepts should be differentiated. When Arimoto or causal-only gains are used, the asymptotic stability condition is defined as:

$$|1 - \gamma_{ii} h_1| < 1, i = 1, \cdots, n. \quad (2)$$

When non-causal gains are used in the ILC learning gain matrix the asymptotic stability condition becomes:

$$\rho(I - H\Gamma) < 1, \quad (3)$$

where ρ represents the spectral radius of $(I - H\Gamma)$.

On the other hand, monotonic converegence is the same for all types of gain and requires:

$$\|I - H\Gamma\|_i$$

where $\| \cdot \|_i$ represents the induced operator norm in the topology of interest. In this paper we will consider the standard l_1 and l_∞ norm topologies.

In the following analysis, we consider four different cases:

1) Arimoto gains with causal LTI gains
2) Arimoto gains with causal LTV gains
3) Arimoto gains with both causal and non-causal LTI gains
4) Arimoto gains with causal and non-causal LTV gains

First, we note the following observations, presented without proof:

Lemma 2.1: In Case 1, the minimum of $\| I - H\Gamma\|_1$ and $\| I - H\Gamma\|_\infty$ occurs if and only if Γ is exactly equal to the inverse of H.

Lemma 2.2: In Case 2, Case 3 and Case 4, the minimum of $\| I - H\Gamma\|_1$ and the minimum of $\| I - H\Gamma\|_\infty$ are zeros if and only if Γ is exactly equal to the inverse of H.

Remark 2.1: From Lemma 2.1 and Lemma 2.2, we conclude that the best structure of Γ is the inverse of H. This is a necessary and sufficient condition. So, if the structure of the learning gain matrix is not fixed *apriori*, it is expected that the optimal Γ of the super-vector ILC would be the inverse of H. However, in super-vector ILC, it is unrealistic to assume that we know all Markov parameters exactly. Moreover, it is not advisable to use the inverse of H as the ILC learning gain matrix, because it is expensive to implement the inverse of H in the control loop when H is ill-conditioned. Therefore, we wish to use the LMI technique when the ILC gain has a fixed structure.

Now, we analyze the learning gain matrix with a fixed band size. First, we compare LTI and LTV cases. We use following definitions:

Definition 2.4: An LTI learning gain matrix with fixed band size is denoted as Γ_{LTI}, and an LTV learning gain matrix with the same band size as Γ_{LTI} is denoted as Γ_{LTV}.

Definition 2.5: When Γ is fixed as Γ_{LTI}, the minimum of $\| I - H\Gamma_{LTI} \|$ is denoted by J_I^*; and when Γ is fixed as Γ_{LTV}, the minimum of $\| I - H\Gamma_{LTV} \|$ is denoted by J_V^*.

Then, using Definition 2.4 and Definition 2.5, we have following theorem, also stated without proof, from the fact that $\Gamma_{LTI} \subset \Gamma_{LTV}$.

Theorem 2.1: If the same band size ILC gain matrices are used in Γ_{LTI} and Γ_{LTV}, the following inequality is satisfied:

$$J_V^* \leq J_I^*$$

Remark 2.2: Theorem 2.1 relates the optimality between Case-1 and Case-2 and the optimality between Case-3 and Case-4. Also, from Theorem 2.1, the following corollary is immediate.

Corollary 2.1: If the same band size is used in causal ILC and non-causal/causal ILC, then

$$J_N^* \leq J_C^*,$$

where J_N^* is the minimum value using causal, Arimoto, and non-causal learning gains; and J_C^* is the minimum value using only causal and Arimoto gains.

In summary, from Lemma 2.1 and Lemma 2.2, we conclude that the best gain matrix is just the inverse of H with respect to convergence in the l_1 and l_∞ norms. When the band size is fixed, we conclude that LTV is better than LTI from Theorem 2.1, and from Corollary 2.1 that including non-causal terms increases the optimality of the system.

III. LMI Design

In this section, we show how to use LMI techniques for ILC design using several different fixed ILC gain matrix structures. Suppose we wish to satisfy the monotonic convergence condition $\min[\overline{\sigma}(I - H\Gamma)]$. Now, because

$$\sigma[I - H\Gamma] \equiv \lambda([I - H\Gamma][I - H\Gamma]^T).$$

and because:

$$\lambda([I - H\Gamma][I - H\Gamma]^T) \leq \parallel [I - H\Gamma][I - H\Gamma]^T \parallel$$

then by minimizing $\parallel [I - H\Gamma][I - H\Gamma]^T \parallel$, we can limit the upper bound of $\overline{\sigma}(I - H\Gamma)$. Because

$$\min(\parallel [I - H\Gamma][I - H\Gamma]^T \parallel)$$

is a typical matrix inequality problem, the ILC design problem can thus be solved by an LMI. We consider three cases:

1. $LMI-1$: *Not-fixed learning gain matrix*

In $LMI-1$, we do not fix the gain matrix structure, and the LMI algorithm finds the best structure by minimizing $\parallel I - H\Gamma \parallel$ using Γ. A typical LMI is designed as

$$\max\{-x_1^2\}$$

subject to

$$x_1^2 I_{n\times n} > [I - H\Gamma][I - H\Gamma]^T. \quad (4)$$

However, (4) is a quadratic form, so we change (4) to a linear inequality form given by

$$\begin{bmatrix} x_1 I_{n\times n} & [I - H\Gamma] \\ [I - H\Gamma]^T & x_1 I_{n\times n} \end{bmatrix} > 0 \quad (5)$$

The more simplified form is designed according to

$$-x_1^2 I_{2n\times 2n} - \begin{bmatrix} 0 & I_{n\times n} \\ I_{n\times n} & 0 \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} \Gamma \, [\, 0 \quad I_{n\times n} \,] + \begin{bmatrix} 0 \\ I_{n\times n} \end{bmatrix} \Gamma^T \, [\, H^T \quad 0 \,] < 0. \quad (6)$$

Then x_1 is an optimization scalar variable and Γ is an optimization matrix variable.

2. $LMI-2$: *LTI ILC with fixed band size*

As shown in Fig. 2.2 in the next section, the LMI algorithm finds the inverse of H as the optimal gain. This is already an expected result from the analysis of Section II. So, in this case, a big band-size gain matrix is indispensable. However, as already commented, the big band-size gain matrix is not practical. Thus, we design using LMIs by forcing a fixed gain matrix structure as shown in Fig. 1. For convenience, we use following learning gain matrix notation:

$$\Gamma = \begin{bmatrix} \gamma_p & \gamma_N^1 & \gamma_N^2 & \cdots & \gamma_N^{n-1} \\ \gamma_C^1 & \gamma_p & \gamma_N^1 & \cdots & \gamma_N^{n-2} \\ \gamma_C^2 & \gamma_C^1 & \gamma_p & \cdots & \gamma_N^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_C^{n-1} & \gamma_C^{n-2} & \gamma_C^{n-3} & \cdots & \gamma_p \end{bmatrix},$$

where subscripts C, p, and N mean causal, Arimoto (proportional terms), and non-causal gains, respectively. The total number of variables in the learning gain matrix is $2n-1$. As already explained, if only γ_p is used in Γ, the band size is 1; if γ_p, γ_C^1, and γ_N^1 are used, the band size is 2, and in the same way, if γ_p, $\gamma_C^1, \cdots, \gamma_C^k$, and $\gamma_N^1, \cdots, \gamma_N^k$ are used, the band size is $k+1$. To design a linear LMI, we change $I - H\Gamma$ to

$$I_{n\times n} - [\gamma_C^{n-1} H_C^{n-1} + \cdots + \gamma_C^1 H_C^1 + \gamma_p H_p + \gamma_N^1 H_N^1 + \cdots + \gamma_N^{n-1} H_N^{n-1}], \quad (7)$$

where $H_C^k, k = 1, \cdots, n-1$ are Markov matrices corresponding to causal gains; H_p is a Markov matrix corresponding to Arimoto gains; and $H_N^k, k = 1, \cdots, n-1$ are Markov matrices corresponding to non-causal gains. For example, they are expressed as:

$$H_C^1 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ h_1 & 0 & 0 & \cdots & 0 \\ h_2 & h_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{n-1} & h_{n-2} & h_{n-3} & \cdots & 0 \end{bmatrix}$$

$$H_N^1 = \begin{bmatrix} 0 & h_1 & 0 & \cdots & 0 \\ 0 & h_2 & h_1 & \cdots & 0 \\ 0 & h_3 & h_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & h_n & h_{n-1} & \cdots & h_2 \end{bmatrix}$$

Then, (7) is linear with respect to the learning gains. So the LMI method can be applied to the above expression, which gives a way for LMI-based LTI ILC. Finally, we change (7) into an LMI form given by

$$-x_1^2 I_{2n\times 2n} - \begin{bmatrix} 0 & I_{n\times n} \\ I_{n\times n} & 0 \end{bmatrix} + M_1 + M_2 + M_3 < 0, \quad (8)$$

with

$$M_1 = \begin{bmatrix} 0_{n\times n} & H_p \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma_p + \gamma_p \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ H_p^T & 0_{n\times n} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0_{n\times n} & H_C^1 \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma_C^1 + \gamma_C^1 \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ (H_C^1)^T & 0_{n\times n} \end{bmatrix} + \cdots + \begin{bmatrix} 0_{n\times n} & H_C^{n-1} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma_C^{n-1} + \gamma_C^{n-1} \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ (H_C^{n-1})^T & 0_{n\times n} \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 0_{n\times n} & H_N^1 \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma_N^1 + \gamma_N^1 \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ (H_N^1)^T & 0_{n\times n} \end{bmatrix} + \cdots + \begin{bmatrix} 0_{n\times n} & H_N^{n-1} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma_N^{n-1} + \gamma_N^{n-1} \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ (H_N^{n-1})^T & 0_{n\times n} \end{bmatrix},$$

where M_1 denotes Ariomoto gain components, M_2 the causal gain components, and M_3 the non-causal gain components. Markov matrices are calculated from the algorithms in Table I, with the proportional term Markov matrix $H_p = H$. Note the MATLAB substitution notation is used in algorithms of Table I to Table III.

TABLE I

ALGORITHMS FOR MARKOV MATRICES OF LTI ILC

LTI ILC algorithm
for $j = 1 : 1 : n-1$
$H_C^j(:, 1 : n-j) = H(:, j+1 : n)$
$H_C^j(:, n-j+1 : n) = 0$
$H_N^j(:, j+1 : n) = H(j, 1 : n-j)$
$H_N^j(:, 1 : j) = 0$
end

Note that the LTI ILC algorithm includes causal gains, non-causal gains, and Arimoto gains. Thus, we have more flexibility to find the better learning gain matrix (Theorem 2.1).

3. $LMI-3$: *LTV ILC with fixed band size*

In LTV ILC, we use following learning gain matrix notation:

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}.$$

Like LTI ILC case, the band size is a design parameter. With the expanded $I - H\Gamma$ in Lemma 2.2, and using the same procedure as LTI ILC, an LMI is designed as

$$-x_1^2 I_{2n\times 2n} - \begin{bmatrix} 0 & I_{n\times n} \\ I_{n\times n} & 0 \end{bmatrix} + \sum_{j=1}^{n}\sum_{i=1}^{n} [H_u \gamma_{ij} + \gamma_{ij} H_l] < 0, \quad (9)$$

where

$$H_u = \begin{bmatrix} 0_{n\times n} & H_{ij} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}; \; H_l = \begin{bmatrix} 0_{n\times n} & 0 \\ H_{ij}^T & 0_{n\times n} \end{bmatrix}.$$

Note that H_{ij} are Markov matrices corresponding to the gains γ_{ij}. If the band size is fixed as m, the algorithms of Table II and Table III are used to make LMI constraints. Then, the final Σ_1 in Table II and the final Σ_2 in Table III are summed to make linear matrix inequality constraints according to (note that initially Σ_1 and Σ_2 are zero):

$$-x_1^2 I_{2n\times 2n} - \begin{bmatrix} 0 & I_{n\times n} \\ I_{n\times n} & 0 \end{bmatrix} + \sum\nolimits_1 + \sum\nolimits_2 < 0. \quad (10)$$

TABLE II

ALGORITHMS FOR LMI CONSTRAINTS OF LTV CAUSAL AND ARIMOTO GAINS

LTV Causal and Arimoto
for $i = 1 : 1 : m$
for $j = 1 : 1 : i$
for $k = 1 : 1 : n-j+1$
$l = k + j - 1$
$\gamma' = \gamma_{kl}$
$R(1 : n, 1 : n) = 0$
$R(1 : n, l) = H(1 : n, k)$
$\Sigma_1 = \Sigma_1 + \begin{bmatrix} 0_{n\times 0} & R \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix} \gamma' + \gamma' \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ R^T & 0_{n\times n} \end{bmatrix}$
end
end
end

IV. SIMULATION ILLUSTRATION

In this section, the optimal learning gain matrices for four different cases are designed based on Section III algorithms. We use the following unstable system:

$$x_{k+1} = \begin{bmatrix} -0.50 & 0.00 & 0.00 \\ 1.00 & 2.04 & -1.20 \\ 0.00 & 1.20 & 0.00 \end{bmatrix} x_k + \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} \quad (11)$$

$$y_k = [\,1.0 \quad 2.5 \quad -1.5\,]\, x_k, \quad (12)$$

TABLE III

ALGORITHMS FOR LMI CONSTRAINTS OF LTV NON-CAUSAL GAINS

LTV Non-causal

```
for i = 1 : 1 : m
    for j = 1 : 1 : i − 1
        for k = j + 1 : 1 : n
            l = k − j
            γ' = γ_kl
            R(1 : n, 1 : n) = 0
            R(1 : n, l) = H(1 : n, k)
            Σ_2 = Σ_2 + [0_{n×0} R; 0_{n×n} 0_{n×n}] γ' + γ' [0_{n×n} 0_{n×n}; R^T 0_{n×n}]
        end
    end
end
```

which has poles at $[\,1.02 + j0.62, 1.02 - j0.62, -0.50\,]$ and zeros at $[\,0.45, -0.91\,]$. For LMI solutions, the free online Matlab software $SeDuMi$ and $SeDuMiInt$ were used [8], [9].

Test 1*: Without using an LMI* In this test, we just use Arimoto-type gains. From the plant dynamics we know that the first Markov parameter is $h_1 = CB = 1$. So, with $\gamma_p = 0.5$ we know we will achieve asymptotic stability. Fig. 2.1 shows the ILC result for the nominal system above. A sinusoidal reference signal was used. The figure shows the magnitude of the errors at each discrete time at the 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} trials. Clearly the convergence is not monotonic

Test 2*: Case*-1 *and Case*-2 *without fixing the band size* Matching the analysis above from Section II, the LMI solution determined that the inverse of the Markov parameters matrix was the optimal gain matrix. So, the l_1-norm and l_∞-norm are zero. Fig. 2.2 shows that the signal is monotonically converging to the reference signal as iteration number increases.

Test 3*: Case*-1 *and Case*-2 *with fixed causal/Arimoto band size* Fig. 2.3 shows the LMI test result with a learning gain matrix of causal/Arimoto, LTI with a fixed band size of 3. The signal does not converge monotonically. Fig. 2.4 shows the LMI test result with learning gain matrix of causal/Arimoto, LTV with a fixed band size of 3. The LTV case shows much better performance than the LTI case, and the signal converges to the reference signal monotonically.

Test 4*: Case*-3 *and Case*-4 *with fixed causal/Arimoto/non-causal band size* Fig. 2.5 shows the LMI test result with a causal/Arimoto/non-causal band size of 3 and an LTI gain structure. Even though Fig. 2.5 shows much better performance than Fig. 2.3, it is still not monotonically convergent. Still, we can conclude that LTV causal gains have better performance than LTI causal/noncausal gains. Fig. 2.6 shows the test result with a causal/Arimoto/non-causal band size of 3 and an LTV gain structure. Fig. 2.6 shows that the signal converges to the reference signal monotonically. From these figures, we conclude that causal/Arimoto/non-causal LTV has the best performance.

V. CONCLUSION REMARKS

In this paper we used LMI algorithms to find the best learning gain matrix structure in the super-vector ILC framework. From both analysis and LMI tests, we concluded that LTV is better than LTI, and non-causal terms can improve the monotonic convergence of the system. In our future work we will consider two related questions: *(i)* how big should the band size be, a *(ii)* how can the LMI-based design be performed when rather than having knowledge of the matrix H we only have knowledge of upper and lower bounds on H (e.g., the case when H represents an interval plant).

VI. REFERENCES

[1] Kevin L. Moore, "Multi-loop control approach to designing iterative learning controllers," in *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, Florida, USA, 1998, pp. 666–671.

[2] J. J. Hatonen D. H. Owens and K. L. Moore, "An algebraic approach to iterative learning control," *Int. J. Control*, vol. 77, no. 1, pp. 45–54, 2004.

[3] Kevin L. Moore, YangQuan Chen, and Vikas Bahl, "Monotonically convergent iterative learning control for linear discrete-time systems," *in Submitted to Automatica*, pp. 1–16, July 2001.

[4] Stephen Boyd, E. Feron, V. Balakrishnan, and L. El Ghaoui, "History of linear matrix inequalities in control theory," in *Proc. American Control Conference*, Baltimore, Maryland, USA., June 1994, AACC, pp. 31–34.

[5] Didier Henrion, "Course on lmi optimization with applications in control," Lecture Note, 2003.

[6] K. Galkowski, J. Lam, E. Rogers, S. Xu, B. Sulikowski, W. Raszke, and D. H. Owens, "Lmi based stability anlaysis and robust controller design for discrete linear repetitive processes," *International Journal of Robust and Nonlinear Control*, vol. 13, pp. 1195–1211, 2003.

[7] Jian-Ming Xu, Ming-Xuan Sun, and Li Yu, "Lmi-based robust iterative learning controller design for discrete linear uncertain systems," *Journal of Control Theory and Applications*, vol. 3, no. 2, pp. 259–265, 2005.

[8] Jos F. Sturm, "Using sedumi 1.02, a matlab toolbox for optimization over symmectric cones," Free On-line Software, October 2001.

[9] Dimitri Peaucelle, Didier Henrion, Yann Labit, and Krysten Taitz, "User's guide for sedumi interface 1.04," Free On-line Software, September 2002.

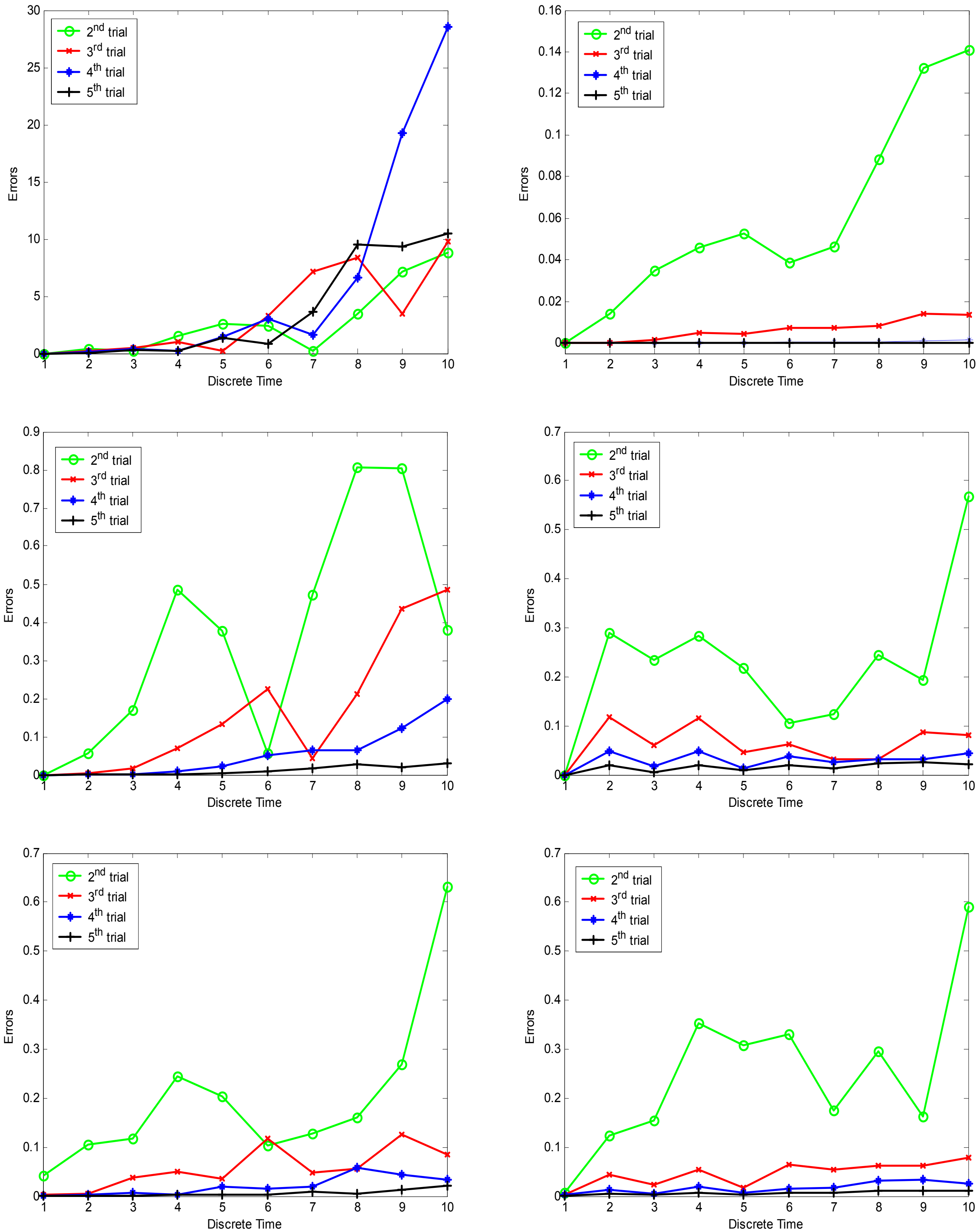

Fig. 2. ILC performances according to various band structures: Fig.2.1, upper-left: no LMI; Fig.2.2, upper-right: using H^1; Fig.2.3, middle-left: causal LTI with band size = 3; Fig.2.4, middle-right: causal LTV with band size = 3; Fig.2.5, bottom-left: non-causal LTI with band size = 3; Fig.2.6, bottom-right: non-causal LTV with band size = 3