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Robust stability condition of an uncertain networked system with delayed data dropout in both forward and feedback channels

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Abstract: This paper provides stability conditions of an uncertain networked control system with both forward control signal loss and feedback output signal loss. It is assumed that both the forward control signal and the output measurement signal are intermittent independently, and the remote plant has a model uncertainty. Also, in this paper, an observer-based controller is used in the remote place because the full states are not accessible. Thus, the main research goal is to analyze the overall stability of this uncertain intermittent networked system. In the second part of this paper, we consider delay effects of both forward signal and feedback signal during the network transfer.

Keywords: Networked control system, observer, data dropout, delay, model uncertainty.

1. INTRODUCTION

Computer network has been popularly used for data transfer during the past two decades. Due to various beneficences, real-time industrial networks such as DeviceNet, Profibus, FireWire, etc. have emerged as new technologies for distributed control applications [1]. Most of these industrial networks have been used for remote control applications and factory automation. The key feature of these industrial networks is to connect sensors, actuators, and controllers as network-wired nodes [2]. This feature has enabled to reduce the system wiring by plug and play devices, increase the system agility, make it ease to diagnose the system, and increase the system reliability. However, although there are many beneficences of using network for control applications, using networks brings about new challenge in the point of control engineering [3]. The main challenge is due to data congestion, which is caused by lack of a universal clock between the supervisor and the remote plant, hardware inherent data delay, and communication constraint such as channel capacity. The direct effects of data congestion are data delay between sender and receiver, and the data loss due to limited traffic line. In fact, there have been numerous efforts to compensate data congestion in the name of networked control system (NCS) [4], [5]. However, main portion of existing works has been devoted to stability analysis of the NCS system with delay [1], [6], [7], [8], [9], [10], [11]. With regard to data dropout, in [12], [13], [14], [15], the data dropout was modelled in the output feedback channel, i.e., only in network from the remote sensor to the main controller. Also, in these works, they have used estimation or filtering techniques for handling only the intermittent output measurement [16], [17], [18], [19], [20].

Thus, so far, in authors' best knowledge, existing literature has not considered both the forward channel (control signal) data dropout and the feedback channel (output measurement) data dropout of an uncertain networked system. Furthermore, even though an observer-based controller design is necessary in most practical applications, including NCS, when the system states are not completely accessible, existing NCS literature has not considered the observer in the remote controller. Thus, the main objective of this paper is to analyze the overall stability of the observer-based uncertain intermittent networked system with data dropout. As the second development of this paper, to represent the time elapse during the network transfer, we will also consider the delay effect of the uncertain intermittent system.

This paper consists of as follows: In Section 2, we provide an analysis and a design, and in Section 4, two examples are given to verify our results. Conclusions are given in Section 5.

2. FORWARD AND FEEDBACK SIGNAL DROPOUTS IN UNCERTAIN SYSTEM

In this problem set-up, it is assumed that the remote plant and the remote controller are connected by a network, which experiences the data dropout. Also, the remote plant is model uncertain and is controlled by a output feedback controller. For the output feedback controller design, we consider an observer. That is, when we consider linear uncertain networked-system, in state-space form such as

$$x(k+1) = (A+\Delta A)x(k) + Bu'(k) + v(k) \quad (1)$$
$$y(k) = Cx(k) + w(k) \quad (2)$$

the remote controller calculates the control signal $u(k)$ by the following observer:

$$\hat{x}(k+1) = A\hat{x}(k) + \overline{\eta}Bu(k) + L(\overline{\gamma}C\hat{x}(k) - y'(k)) \quad (3)$$

$$u(k) = F\hat{x}(k) \quad (4)$$

where $A \in \mathbf{R^{n\times n}}, \mathbf{B} \in \mathbf{R^{n\times m}}$, and $C \in \mathbf{R^{l\times n}}$; ΔA is uncertainty of the remote plant; L and F are control gain matrices; intermittent signals are defined as $u'(k) = \eta u(k)$ and $y'(k) = \gamma y(k)$ with $\eta \in \{0,1\}$ and $\gamma \in \{0,1\}$; $\overline{\eta} = E(\eta)$ and $\overline{\gamma} = E(\gamma)$ are expectations of η and γ; and $v(k)$ and $w(k)$ are zero-mean random noises. Now, we rewrite η and γ such as $\eta = \overline{\eta} + \widetilde{\eta}$ and $\gamma = \overline{\gamma} + \widetilde{\gamma}$ where $\widetilde{\eta}$ and $\widetilde{\gamma}$ are zero-mean random sequences with variances $\sigma_e^2 := E(\widetilde{\eta}^2) = (1-\overline{\eta})\overline{\eta}$ and $\sigma_g^2 := E(\widetilde{\gamma}^2) = (1-\overline{\gamma})\overline{\gamma}$. Since η and γ represent the data dropout in forward channel and feedback channel respectively, they are not correlated each other. Next, defining $e(k) = x(k) - \hat{x}(k)$, we have an augmented system:

$$X(k+1) = (H+\Delta H)X(k) + \widetilde{\eta}JX(k) + \widetilde{\gamma}OX(k) + Qn(k) \quad (5)$$

where

$$X(k) := \begin{bmatrix} e(k) \\ x(k) \end{bmatrix};\ H := \begin{bmatrix} A+\overline{\gamma}LC & 0 \\ -\overline{\eta}BF & A+\overline{\eta}BF \end{bmatrix}$$

$$\Delta H := \begin{bmatrix} 0 & \Delta A \\ 0 & \Delta A \end{bmatrix};\ J := \begin{bmatrix} -BF & BF \\ -BF & BF \end{bmatrix}$$

$$O := \begin{bmatrix} 0 & LC \\ 0 & 0 \end{bmatrix};\ Q := \begin{bmatrix} I & L \\ I & 0 \end{bmatrix}; n(k) := \begin{bmatrix} v(k) \\ w(k) \end{bmatrix}$$

For the stability analysis, we need the following definition:

Definition 2.1: System (5) is mean square stable (MSS) if $\|E(X(k)) - q\| \to 0$ as $k \to \infty$ and $\|E(X(k)X(k)^T) - Q\| \to 0$ as $k \to \infty$, where q and Q are fixed constant vector and matrix.

For our main result, we also need the following lemma.

Lemma 2.1: Assuming $\Delta A = 0$, the intermittent networked-system is MSS if and only if there exists positive definite matrix $P = P^T > 0$ such that

$$P - HPH^T - \sigma_e^2 JPJ^T - \sigma_g^2 OPO^T > 0. \quad (6)$$

Proof: From Theorem 1 of Kubrusly and Costa [21], since $E(\widetilde{\eta}\widetilde{\gamma}) = 0$, $E(\widetilde{\eta}) = 0$, and $E(\widetilde{\gamma}) = 0$, the proof is immediate. ■

For the existence test of P, we need an operator $\boldsymbol{f} : M(n\times n) \mapsto V(n^2)$, where $M(n\times n)$ is $n\times n$ matrix and $V(n^2)$ is an array with length of n^2 [22]. That is, for $n\times n$ matrix $M = [m_1, \cdots, m_n]$, we have $\boldsymbol{f}(M) = [m_1^T, m_2^T, \cdots, m_n^T]^T$. From [22], when we define $\mathcal{A}$ such as: $\mathcal{A} := \mathcal{H}\otimes\mathcal{H} + \sigma_\rceil^\in \mathcal{J}\otimes\mathcal{J} + \sigma_\}^\in \mathcal{O}\otimes\mathcal{O}$, where $\otimes$ is Kronecker product, if $\rho(\mathcal{A}) < \infty$, then there always exists $P = P^T > 0$ such that (6) holds. Furthermore, for any $S = S^T > 0$, P is calculated as: $P = \boldsymbol{f}^{-1}[(I-\mathcal{A})^{-\infty}\boldsymbol{f}(\mathcal{S})]$. Now, based on lemma given above, we develop a stability condition of an uncertain data dropout networked-system.

Theorem 2.1: Without model uncertainty, let us assume that there exists $P = P^T > 0$ such that Lemma 2.1 holds. If there exists $\alpha > 0$ and if

$$\|\Delta A\| < \frac{1}{\sqrt{2}}\left(-\|HP\|\|P\|^{-1} + \sqrt{[\|HP\|\|P\|^{-1}]^2 + \alpha\|P\|^{-1}}\right),$$

then the remote plant and controller system is robust MSS.

Proof: From Lemma 2.1, if and only if there exists $P = P^T$ such that $P - (H+\Delta H)P(H+\Delta H)^T - \sigma_e^2 JPJ^T - \sigma_g^2 OPO^T > 0$, then the model uncertain data dropout system is robust MSS. Let us say that, for nominal system, there exists $P = P^T$ and $\alpha > 0$ such that $P - HPH^T - \sigma_e^2 JPJ^T - \sigma_g^2 OPO^T = \alpha I$. Then, the following inequality is obtained:

$$HP\Delta H^T + \Delta HPH^T + \Delta HP\Delta H^T < \alpha I. \quad (7)$$

Now, making operator 2-norm to both sides, we have

$$\|\Delta H\|^2\|P\| + 2\|\Delta H\|\|HP\| - \alpha < 0. \quad (8)$$

Since $\|\Delta H\| = \sqrt{2}\|\Delta A\|$, we have the desired condition from (8). ■

Theorem 2.2: Let us assume that $I - HH^T - \sigma_e^2 JJ^T - \sigma_g^2 OO^T = S^* > 0$. If

$$\|\Delta A\| < \frac{1}{\sqrt{2}}\left(-\|H\| + \sqrt{\|H\|^2 + \|S^*\|}\right),$$

the system is then robust MSS.

Proof: From Lemma 2.1, simply selecting $P = I$, if $I - HH^T - \sigma_e^2 JJ^T - \sigma_g^2 OO^T = S^* > 0$, then nominal system is MSS. Now, adding ΔH, the robust MSS condition is given as:

$$S^* > H\Delta H^T + \Delta HH^T + \Delta H\Delta^T.$$

Therefore, by making operator norm to both sides, we obtain the desired inequality. ■

Next, for a designing purpose, we may consider a suboptimal controller. From Theorem 22, we need to minimize $\|HH^T + \sigma_e^2 JJ^T + \sigma_g^2 OO^T\|$. For convenience, denoting $S := HH^T + \sigma_e^2 JJ^T + \sigma_g^2 OO^T$, $U := LC$, $V := BF$, $m_{11} = (1 + 2\overline{\gamma}\pi + \overline{\gamma}\pi^2 + 2\sigma_e^2\xi^2)I_{n\times n}$, and we have a matrix (see the equation (9) at the next page).

Now, denoting $\xi := -\frac{4-2\overline{\eta}-4\overline{\gamma}^{-1}+4\overline{\gamma}^{-1}\overline{\eta}}{8(1-\overline{\eta})^4\overline{\eta}^4+8(1-\overline{\eta})^2\overline{\gamma}^{-1}+4\overline{\eta}^2}$, $\pi := \overline{\gamma}^{-1}[2(1-\overline{\eta})\xi - 1]$, $m_{11} = 1 + 2\overline{\gamma}\pi + \overline{\gamma}\pi^2 + 2\sigma_e^2\xi^2$, $m_{22} = \overline{\eta}^2\xi^2 + (1+\overline{\eta}\xi)^2 + 2\sigma_2^2\xi^2$, we make the following theorem:

Theorem 2.3: If the nominal remote plant

$$x(k+1) = Ax(k) + Bu(k) \quad (10)$$

$$y(k) = Cx(k) \quad (11)$$

is controllable and observable, and if $\vartheta < 1/\|A\|^2$ where $\vartheta = \max\{m_{11}, m_{22}\}$, then the controllers designed by $V = \xi A$ and $U = \pi A$ guarantee the MSS of the overall system with an uncertainty, $\|\Delta A\|$, of more than $\frac{1}{\sqrt{2}}\left(-\|H\| + \sqrt{\|H\|^2 + \|S^*\|}\right)$.

$$S = \begin{bmatrix} AA^T + \overline{\gamma}AU^T + \overline{\gamma}UA^T + \overline{\gamma}^2UU^T + 2\sigma_e^2VV^T + \sigma_g^2UU^T & -\overline{\eta}AV^T - \overline{\gamma\eta}UV^T + 2\sigma_e^2VV^T \\ -\overline{\eta}VA^T - \overline{\eta\gamma}VU^T + 2\sigma_e^2VV^T & \overline{\eta}^2VV^T + (A+\overline{\eta}V)(A+\overline{\eta}V)^T + 2\sigma_e^2VV^T \end{bmatrix} \quad (9)$$

Proof: For the proof, first, we change S_{12} term such as $-\overline{\eta}AV^T - \overline{\gamma\eta}UV^T + 2\sigma_e^2VV^T = (-\overline{\eta}A - \overline{\gamma\eta}U + 2\sigma_e^2V)V^T$. Then, we know that U give as

$$U = (\overline{\gamma\eta})^{-1}(2\sigma_e^2V - \overline{\eta}A) \quad (12)$$

makes $\|S_{12}\| = 0$. Thus, inserting $V = \xi A$ into (12), we obtain $U = \pi A$. Next, to minimize $\|S\|$, we use the trace of S such as

$$\begin{aligned} \text{trace}(S) &= \text{trace}[AA^T + \overline{\gamma}AU^T + \overline{\gamma}UA^T + \overline{\gamma}^2UU^T \\ &\quad +2\sigma_e^2VV^T + \sigma_g^2UU^T + \overline{\eta}^2VV^T \\ &\quad + (A+\overline{\eta}V)(A+\overline{\eta}V)^T + 2\sigma_e^2VV^T]. \end{aligned} \quad (13)$$

Denoting $\alpha = (\sigma_g^2 + \overline{\gamma}^2)(\overline{\gamma\eta})^{-2}$ and substituting (12) into (13), and making $\frac{\partial \text{trace}(S)}{\partial V}$, we have

$$\begin{aligned} 4\sigma_e^2\overline{\gamma}(\overline{\gamma\eta})^{-1}A \quad &+8\alpha\sigma_e^4V - 4\alpha\overline{\eta}\sigma_e^2A + 2(4\sigma_e^2 + \overline{\eta}^2)V \\ &+2\overline{\eta}A + 2\overline{\eta}^2V. \end{aligned} \quad (14)$$

Simplifying, we have

$$\begin{aligned} \frac{\partial \text{trace}(S)}{\partial V} &= (4 - 2\overline{\eta} - 4\overline{\gamma}^{-1} + 4\overline{\gamma}^{-1}\overline{\eta})A + \\ &\quad (8(1-\overline{\eta})^4\overline{\eta}^4 + 8(1-\overline{\eta})^2\overline{\gamma}^{-1} + 4\overline{\eta}^2)V. \end{aligned}$$

Then, equalizing $\frac{\partial \text{trace}(S)}{\partial V} = 0$ and solving this equality, we obtain $V = \xi A$. Now, since the nominal system is controllable and observable, we can find F and L such that $BF = \xi A$ and $LC = \pi A$. We insert V and U into S, which changes S in a diagonal matrix form (see the equation (15) at the next page).

Finally, since $m_{11} > 0$, $m_{22} > 0$, and $\vartheta < 1/\|A\|^2$, $\|S\| = \|HH^T + \sigma_e^2JJ^T + \sigma_g^2OO^T\| < 1$. Thus, the proof can be completed by Theorem 2.2. ■

From Theorem 2.3, it is better to make ϑ smaller. However, ϑ depends on both $\overline{\gamma}$ and $\overline{\eta}$. Fig. 1 shows ϑ vs. $\overline{\gamma}$ and $\overline{\eta}$. From this plot, we see that the system will be mostly stable with $\overline{\gamma} = \overline{\eta} = 1$ (i.e., when there are no intermittent signals). Also, even though ϑ increases as $\overline{\eta}$ decreases, ϑ increases more rapidly as $\overline{\gamma}$ decreases. Thus, from this result, it can be concluded that the output measurement loss damages to the system stability more seriously than the control signal loss.

3. DELAYED NETWORK CONSIDERATION

In this section, it is assumed that the data packet is delayed during the network transmission. So, the remote plant and the remote controller are represented as:

- Remote Plant:

$$x(k+1) = (A+\Delta A)x(k) + B\eta u(k-l) + v(k) \quad (16)$$

$$y(k) = Cx(k) + w(k) \quad (17)$$

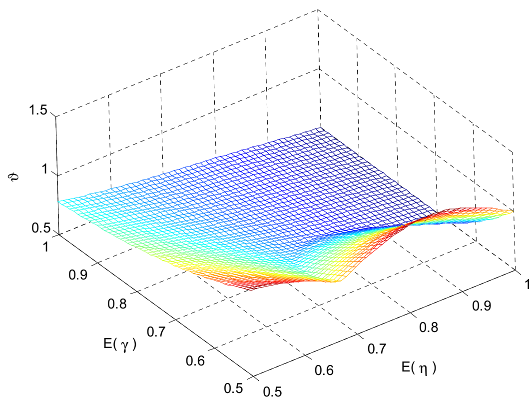

Fig. 1 Plot ϑ vs. $E(\gamma) = \overline{\gamma}$ and $E(\eta) = \overline{\eta}$.

- Remote Controller:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + \overline{\eta}Bu(k) \\ &\quad + L(\overline{\gamma}C\hat{x}(k) - \gamma y(k-l)) \end{aligned} \quad (18)$$

$$u(k) = F\hat{x}(k) \quad (19)$$

where l is the delayed amount in the sampling time interval. That is, ΔT is the sampling time of the networked system, $u(k-l)$ means that the control signal is delayed by the amount of $l \times \Delta T$. Similarly, $y(k-l)$ means that the output signal is delayed by the amount of $l \times \Delta T$. It is assumed that the delay is deterministic and the forward channel and feedback channel have the same delay. Now, defining $e(k) = x(k) - \hat{x}(k)$ and introducing augmented state vectors

$$X(k+1) := \begin{bmatrix} e(k+1) \\ x(k+1) \end{bmatrix}; \quad X(k) := \begin{bmatrix} e(k) \\ x(k) \end{bmatrix},$$

we can represent the delayed network system, in a state-space form such as

$$\begin{aligned} X(k+1) &= (H + \Delta H)X(k) + H_dX(k-l) \\ &\quad +Qn(k) \end{aligned} \quad (20)$$

where

$$H = \begin{bmatrix} A + \overline{\eta}BF + \overline{\gamma}LC & -\overline{\eta}BF - \overline{\gamma}LC \\ 0 & A \end{bmatrix}$$

$$\Delta H = \begin{bmatrix} \Delta A & 0 \\ 0 & \Delta A \end{bmatrix}; \quad H_d = \begin{bmatrix} -\eta BF & \gamma LC + \eta BF \\ -\eta BF & \eta BF \end{bmatrix}$$

$$Q := \begin{bmatrix} I & L \\ I & 0 \end{bmatrix}; \quad n(k) := \begin{bmatrix} v(k) \\ w(k-l) \end{bmatrix}.$$

Here, we define four different cases (i.e., $H_d(i), i \in \{1,2,3,4\}$) according to η and γ such as:

$$H_d(1) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad H_d(2) = \begin{bmatrix} -BF & BF \\ -BF & BF \end{bmatrix}$$

$$\begin{bmatrix} m_{11}AA^T & 0 \\ 0 & m_{22}AA^T \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} ((1+2\overline{\gamma}\pi+\overline{\gamma}\pi^2+2\sigma_e^2\xi^2)I_{n\times n} & 0 \\ 0 & (\overline{\eta}^2\xi^2+(1+\overline{\eta}\xi)^2+2\sigma_2^2\xi^2)I_{n\times n} \end{bmatrix} \times \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}^T. \quad (15)$$

$$H_d(3) = \begin{bmatrix} 0 & LC \\ 0 & 0 \end{bmatrix}; \; H_d(4) = \begin{bmatrix} -BF & LC+BF \\ -BF & BF \end{bmatrix}$$

As already commented, η and γ are random variables. Thus, the variation of $H_d(i)$ can be modeled by Markov jump. Also, it is assumed that the probability of change from i-th state to j-th state, denoted as p_{ij}, is known.

Then, as a main development of this paper, we can make the following theorem:

Theorem 3.1: For any L and F, if there exist $P(i) = P(i)^T > 0$ and $Q(i) = Q(i)^T > 0$ for all $i \in \{1,2,3,4\}$ such that $\lambda_{max}\left[Q(i) - H_d(i)^T\overline{P(i)}H_d(i)\right] > 0$ and $\lambda_{\max}(-P(i)+Q(i))+\lambda_{\max}((1+\|\Delta A\|/\|H\|)H^TJ(i)H) +\|J(i)\|\|\Delta A\|(\|H\| + \|\Delta A\|) < 0$ where $\overline{P(i)} = \sum_{j=1}^4 p_{ij}P(j)$ and $J(i) = \overline{P(i)} + \overline{P(i)}H_d(i)(Q(i) - H_d(i)^T\overline{P(i)}H_d(i))^{-1}H_d(i)^T\overline{P(i)}$, then the delayed networked system (16) - (19) with control gain matrices L and F is mean square stable.

Proof: From (20), let us use the following Lyapunov equation:

$$V_{i_k}(X_k) = X_k^TP(i_k)X_k + \sum_{j=1}^{l} X_{k-j}^TQ(i_k)X_{k-j}, \quad i_k \in \{1,2,3,4\} \quad (21)$$

Following [23], we can have the equation (22) (see the next page).

Thus, if $[Q(i) - H_d(i)^T\,\overline{P(i)}H_d(i)] > 0$, then $\overline{P(i)} + \overline{P(i)}H_d(i)\left(Q(i) - H_d(i)^T\overline{P(i)}H_d(i)\right)^{-1}H_d(i)^T\overline{P(i)} > 0$. Here, using Lemma 4.1 of [24], we obtain an inequality (see equation (23) at the next page). This completes the proof. ■

If conditions of Theorem 3.1 hold, then we can also find the upper boundary of $\|\Delta\|$ such as:

Corollary 3.1: If there exist L, F, $P(i)$, and $Q(i)$ such that all conditions of Theorem 3.1 are satisfied, then the upper boundary of $\|\Delta A\|$ is calculated as shown in equation (24) (see the next page).

4. SIMULATION ILLUSTRATION

4.1 Example-1

For simulation verification, we use the following example (A- matrix was obtained from [18]):

$$A = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & -0.5 \end{bmatrix}; \; B = \begin{bmatrix} 0.5 \\ 0.0 \end{bmatrix}; \; C = [0.0 \quad 0.5]$$

The uncertainty, data dropout, and random noises are generated using MATLAB command $rand$. In this test, $\overline{\eta} = \overline{\gamma} = 0.8$; that is, there are 20 percent data dropouts in forward channel and feedback channel, respectively. In the design of gain matrices, L and F are selected as $L = [-1.0, 0.1]^T$ and $F = [0.1, -1.0]$ to satisfy the condition of Theorem 2.2. Using these gain matrices, we calculate $\frac{1}{\sqrt{2}}\left(-\|H\| + \sqrt{\|H\|^2+\|S^*\|}\right) = 0.3011$. Therefore, if $\|\Delta A\| < 0.3011$, then uncertain data dropout networked-system is considered robust MSS. From the numerous tests, for example, with the following uncertain matrix

$$\Delta A = \begin{bmatrix} 0.0719 & 0.0785 \\ 0.0797 & -0.2400 \end{bmatrix}$$

which is $\|\Delta A\| = 0.2590$, we have the left plot of Fig. 2. As an unstable case, for example, with the following uncertainty

$$\Delta A = \begin{bmatrix} 0.4491 & 0.1852 \\ 0.1884 & -0.1354 \end{bmatrix}$$

which is $\|\Delta A\| = 0.5037$, we have the right plot of Fig. 2.

4.2 Example-2

With $\overline{\eta} = \overline{\gamma} = 0.8$, let us consider the following system:

$$A = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & -0.5 \end{bmatrix}; \; B = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

From Theorem 2.3, we design F and L such as:

$$F = \begin{bmatrix} -0.4721 & -0.0944 \\ -0.0944 & 0.4721 \end{bmatrix}$$

$$L = \begin{bmatrix} -1.4861 & -0.2972 \\ -0.2972 & 1.4861 \end{bmatrix},$$

from which we have $\vartheta = \max\{m_{11}, m_{22}\} = 0.6012$ and $1/\|A\|^2 = 3.8462$. Now, since

$$\frac{1}{\sqrt{2}}\left(-\|H\| + \sqrt{\|H\|^2+\|S^*\|}\right) = 0.4494,$$

as far as $\|\Delta A\| < 0.4494$, the overall system is MSS.

5. CONCLUSIONS

In this paper, we analyzed mean square stability of a networked-system that has forward control signal loss, feedback measurement loss, and uncertain remote plant. Even though our analysis is simple, the results of this paper provide several important understanding about the

$$
\begin{aligned}
E\{V_{i_{k+1}}(X_{k+1})|X_k, i_k\} - V_{i_k}(X_k) \quad = \quad & X_k^T \Big[-P(i) + (H+\Delta H)^T \overline{P(i)} (H+\Delta H) + \\
& (H+\Delta H)^T \overline{P(i)} H_d(i) \left(Q(i) - H_d(i)^T \overline{P(i)} H_d(i)\right)^{-1} \\
& \times H_d(i)^T \overline{P(i)} (H+\Delta H) + Q(i)\Big] X_k \\
& - \left[X_k^T (H+\Delta H)^T \overline{P(i)} H_d(i) \left(Q(i) - H_d(i)^T \overline{P(i)} H_d(i)\right)^{-1} - X_{k-l}^T\right]^T \\
& \times \left[Q(i) - H_d(i)^T \overline{P(i)} H_d(i)\right] \\
& \times \left[X_k^T (H+\Delta H)^T \overline{P(i)} H_d(i) \left(Q(i) - H_d(i)^T \overline{P(i)} H_d(i)\right)^{-1} - X_{k-l}^T\right]
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
& \lambda_{\max}\Big(-P(i) + (H+\Delta H)^T \overline{P(i)} (H+\Delta H) + (H+\Delta H)^T \overline{P(i)} H_d(i) \left(Q(i) - H_d(i)^T \overline{P(i)} H_d(i)\right)^{-1} \\
& H_d(i)^T \overline{P(i)} (H+\Delta H) + Q(i)\Big) \\
& \quad < \lambda_{\max}(-P(i) + Q(i)) + \lambda_{\max}\left((H+\Delta H)^T J(i) (H+\Delta H)\right) \\
& \quad < \lambda_{\max}(-P(i) + Q(i)) + \lambda_{\max}\left((1 + \|\Delta H\|/\|H\|) H^T J(i) H\right) + \|J(i)\| \|\Delta H\| \left(\|H\| + \|\Delta H\|\right) \\
& \quad = \lambda_{\max}(-P(i) + Q(i)) + \lambda_{\max}\left((1 + \|\Delta A\|/\|H\|) H^T J(i) H\right) + \|J(i)\| \|\Delta A\| \left(\|H\| + \|\Delta A\|\right)
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
& \|\Delta A\| < \\
& -\frac{\lambda_{\max}(H^T J(i) H)/\|H\|}{2\|J(i)\|} + \frac{\sqrt{(\lambda_{\max}(H^T J(i) H)/\|H\|)^2 - 4\|J(i)\|(\lambda_{\max}(-P(i) + Q(i)) + \lambda_{\max}(H^T J(i) H))}}{2\|J(i)\|}
\end{aligned}
\tag{24}
$$

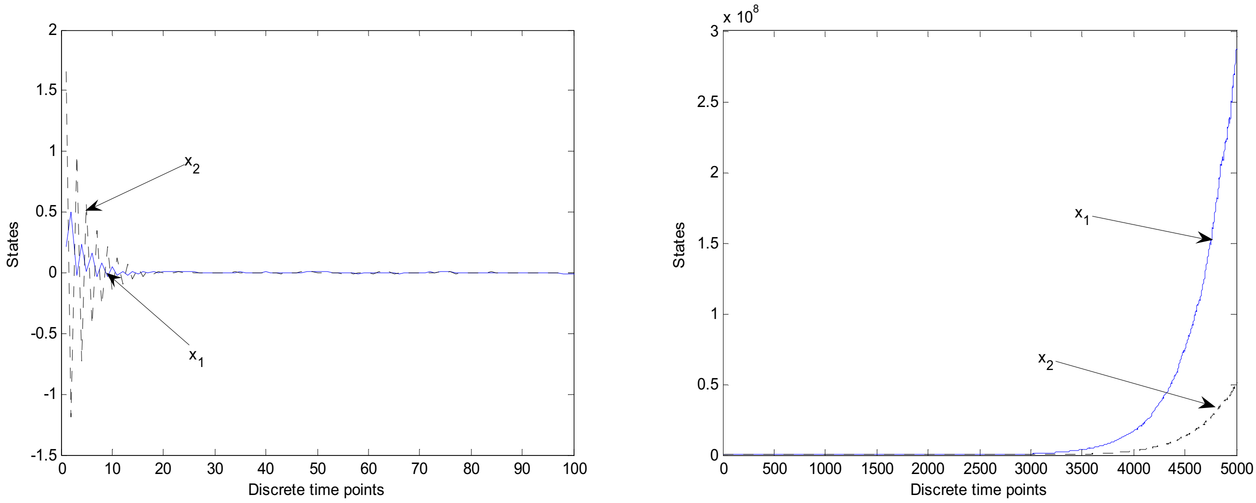

Fig. 2 Test results. Left: Stable case. Right: Unstable case.

networked-system. First, in existing works, intermittent data dropout problem of the output measurement has been considered. However, from our analysis, the forward control signal loss should be also considered carefully (see Fig. 1). Second, there is a relationship between the signal loss and the model uncertainty (see Theorem 2. .1, Theorem 22, and Theorem 23). Third, there could exist optimal controllers for L and F, and from our suboptimal design, it is shown that the output feedback signal loss could be more damageable to the overall system stability (see Fig. 1). Although our results are very simple, authors believe that this paper has firstly presented a relationship between intermittent data dropouts and model uncertainty of the remote plant, and an analytical method for designing a suboptimal controller of an uncertain data dropout networked-system.

As the second development of this paper, a stability analysis of networked control system with delay as well as data dropout was developed. For this development, we discovered a matrix inequality condition which was further simplified based on maximum eigenvalue. In our future efforts, we will provide a simulation illustration and will verify the results using an actual experimental test.

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