Abstract—This paper focuses on modeling and parameter identification of Wiener systems with the ultimate aim of compensating for the nonlinearity in those systems. Four-segment polynomial approximations are investigated for the nonlinear part of the Wiener systems and are shown to perform better than global and two-segment approximations. A special type of polynomial expression is also proposed that makes the analytical inverse of the nonlinearity possible. The idea behind finding inverse of the nonlinearity is to compensate for the nonlinearity introduced into the closed loop systems because of nonlinear sensors.

Index Terms—Wiener systems, invertible nonlinearity, four-segment nonlinearity, sensor linearization, undistortion.

I. INTRODUCTION

Many sensors connected to linear dynamic systems have nonlinearities such as distortion, deadband, saturation etc., which affect the closed loop performance of such systems [1]. These systems, linear plant followed by nonlinear sensor, can be represented by a Wiener model (Fig. 1). Although Hammerstein models (instantaneous nonlinear distortion followed by a linear plant) have attracted more attention ([2], [3], [4]), not much research has been done on Wiener model identification. The importance of the study of Wiener system identification lies in its use for improving the performance of control systems with nonlinear sensors while Hammerstein models can be used to model nonlinear actuators.

Identification of Wiener models is more tedious due to parameter nonlinearities. Different approaches have been taken for Wiener system identification. The method given in [5] assumes the form of nonlinearity to be known and uses the model of nonlinear part to estimate the linear part. [6] presents recursive algorithms for separate identification of nonlinear and dynamic parts. [7] shows identification of Wiener systems using spline nonlinearities using separable least-squares to estimate the model parameters. Another approach to identifying a Wiener model without having to deal with nonlinear parameters is to estimate the intermediate variable $x(t)$ as shown in [8]. An iterative algorithm has been presented in [8] to estimate the intermediate variable and the nonlinearity is approximated as a two-segment polynomial. Using two-segment polynomials gives you freedom of choosing lower degree polynomials for each segment instead of choosing one polynomial of higher degree for the entire data range.

The most straightforward method to compensate for the nonlinearities, as suggested in [9], is to pass the output of the nonlinear system through its inverse. Both [9] and [10] show the use of hyperbolic splines for characterizing and inverting static nonlinearity. Although hyperbolic splines show good performance in curve fitting and have analytical inverse, they cannot be used for Wiener system identification because of inseparability of parameters in input-output equation. In [11], piece-wise orthogonal polynomials are used as the “linearizer” for the hard-disk read/write head linearization process.

This paper extends the idea of using two-segment polynomial approximations of nonlinearity, from [8], to four-segment polynomials. It is shown that four-segment polynomials can give better approximation of nonlinearity as compared to global (one-segment) and two-segment polynomials in case of hard nonlinearity such as saturation. One more idea presented in this paper is a special type of polynomial approximation which has an analytical inverse. The motivation behind using analytical invertible polynomial is to correct the nonlinear distortion caused by the sensor. After the nonlinearity model has been found out, its inverse can be incorporated into the feedback path of a closed loop system to correct the distorted feedback signal from the sensor in real-time.
II. STATIC NONLINEARITY APPROXIMATION

A. Four-segment polynomial

It is tough to approximate hard nonlinearities such as deadzone and saturation with polynomials. For higher accuracy, polynomials with higher degree are needed. It has been shown in [8] that using piecewise polynomials reduces the degree of polynomial required while providing good accuracy. We extend that idea from two-segment polynomials to four-segment polynomials. The motivation behind using four-segment polynomials is that nonlinearities like saturation may be better expressed in terms of four-segment polynomials. Let us assume the output of the system $y$ depends on the value of intermediate variable $x$ and can be written as

$$y = \begin{cases} f_1(x) & \text{if } x \geq 0 \text{ and } x \leq 0.5 \\ f_1(x) + f_2(x) & \text{if } x < -0.5 \\ f_1(x) + f_3(x) & \text{if } x \geq -0.5 \text{ and } x < 0 \\ f_1(x) + f_4(x) & \text{if } x > 0.5 \end{cases}$$

(1)

For the sake of simplicity the nonlinear map has been defined as the sum of a global function and one local function in each segment.

We can define a switching function $h(x)$ as

$$h(x) = \begin{cases} 1 & \text{if } x < -0.5 \\ 2 & \text{if } x \geq -0.5 \text{ and } x < 0 \\ 3 & \text{if } x \geq 0 \text{ and } x \leq 0.5 \\ 4 & \text{if } x > 0.5 \end{cases}$$

(2)

Now the relationship between inputs and outputs of the assumed nonlinearity can be given as follows:

$$y = f_1(x) + f_2(x)h_1(x) + f_3(x)h_2(x) + f_4(x)h_3(x)$$

(3)

where

$$h_1(x) \equiv (2 - h(x))(3 - h(x))(4 - h(x))(1/6),$$

$$h_2(x) \equiv (1 - h(x))(3 - h(x))(4 - h(x))(-1/2),$$

$$h_3(x) \equiv (1 - h(x))(2 - h(x))(3 - h(x))(-1/6).$$

Here we can assume that nonlinear functions $f_i(x)(i = 1, \ldots, 4)$ can be approximated by proper polynomials of the same degree $r$, where

$$f_i(x) = \sum_{k=1}^{r} f_{ik} x^k$$

(4)

and $f_{i1} \neq 0$.

The four-segment polynomial proposed here can be considered as an extension of global and two-segment (one segment each for positive and negative inputs) polynomial approximations. The four-segment approximation may give more accurate identification of the nonlinear mapping which is an important step toward compensation of nonlinearity.

B. Analytically invertible polynomial

Having nonlinear sensors in real-time control loops is not at all desirable. Thus, the ultimate goal of nonlinear system identification process should be to find a model which helps in removing distortion due to nonidealities. The polynomial chosen for this purpose should meet the following requirements:

1) The polynomial should have an analytical inverse.

2) Computation of inverse should be simple enough so as not to act as a performance overhead on the real-time performance of the system.

The following polynomial, proposed in this paper, satisfies the above requirements

$$y = ax + bx^2 + cx|x|.$$  

(5)

The speciality of this polynomial is that it contains the same number of even and odd terms as a cubic polynomial. Fitting accuracy of this “special” polynomial can be shown to be comparable with (if not better than) that of a cubic polynomial. The fact that its inverse is easier to compute than a cubic polynomial makes it a good candidate to be considered for online realtime nonlinearity compensation purposes.

III. WIENER MODEL IDENTIFICATION

A. System equations

Identification of Wiener model, as shown in Fig. 1, involves identifying parameters of both the linear dynamic system and the static nonlinearity that are connected in cascade. The input-output equation for the linear part of the Wiener model can be given as

$$x(t) = A(q^{-1})u(t) + [1 - B(q^{-1})]x(t)$$

(6)

where $u(t)$ and $x(t)$ are inputs and intermediate variables, respectively, of the Wiener model and $A(q^{-1})$ and $B(q^{-1})$ are functions of unit delay operator $q^{-1}$

$$A(q^{-1}) = a_0 + a_1 q^{-1} + \ldots + a_m q^{-m}$$

$$B(q^{-1}) = 1 + b_1 q^{-1} + \ldots + b_n q^{-n}.$$  

(7)

(8)

Let the static nonlinearity be asymmetric as shown in (1) and let it be represented by the four-segment polynomial

$$y(t) = \sum_{k=1}^{r} f_{1k} x^k(t) + \sum_{k=1}^{r} f_{2k} x^k(t)h_1(x(t))$$

$$+ \sum_{k=1}^{r} f_{3k} x^k(t)h_2(x(t)) + \sum_{k=1}^{r} f_{4k} x^k(t)h_3(x(t))$$

(9)

Although the output of linear block is the same as input to the nonlinear block, equation (6) cannot be directly substituted into equation (9). This would result in a complex input-output equation for the Wiener model and the parameters of linear and nonlinear blocks would become inseparable. The key term separation method used in [8] can be used here.
As shown in [8], if we assume $f_{11} = 1$, the variable $x(t)$ can be separated as a key term of the nonlinear mapping and thus (9) becomes

$$y(t) = x(t) + \sum_{k=2}^{r} f_{1k} x^k(t) + \sum_{k=1}^{r} f_{2k} x^k(t) h_1(x(t))$$

$$+ \sum_{k=1}^{r} f_{3k} x^k(t) h_3(x(t)) + \sum_{k=1}^{r} f_{4k} x^k(t) h_3(x(t))$$

(10)

The system output can be given by substituting the expression for $x(t)$ from (6) for the key term only

$$y(t) = A(q^{-1})u(t) + [1 - B(q^{-1})]x(t)$$

$$+ \sum_{k=2}^{r} f_{1k} x^k(t) + \sum_{k=1}^{r} f_{2k} x^k(t) h_1(x(t))$$

$$+ \sum_{k=1}^{r} f_{3k} x^k(t) h_3(x(t)) + \sum_{k=1}^{r} f_{4k} x^k(t) h_3(x(t))$$

(11)

The system model, given by equations (6) and (11), is a special form of Wiener model with four-segment nonlinearity which itself is an extension of the idea of two-segment polynomial nonlinearity.

B. Identification algorithm

As the aim of this paper is to propose two new forms of polynomial approximation for Wiener model identification, we will employ the iterative technique for estimation of intermediate variable $x(t)$ as in [8]. The Wiener model described by equation (11) can be written as

$$y(t) = \Phi^T(t) \cdot \Theta$$

(12)

where the data vector $\Phi^T(t)$ is defined as

$$\Phi^T(t) = [u(t), u(t-1), \ldots, u(t-m), x(t-1), \ldots, x(t-n), x^2(t), \ldots, x^r(t), x(t) h_1(x(t)), x(t) h_2(x(t)), \ldots, x(t) h_r(x(t)),$$

$$x(t) h_3(x(t)), \ldots, x^r(t) h_3(x(t))]$$

(13)

and the parameter vector is given by

$$\Theta^T = [a_0, a_1, b_1, \ldots, b_m, f_{11}, \ldots, f_{1r}, f_{21}, f_{22}, \ldots, f_{2r}, f_{31}, f_{32}, \ldots, f_{3r}, f_{41}, f_{42}, \ldots, f_{4r}]$$

(14)

The iterative algorithm used here makes use of the preceding estimates of the model parameters for estimating intermediate variable. The estimated variable for the $s$th iteration can be written as

$$x^s(t) = \sum_{i=0}^{m} a_i^s u(t-i) - \sum_{j=1}^{n} b_j^s x(t-j)$$

(15)

The error, that we seek to minimize, between actual and estimated outputs is obtained from (12) as

$$e(t) = y(t) - \Phi^s T(t) \cdot \Theta^{s+1}$$

(16)

where $\Phi^s(t)$ is the data vector with estimates of intermediate variable for the $s$th iteration and $\Theta^{s+1}$ is the parameter estimate for the $(s+1)$th iteration.

The iteration steps ([8]) are summarized here to make the paper self-containing.

1) The initial estimates of the parameters of linear block are made and are used to calculate initial estimates of the intermediate variable from (15).
2) Minimizing a proper criterion based on estimation error from (16) the estimates of both linear and nonlinear block parameters $\Theta^{s+1}$ are made using $\Phi^s$ the $s$th estimates of intermediate variable. (In this paper we have used least-squares minimization).
3) Using (15) the estimates of $x(t)^{s+1}$ are evaluated by means of the recent estimates of model parameters.
4) If the estimation criterion is met the procedure ends, else it continues by repeating steps 2) and 3).

IV. SIMULATION RESULTS

MATLAB was used to test the proposed Wiener system identification and nonlinearity compensation scheme. For comparing the performance of two-segment and four-segment polynomial approximations, same system model as used in [8] was used.

A. System identification using cubic polynomial

The linear dynamic part of the Wiener model to identified is given by the difference equation:

$$x(t) = u(t - 1) + 0.5 u(t - 2) - 0.2 x(t - 1) + 0.35 x(t - 2)$$

(17)

and the input-output characteristic for nonlinear block, shown in Fig. 2, is given by

$$y(t) = \left\{ \begin{array}{ll}
x(t)/3[0.1 + 0.9 x^2(t)]^{1/2}, & \text{for } x(t) \geq 0 \\
-x^2(t)[1 - \exp(0.7x(t))]/3, & \text{for } x(t) < 0.
\end{array} \right. $$

(18)

The excitation signal chosen for system identification was 1000 samples of uniformly distributed random input. Least-squares estimates were found for the parameters from the inputs and simulated output data.

To compare the accuracy of both the models, an input signal $u(t) = 0.5 \sin(\pi t/2)$ was passed through identified and actual systems and output responses for both models of nonlinearity were compared (Fig. 3). Here only the output signals are shown as the emphasis is more on matching the output of identified system with the actual system. About 15-20 iterations were required for the $l_2$ norm of error , given by (16), to converge. Error norms for both nonlinear models are listed in Table I. It can be said, from the given plots and error figures, that four-segment nonlinearity model gives better approximation in Wiener model identification.
TABLE I

<table>
<thead>
<tr>
<th>Nonlinearity model</th>
<th>$l_2$</th>
<th>$l_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-segment</td>
<td>0.122</td>
<td>0.0449</td>
</tr>
<tr>
<td>Four-segment</td>
<td>0.0444</td>
<td>0.015</td>
</tr>
</tbody>
</table>

B. System identification using the proposed “special” polynomial

Same system model as in previous section is used for the test plant. The special form of polynomial given in (5) is used to approximate the nonlinear part of the model. As can be seen from Fig. 4, the invertible polynomial may not provide the accuracy of cubic polynomial but it can be considered good enough if it is being used for undistortion purposes. The following table shows comparison of error norms for two- and four-segment polynomials.

TABLE II

<table>
<thead>
<tr>
<th>Nonlinearity model</th>
<th>$l_2$</th>
<th>$l_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-segment</td>
<td>0.575</td>
<td>0.1338</td>
</tr>
<tr>
<td>Four-segment</td>
<td>0.1816</td>
<td>0.056</td>
</tr>
</tbody>
</table>

C. Nonlinearity compensation

Invertible model of nonlinearity, given by (5), can be used in the identification process to ultimately use it for correction of distorted signal. The nonlinearity used in actual model for this purpose is shown in Fig. 5 while the linear dynamic part of the system remains the same as given in (17). This type of nonlinearity is similar to the one found in clamp-on ammeters [12]. Correcting such distortions can significantly improve the range of measuring instrument.

Wiener system identification process was carried out in a similar way as described in section IV-A. Instead of cubic polynomial, the special polynomial from (5) was used for four-segment approximation. To test the fitting accuracy and correction performance of proposed polynomial, a sinusoidal input was applied to Wiener system with nonlinearity shown in Fig. 5. The distorted signal was then passed through the inverse of nonlinearity. The output of identified system and corrected output are shown in Fig. 6. $l_2$ norm of error between intermediate variable $x(t)$ in Wiener model and the corrected signal is 0.8591 and $l_\infty$ norm for the same error is 0.0812.

V. Conclusion

A new approach for Wiener model identification using four-segment polynomial nonlinearity is presented in this paper. It is shown that four-segment polynomials provide better approximation for nonlinear block characteristics of a Wiener model.
Nonlinear system identification is pursued with the purpose of identifying and compensating for sensor nonlinearities. The approach of passing the distorted signal through the inverse mapping of nonlinearity is used for undistortion. A special type of polynomial having an analytical inverse is presented for this purpose. Advantage of this type of polynomial over cubic polynomial is its inverse is easier to compute and hence is better suited for real-time control purposes. As suggested in the last section, this idea can be used to extend the operating range of sensors.

Future research on this topic may involve identifying knot points for optimal approximation using four-segment nonlinearity. Additional knot constraints can also be added to reduce jumps at knot points as can be seen in Fig. 6(c). Robustness of closed loop systems involving inverse mapping also needs to be investigated.

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Fig. 6. Identification and undistortion with invertible nonlinearity model.