Fractional-order integral and derivative controller design for temperature profile control

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Abstract—This paper presents a strategy to tune a fractional order integral and derivative controller satisfying gain and phase margins. The closed-loop system designed has a feature of robustness to gain variations with step responses exhibiting a nearly iso-damping property. This paper aims to apply the tuning procedure proposed to temperature control at selected points in Quanser’s heat flow experimental platform. From the comparison with the traditional PI/PID controller based on Ziegler Nichol’s tuning method, the effectiveness and validity of the proposed methodologies are illustrated.

Index Terms—Temperature control, fractional-order \( I^{\alpha} D^{\beta} \) control, heat flow

I. INTRODUCTION

Temperature control arises in many engineering fields. For example, in cryogenic applications [1], the precision spatial temperature control via spatial heating and spatial temperature sensing is important, and in process industry [2], the most common control task is to achieve the precise temperature profile. There are basically three types of temperature control tasks: temperature set point regulation, temperature profile tracking, and temperature uniformity control. For temperature set-point control, single loop commercial off-the-shelf temperature controllers can usually be used. Temperature profile tracking is to raise the temperature according to a prescribed temperature time history, which is required for applications such as precision heat treatment for materials, batch chemical reactors, etc. The temperature uniformity control is to achieve uniformly-spatially-distributed temperature profile. The objective of this paper is to design a controller for temperature profile tracking at the spatially distributed places; thus, to regulate the temperatures at given sensor locations to ensure the temperatures be close to the desired values. For temperature control, usually, On/Off control, proportional control, and traditional PID controller have been widely used. In this paper, as a new control scheme for temperature regulation, we suggest using fractional-order PID controller for a more accurate temperature profile tracking of the spatially distributed heat flow. An actual experimental task is performed using the heat flow (HFE) of QUANSER.

This paper consists of as follows. Section II briefly summarizes the role of integer order PID controller in temperature control. Section III lists basic definitions in fractional-order calculus and merits of fractional-order controller. Also in this section, we present a tuning method for fractional \( I^{\alpha} D^{\beta} \) controller. In Section IV an extensive comparison of existing integer order solutions and fractional order solutions is presented. Finally, Section V concludes this paper with some remarks on the achieved results and ideas for future work.

II. TEMPERATURE CONTROL

Since in temperature control, it is difficult to find the response time constant, On/Off control scheme is usually used so as to regulate the output temperature within the dead band. However, in On/Off control, the output will be oscillatory around the set-point; so an accurate temperature profile is not achieved. For a more accurate temperature regulation, closed-loop feedback control schemes are required. Most popular control scheme is PID control, because it does not require the plant model and practically it is easily implementable. In traditional PID control, the proportional term, integral term, and derivative term have different effects to the heat flow, temperature, and speed. For the temperature control, it is usually recommended to use full PID control, but with the accurately-tuned-control gains. Various PID tuning methods for temperature control have been developed by the many industries. For the reliable temperature control, however there are some basic environmental requirements. For examples, the heater should apply enough power and the temperature sensors should

1see “http://www.omega.com/prodinfo/temperaturecontrollers.html”
2“www.quanser.com”
3For more detail, see “http://www.w-dhave.imt.co.th/index/”
be spatially distributed in appropriate places. Tuning the controller means that we select the proportional, integral, and derivative gains with a particular purpose. In fact, tuning PID gains for the temperature control requires some physical interpretation about the system. Thus, it is necessary to understand the effects of the proportional, integral, and derivative terms to the system. In temperature control, individual PID gain has the following characteristics:

- Proportional gain: it requires more power proportional to the error between sensor temperature and the desired trajectory profile. The proportional gain is used for On/Off control. That is, when the output is within the proportional band, the power is off; but when it is out of the dead band, the power is on. If the temperature is below set point, the output will be on longer; if the temperature is too high, the output will be off longer.\(^5\)
- Integral gain: it provides a control signal that is proportional to the accumulated error. So, this integral term is for slow mode reaction and forces the steady-state error to zero for a step response. In temperature control, it adjusts the temperature to set point after stabilization.
- Derivative gain: it provides the control force proportional to the rate of change of the output error. So, this derivative term is for fast mode reaction and yields large signal with the high-frequency control errors and with the rise or fall of system temperature.

From a literature survey, it is shown that there have been numerous applications of PID controller or fuzzy/neural network-based PID controller for the temperature control of various engineering objects. As some examples, Peter Galan showed that a fuzzy logic for enhancing PID controller is necessary for the satisfactory temperature profile tracking of injection moulding processes\(^6\) and Dihac et al. used PID controller for a rapid thermal processor control [3]. Lin et al. proposed a neural fuzzy inference network for the temperature control of a water bath system and compared the performance with the PID control [4]. Juang and Chen proposed TSK-type recurrent neural fuzzy network based on the direct inverse control configuration, which does not require the priori knowledge of the plant order [5] and Ramos et al. used PID controller to control the temperature of the bath [6]. However, there has been no trial of using fractional-order PID controller for the temperature control of the spatially distributed system. In the next section, we briefly summarize fractional-order \(I^\alpha D^\beta\) control and its benefits in temperature control.

### III. Fractional-Order \(I^\alpha D^\beta\) Control

After Newton and Leibniz discovered calculus in the 17th century, fractional-order calculus has been studied as an alternative calculus in mathematics [7]. As claimed in [8], fractional order calculus will play an important role in mechatronic and biological systems.

Fractional order dynamic system and controls are relatively new research areas in control engineering. From early 90’s, there has been steadily research in these areas as shown in [9], [10], [11], [12], [13], [14], [15], [16] even though some pioneering works can be traced back to [17], [18], [19], [20].

Traditional PID control method is a most popular control approach where integrator and derivative are integer order. Recently, in fractional order calculus community, a trend of using non-integer integrator or non-integer derivative for the accurate profile tracking in controlled-output has appeared, which is so-called fractional-order PID control. In the following subsections, we briefly review this fractional-order PID control.

#### A. Fractional-order calculus

In this paper, fractional integral is defined as:

\[
I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad t > 0, \quad \alpha \in \mathbb{R}^+ \tag{1}
\]

and for the fractional derivative, Caputo derivative is used, which is defined as:

\[
D^\alpha f(t) = \frac{d^n}{dt^n} \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+n-1}} d\tau, \quad (n-1) < \alpha \leq n \tag{2}
\]

where Euler's Gamma function is given as

\[
\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \quad x > 0 \tag{3}
\]

with the special case when \(x = n:\)

\[
\Gamma(n) = (n-1)(n-2)\ldots(2)(1) = (n-1)!. \tag{4}
\]

Now, using the Laplace transformation, we have \(s^{-\mu} F(s) = I^\mu f(t), \quad \mu = \alpha, \) and \(s^{\lambda} F(s) = \frac{d^\alpha f(t)}{dt^\alpha}.
\)

Then, the fractional PID controller can be written as:

\[
C(s) = K_p + K_i s^{-\mu} + K_d s^\lambda. \tag{5}
\]

If we take \(\mu = \lambda = 1,\) then we obtain the classic PID controller; with \(\mu = 0,\) it is the PD controller, and if \(\lambda = 0,\) it is the PI controller. Also, we will consider fractional order \(I^\alpha D^\beta\) controller which is an special case of fractional PID controller with \(K_p = 0.\)

#### B. Merit of using fractional-order controller

As explained in [21], the idea of using fractional-order controllers for the dynamic system control belongs to [22], [11] and generalized fractional-order PID controller was proposed by Podlubny [23]. Advantages of using fractional-order PID controller have been introduced in a number of publications. In [24], it was claimed that, out of the following specifications:

- No steady-state error
- Phase margin and gain crossover frequency specifications

5http://www.omega.com/prodinfo/temperaturecontrollers.html

6http://www.manufacturing.net/cfl/article/CA408369.html

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2008 Chinese Control and Decision Conference (CCDC 2008)
Gain margin and phase crossover frequency specifications
Robustness to variations in the gain of the plant
Robustness to high frequency noise
Good output disturbance rejection

five specifications can be met by the closed-loop system, because the fractional-order PID controller (4) has five tuning parameters (i.e., $K_p, K_i, K_d, \lambda, \alpha$). In [25], some observations about fractional-order PID control were given. Particularly, by varying $\alpha$ from 1 to $\infty$, it was shown that there could be a constant increment in the slope of the magnitude curve varying between $-20$ dB/dec and 0 dB/dec and could be a constant delay in the phase response varying between $-\frac{\pi}{2}$ and 0. Similarly, by changing $\lambda$ from zero to 1, we can change the amount of phase lead and the slope of magnitude response. Vinagre et al. provided frequency domain analysis to illustrate the superiority of the fractional-order PID controller applied to both the fractional dynamic system and the integer dynamic system [26]. In [21], it was claimed that fractional-order PID controller is an adequate controller for the fractional-order mathematical models and it is less sensitive to shifts of parameters of a controlled-system and to variations of parameters of the controller. Particularly, in [10], it was illustrated that the fractional-order PID controller is a suitable way for the control of the fractional system. Hwang et al. [27] proposed the fractional-order band-limited compensator, which has the similar response as the fractional-order PD controller but has the less sensitivity to the high frequency noise. Leu et al. [28] provided a differential evolution algorithm to search for optimal fractional-order PID parameters to meet the phase margin and the gain margin specifications. Thus, fractional-order controller could be beneficial for temperature control since there are often variations in parameters in heat flow systems, and most of desired specifications, which are not readily achieved simultaneously by traditional PID controller, can be ensured by fractional-order closed-loop dynamics.

C. Fractional-Order $I^\alpha D^\beta$ control

Let us suppose that the user-defined gain margin $A_m$, and phase margin $\phi_m$, are given as:

$$|C(j\omega_p)G(j\omega_p)| = \frac{1}{A_m},$$  \hspace{1cm} (5)

$$\phi_m = \pi + \angle C(j\omega_g)G(j\omega_g) \hspace{1cm} (6)$$

where $\omega_p$ and $\omega_g$ are the phase and gain crossover frequencies of the open-loop system. Then the gain relations are given as:

$$\frac{KK_i}{\omega_g^\alpha} = 1 \hspace{1cm} (7)$$

$$\frac{\omega_p^\alpha}{KK_i} = A_m \hspace{1cm} (8)$$

and the phase relations are given as:

$$\phi_m = \pi - L\omega_g - \alpha \frac{\pi}{2} \hspace{1cm} (9)$$

$$\pi = \alpha \frac{\pi}{2} + L\omega_p \hspace{1cm} (10)$$

Therefore we obtain:

$$A_m = \left(\frac{\omega_p}{\omega_g}\right)^\alpha \hspace{1cm} (11)$$

$$\frac{\omega_p}{\omega_g} = \frac{\pi - \alpha \frac{\pi}{2}}{\pi - \phi_m - \alpha \frac{\pi}{2}}, \hspace{1cm} (12)$$

which yields

$$A_m = \left(\frac{\pi - \alpha \frac{\pi}{2}}{\pi - \phi_m - \alpha \frac{\pi}{2}}\right)^\alpha. \hspace{1cm} (13)$$

Thus, it is shown that given gain and phase margins desired, the above equation (13) can be solved for fractional order $\alpha$ using numerical classical approach in MATLAB. Once $\alpha$ is calculated, then fractional-order $I^\alpha D^\beta$ controller is obtained by

$$C(s) = K_i \frac{T s + 1}{s^\alpha} \hspace{1cm} (14)$$

Once the value of $\alpha$ is obtained, the corresponding values of $(\omega_g, \omega_p, K_i, K_d)$ are obtained as:

$$\omega_g = \frac{\pi - \phi_m - \alpha \frac{\pi}{2}}{L} \hspace{1cm} (15)$$

$$\omega_p = \frac{\alpha - \frac{\pi}{2}}{L} \hspace{1cm} (16)$$

$$K_i = \frac{\omega_p^\alpha}{K} \hspace{1cm} (17)$$

$$K_d = TK_i \hspace{1cm} (18)$$

IV. Comparisons Results

In this section an extensive comparison of existing integer order solutions and fractional order solutions is made. Quanser HFE system (see Fig. 1) consists of a duct equipped with a heater and a blower at one end and three temperature sensors located along the duct. The power delivered to the heater is controlled using an analog signal. For the analog signal generation and measurement, we use Quanser analog input/output libraries via MATLAB/Simulink/RTW. The fan speed of HFE is controlled using an analog signal. Fan speed is measured using a tachometer and is an input signal. Fig. 1 shows the set of test equipment: computer, software, AD/DA converter and QUANSER HFE. HFE system includes built-in power module, analog signals for fan speed and power; an onboard tachometer to design speed control; and fast settling platinum temperature transducers (3 sensors along the duct) to measure the temperature.

For integer order case both PI and PID controllers are considered based on Ziegler Nichols’ tuning method. The Ziegler Nichols’ tuning formulae for a PI controller is given
as:

\[
K_p = \frac{0.9T}{KL} \quad (19)
\]

\[
K_i = \frac{K_p}{3L} \quad (20)
\]

For a PID controller the gain values are:

\[
K_p = \frac{1.2T}{KL} \quad (21)
\]

\[
K_i = \frac{K_p}{2L} \quad (22)
\]

\[
K_d = \frac{K_p}{L} \quad (23)
\]

The other controller compared in this paper is a fractional PI controller based on FMIGO tuning method (Ms constrained integral gain optimization) outlined as:

\[
C(s) = K_p^* + \frac{K_i^*}{s^\alpha} \quad (24)
\]

where \(K_p^*\) and \(K_i^*\) and \(\alpha\) values are given by:

\[
K_p^* = \frac{0.2978}{K(\tau + 0.000307)} \quad (25)
\]

\[
K_i^* = \frac{0.8578}{\tau^2 + 3.402\tau + 2.405} \quad (26)
\]

\[
\alpha = \begin{cases} 
0.7, & \text{if } \tau < 0.1 \\
0.9, & \text{if } 0.1 \leq \tau < 0.4 \\
1.0, & \text{if } 0.4 \leq \tau < 0.6 \\
1.1, & \text{if } \tau \geq 0.6
\end{cases} \quad (27)
\]

The controller performances for all the three sensors are compared in real time as shown in Fig. 2, Fig. 3 and Fig. 4. As is clear from the figures, the fractional controllers outperform simple integer order PI/PID controller. The new controller \((FO-ID)\) is tuned better in comparison to Ms constrained FO-PI controller for sensor 1 whereas for sensor 2 and sensor 3 both the fractional order controllers perform more or less the same.

The same results in simulink environment are shown in Fig. 5, Fig. 6 and Fig. 7. The simulation and actual lab results showed some mis-matchings indicating that the linear model was unable to model the system nonlinearities.

V. CONCLUSIONS

The objective of this paper is to introduce a novel analytical tuning method for a fractional integral derivative, i.e., \((\mathbf{I}^\alpha \mathbf{D}_\mathbf{t}^\beta)\) controller, and apply the results to analyze the heat flow in HFE module. The performance of new FO-ID controller was compared to the integer-order PI/PID controller and FO-PI controller. Our observation is that
the resulting closed loop system has the desirable feature of being robust to gain variations with step responses exhibiting a nearly iso-damping property.

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